Math 221 - Test 4 - Part 1/2 - Fall 2016

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. You **can not** use a graph to justify your answer. Each exercise is worth 10 points.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the most general antiderivative of the function $f(x) = x^2e^{-x^3+8} + \cos(2\pi x)$, then the particular antiderivative $F(x)$ that satisfies the condition $F(2) = -1$.

$$F(x) = \int f(x)\,dx = \int x^2 e^{-x^3+8} + \cos(2\pi x)\,dx =$$

$e^u: u = -x^3+8$, $u' = -3x^2$ $\cos u: u = 2\pi x$, $u' = 2\pi$

$$= \frac{1}{-3}\int -3x^2 e^{-3x^3+8}\,dx + \frac{1}{2\pi}\int 2\pi\cos(2\pi x)\,dx =$$

$$= \boxed{-\frac{1}{3}e^{-x^3+8} + \frac{1}{2\pi}\sin(2\pi x) + C} \quad \text{M.G.A.}$$

PLUG CONDITION: $-1 = F(2) = -\frac{1}{3}e^{-(2)^3+8} + \frac{1}{2\pi}\sin(4\pi) + C \Rightarrow$

$\Rightarrow C = \frac{1}{3} - 1 = -\frac{2}{3}$. THEN $F(x) = -\frac{1}{3}e^{-x^3+8} + \frac{1}{2\pi}\sin(2\pi x) - \frac{2}{3}$

2. Solve the second order ODE
$f''(x) = 2e^x + 3\sin x$, subject to the conditions $f(0) = 0$ and $f(\pi) = 0$.

$$f'(x) = \int f''(x)\,dx = \int 2e^x + 3\sin x\,dx = 2e^x - 3\cos x + C_1$$

$$f(x) = \int f'(x)\,dx = \int 2e^x - 3\cos x + C_1 = 2e^x - 3\sin x + C_1 x + C_2$$

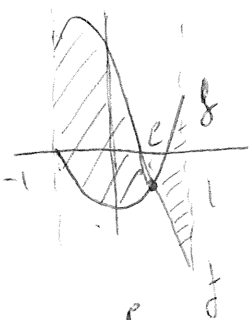
PLUG CONDITIONS:

$$\begin{cases} 0 = f(0) = 2 + C_2 \Rightarrow C_2 = -2 \\ 0 = f(\pi) = 2e^\pi + \pi C_1 + C_2 \Rightarrow C_1 = \frac{2}{\pi}(1 - e^\pi) \end{cases}$$

THEN

$$f(x) = 2e^x - 3\sin x + \frac{2}{\pi}(1 - e^\pi)x - 2$$

3. Compute the area of the region enclosed by the graphs of $f(x) = -4x^2 - 7x + 2$ and $g(x) = 3x^2 + 2x - 1$ over the interval $[-1, 1]$.



$$\text{AREA} = \int_{-1}^{1} |f-g|\,dx = \int_{-1}^{c} f-g\,dx + \int_{c}^{1} g-f\,dx.$$

c IS SOLUTION OF $f(x)-g(x)=0$, THAT IS

$$-4x^2-7x+2-3x^2-2x+1=0 \Rightarrow -7x^2-9x+3=0$$

$$x = \frac{9 \pm \sqrt{81+84}}{-14} = \frac{-9 \pm \sqrt{165}}{14} \Rightarrow c = \frac{\sqrt{165}-9}{14} \approx .27$$

$$\text{AREA} = \int_{-1}^{c} -7x^2-9x+3\,dx + \int_{c}^{1} 7x^2+9x-3\,dx = \left[-\frac{7}{3}x^3-\frac{9}{2}x^2+3x\right]_{-1}^{c} +$$

$$+ \left[\frac{7}{3}x^3+\frac{9}{2}x^2-3x\right]_{c}^{1} = -\frac{7}{3}+\frac{9}{2}+3+\frac{7}{3}+\frac{9}{2}-3+\left(-\frac{7}{3}c^3-\frac{9}{2}c^2+3c\right)\cdot 2$$

$$= \frac{1}{196}\left(55\sqrt{165}+1143\right) \approx 9.4362$$

4. $\int_{-1}^{2} x^3 - \cos(2x+3)\,dx =$

$u = 2x+3$
$u' = 2$

$$= \left[\frac{x^4}{4}\right]_{-1}^{2} - \frac{1}{2}\left[\sin(2x+3)\right]_{-1}^{2} = 4 - \frac{1}{4} - \frac{1}{2}(\sin 7 - \sin 1)$$

$$= \frac{15}{4} - \frac{1}{2}\sin 7 + \frac{1}{2}\sin 1 \approx 3.8422$$

5. A virus is spreading, infecting at a rate modeled by $I'(t) = 400\frac{(200-3t^2)}{(3t^2+5)^3}$ individuals per day, where $I(t)$ is the number of new people infected t days after the first case has been recorded. What is the net-change in new cases recorded during the first week? (Once you setup this problem you can use technology.)

FIRST WEEK: $1 \leq t \leq 7$

NET CHANGE: $I(7) - I(1) = \int_1^7 I'(t)\,dt$

$$= \int_1^7 400 \frac{(200-3x^2)}{(3x^2+5)^3}\,dx = 67 + \frac{1}{3}$$

THEN 67 NEW CASES

6. A rocket is launched from a vertical position while at rest and 50 feet above the sea level. Immediately after launch the rocket's acceleration is modeled by $a(t) = t^2 + t - 1$, where t is the time in seconds after the rocket is launched. After 10 seconds the acceleration is modeled by $a(t) = t + 2$. Twenty seconds after launch the rocket is only subject to the gravitational force and it starts falling as a free object.

(a) Express the height of the rocket as a function of the time (in seconds).

(b) Will the rocket reach the 5,000 feet altitude? If it does, how long does it take?

(a)

$$a(t) = \begin{cases} t^2+t-1, & 0 \le t < 10 \\ t+2, & 10 \le t < 20 \\ -32, & t \ge 20 \end{cases}, \quad (\text{AT REST} \Rightarrow)\ V(0) = s'(0) = 0, \ s(0) = 50$$

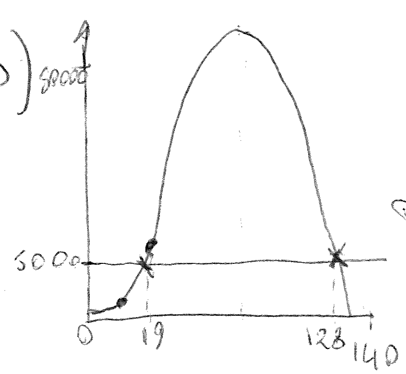
$\boxed{0 \le t < 10}$ $V(t) = \int a(t)\,dt = \frac{t^3}{3} + \frac{t^2}{2} - t + C \Rightarrow 0 = V(0) = C \Rightarrow V(t) = \frac{t^3}{3} + \frac{t^2}{2} - t \Rightarrow s(t) = \int v(t)\,dt = \frac{t^4}{12} + \frac{t^3}{6} - \frac{t^2}{2} + C$ AND $50 = s(0) = C \Rightarrow$ $\boxed{s(t) = \frac{1}{12}t^4 + \frac{1}{6}t^3 - \frac{1}{2}t^2 + 50}$

$\boxed{10 \le t < 20}$ $V(10) = \frac{10^3}{3} + \frac{10^2}{2} - 10 = \frac{1120}{3}$; $s(10) = 1000$

$V(t) = \int t+2\,dt = \frac{t^2}{2} + 2t + C$ AND $\frac{1120}{3} = V(10) = 70 + C \Rightarrow C = \frac{910}{3} \Rightarrow V(t) = \frac{1}{2}t^2 + 2t + \frac{910}{3} \Rightarrow s(t) = \frac{1}{6}t^3 + t^2 + \frac{910}{3}t + C$ AND $1000 = s(10) = 3300 + C \Rightarrow C = -2300 \Rightarrow$ $\boxed{s(t) = \frac{1}{6}t^3 + t^2 + \frac{910}{3}t - 2300}$

$\boxed{t \ge 20}$ $V(20) = \frac{1630}{3}$; $s(20) = 5500$; $v(t) = -32t + C$ AND $\frac{1630}{3} = V(20) = -640 + C \Rightarrow C = \frac{3550}{3} \Rightarrow V(t) = -32t + \frac{3550}{3} \Rightarrow s(t) = -16t^2 + \frac{7100}{3}t + C$ AND $5500 = s(20) = \frac{122800}{3} + C \Rightarrow C = -\frac{106300}{3}$ AND $\boxed{s(t) = -16t^2 + \frac{7010}{3}t - \frac{106300}{3}}$

(b)



$s(t) = 5000$ IN TWO TIME-FRAMES: $10 \le t < 20$ AND $t \ge 20$

AT ABOUT 19 SECONDS AND 128 SECONDS

WITH GGB.

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7. $\int_0^{\frac{1}{2}} x \sin\left(\underbrace{3\pi x^2 - 2}\right)\,dx = \frac{1}{6\pi}\int_0^{\frac{1}{2}} 6\pi x \sin(3\pi x^2 - 2)\,dx$

$u = 3\pi x^2$
$u' = 6\pi x$

$$= \frac{1}{6\pi}\left[-\cos(3\pi x^2 - 2)\right]_0^{\frac{1}{2}}$$

$$= \frac{1}{6\pi}\left(-\cos\left(\frac{3}{4}\pi - 2\right) + \cos(-2)\right) \approx -.0718$$