MAT 221 - Fall 2014 - Exam 1 - Lab (at most 20 minutes)

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Instructor: Dr. Francesco Strazzullo Name KSX

Instructions. You are expected to use a graphing calculator or a software to complete this part. You can use our class notes. Sketch any graph that you use, **approximating up to the third decimal place.** Each problem is worth 10 points.

SHOW YOUR WORK NEATLY, PLEASE.

1. Consider the function $f(x) = 2x^3 + \cos(2\pi x)$.

   (a) Complete the following table.

a	-.1	-.001	0	.001	.1
$f'(a)$	.1289	.0007	0	-.0007	-.0089

   (b) Compute the slope-intercept form of the equation of the tangent and the normal lines to $f(x)$ at $x = 0$.

USING (a): $f'(0) = \frac{2}{10^6}$; $f(0) = 1$

TANGENT LINE: $L(X) = f(a) + f'(a) \cdot (X - a) = 1$

NORMAL LINE: $N(X) = f(a) - \frac{1}{f'(a)} (X - a)$ BUT HERE $L(X)$ IS HORIZONTAL, THEREFORE $N(X)$ MUST BE THE VERTICAL LINE THROUGH $(a, f(a))$; $X = a$. THEREFORE N: $X = 0$

   (c) Use the previous step to complete the following table and check the local linearity of $f(x)$ at $x = 0$, approximating up to the third decimal place.

x	-.1	-.001	0	.001	.1
$f(x)$	.998	1	1	1	1.002
$L(x)$	1	1	1	1	1

$f(x) \approx L(X)$ TO THE TENTHS.

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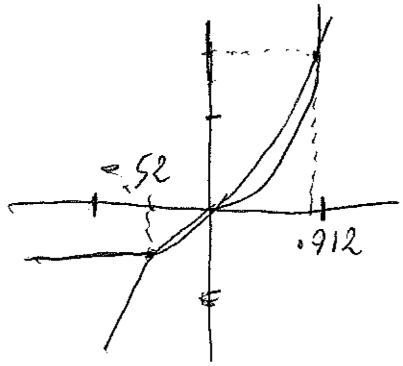
2. Find in the interval $[-1, 1]$ the solutions of the equation $x + \ln(1 + x^2) = 2x^3$.

GRAPHICALLY:



SOLUTIONS IN $[-1, 1]$: $X = 0$, $X \approx -.52$, $X \approx .912$

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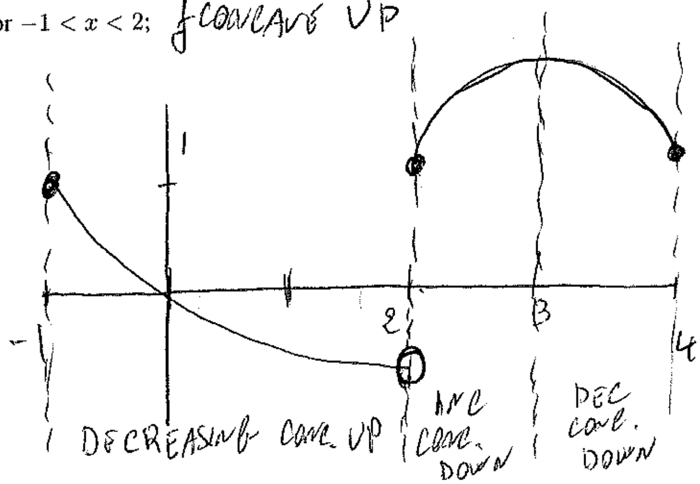
MAT 221- Fall 2014 - Exam 1 - Conceptual

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. You are expected to use a graphing calculator or a software, but **not a CAS**. You can use our class notes. Sketch any graph that you use, **approximating up to the third decimal place**. Each problem is worth 10 points. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Over the interval $[-1, 4]$, sketch the graph of a function $f(x)$ such that:

   (a) $f(-1) = f(2) = f(4) = 1$;
   (b) $f'(x) > 0$ for $2 < x < 3$; f INCREASING
   (c) $f'(x) < 0$ for $-1 < x < 2$ and $3 < x < 4$; f DECREASING
   (d) $f''(x) < 0$ for $2 < x < 4$; f CONCAVE DOWN
   (e) $f''(x) > 0$ for $-1 < x < 2$; f CONCAVE UP



2. For $f(x) = 4x^5 + x^3 - 2x + 4$, compute $f'(x)$ and $f''(x)$.

$$f'(x) = 4(5x^{5-1}) + 3x^{3-1} - 2(1x^{1-1}) + 0$$
$$= 20x^4 + 3x^2 - 2$$

$$f''(x) = 20(4x^{4-1}) + 3(2x^{2-1}) - 0$$
$$= 80x^3 + 6x$$

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3. Differentiate the following functions (each problem is worth 10 points):

(a) $h(x) = \frac{2x^2+1}{2+e^x}$

QUOTIENT RULE: $\left(\frac{f}{g}\right)' = \frac{f'g + fg'}{g^2}$

$f = 2x^2+1 \to f' = 4x$

$g = 2+e^x \to g' = e^x$

$$h' = \frac{4x(2+e^x) - (2x^2+1)e^x}{(2+e^x)^2} = \frac{8x+4xe^x-2x^2e^x-e^x}{}$$

$$= \frac{-2x^2e^x + 4(2+e^x)x - e^x}{(2+e^x)^2}$$

(b) $g(t) = (t^2+3)\sqrt[5]{t}$

PRODUCT RULE: $(fg)' = f'g + fg'$

BUT HERE THERE ARE ONLY POWERS OF t:
IT IS FASTER TO EXPAND.

$g(t) = t^2 \cdot t^{\frac{1}{5}} + 3t^{\frac{1}{5}} \Rightarrow g(t) = t^{\frac{11}{5}} + 3t^{\frac{1}{5}}$

$$g'(t) = \frac{11}{5}t^{\frac{6}{5}} + \frac{3}{5}t^{-\frac{4}{5}}$$

$$= \frac{1}{5}t^{\frac{1}{5}}(11t + 3t^{-1})$$

$$= \frac{\sqrt[5]{t}}{5}\left(11t + \frac{3}{t}\right)$$

OR $= \frac{\sqrt[5]{t}}{5}\left(\frac{11t^2+3}{t}\right)$

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4. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

(a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?

10,000 YARDS OF FABRIC ARE SOLD AT 20 DOLLARS PER YARD EACH AND THEIR QUANTITY AT THIS PRICE IS DECREASING AT A RATE OF 350 YARDS PER UNIT OF PRICE (PER \$/YRD)

(b) Assuming the values in part (4a), find $R'(20)$ and interpret your answer.

$$R' = \frac{dR}{dp} = \frac{d}{dp}[p \cdot f(p)] = 1 \cdot f + p \cdot f'$$ BY PRODUCT RULE

$$R'(20) = f(20) + 20 \cdot f'(20) = 10000 + 20(-350)$$
$$= 3000.$$

WHEN THE PRICE IS SET AT 20 DOLLARS PER YARD THE REVENUE IS INCREASING AT 3000 DOLLARS PER UNIT OF PRICE.

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5. If R denotes the *reaction* of the body to some stimulus of strength x, the *sensitivity* S is defined to be the rate of change of the reaction with respect to x. A particular example is that when the brightness x of a light source is increased, the eye reacts by decreasing the area of the pupil. The experimental formula

$$R = \frac{40 + 24x^{0.4}}{1 + 4x^{0.4}}$$

has been used to model the dependence of R on x when R is measured in square millimeters and x is measured in appropriate units of brightness, for simplicity here let's say lumens.

(a) Build a table by computing both the reaction and the sensitivity to respectively 0.5, 10, 100, and 1000 lumens.

X	.5	10	100	1000
R	14.434	9.078	7.296	6.528
S=R'	-5.073	-.112	-.005	-.0002

(b) Comment on the values you found in part (5a): in very few words, what do they mean?

THE LARGER THE INTENSITY (X) THE SMALLER THE REACTION BECOMES, DECREASING MORE AND MORE SLOWLY WITH ALMOST NONE SENSITIVITY.

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