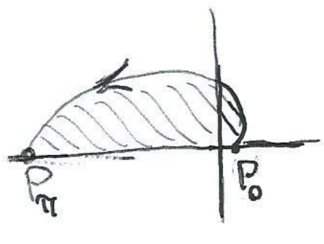
Instructor: Dr. Francesco Strazzullo **Name** KEY

I certify that I did not receive third party help in *completing* this test (sign) ______________________

Instructions. Technology and instructor's notes (including the formula sheets from our book) are allowed on this exam. Each problem is worth 10 points. If you use notes or formula sheets, make a reference. When using technology describe which commands (or keys typed) you used or print out your worksheet.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the area of the region bounded by $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le \pi$, and the x-axis.



X-INTERCEPTS: $y = 0 \Rightarrow \begin{cases} e^t = 0 & \text{NOT POSSIBLE} \\ \sin t = 0 \Rightarrow t = 0, \pi \end{cases}$

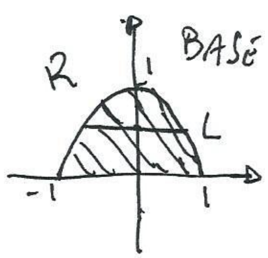
NOTE THAT THE CURVE IS COUNTERCLOCKWISE ORIENTED:

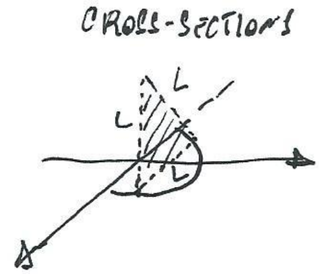
$$A = \int_{P_\pi}^{P_0} y\,dx = \int_\pi^0 e^t \sin t \cdot e^t(\cos t - \sin t)\,dt =$$

$$= \int_\pi^0 e^{2t}\left(\frac{\sin(2t)}{2} - \sin^2 t\right)dt \underset{\uparrow}{=} \frac{1}{4}(e^{2\pi} - 1)$$

GGB, FORMULAS, OR BY PARTS

2. The base of a certain solid is a plane region $\boldsymbol{R}$ enclosed by the x-axis and the curve $y = 1 - x^2$. Each cross-section of the solid perpendicular to the y-axis is an equilateral triangle with its base lying in $\boldsymbol{R}$. Find the volume of the solid.





$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}L\right)L$$

$$A = \frac{\sqrt{3}}{4}L^2$$

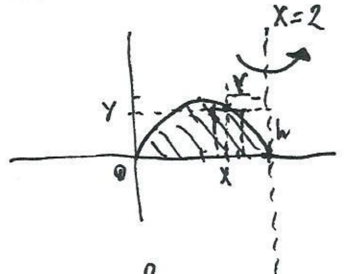
$$L = 2|x| = 2\sqrt{x^2} = 2\sqrt{1-y} \Rightarrow$$

$$\Rightarrow A(y) = \sqrt{3}\,(1-y)\text{, BECAUSE } y < 1.$$

$$V = \int_0^1 A(y)\,dy = \int_0^1 \sqrt{3}(1-y)\,dy = \frac{\sqrt{3}}{2}$$

$$\sqrt{3}\left[y\left(1 - \frac{y}{2}\right)\right]_0^1$$

3. Find the volume of the solid obtained by rotating the region bounded by $y = 2x - x^2$ and the x-axis about the line $x = 2$.



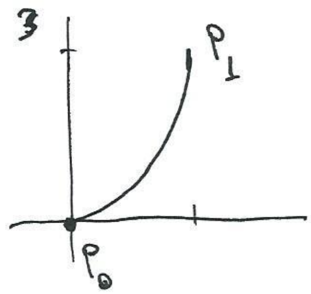
CYL. SHELLS: $V = \int_0^2 2\pi r h \, dx$

$r = 2 - x$; $h = y = 2x - x^2$ $\Rightarrow$

$$\Rightarrow V = \int_0^2 2\pi (2-x)(2x-x^2)\,dx = \int_0^2 2\pi\, x(2-x)^2\,dx = 2\pi \int_0^2 x^3 - 4x^2 + 4x\,dx$$

$$= 2\pi \left[x^2 \left(\frac{x^2}{4} - \frac{4}{3}x + 2 \right) \right]_0^2 = \frac{8}{3}\pi$$

4. A curve is written parametrically as $x = 3t - t^3$, $y = 3t^2$. Find the arc length of the curve from $t = 0$ to $t = 1$



$$L = \int_0^1 \sqrt{(x')^2 + (y')^2}\,dt$$

$$= \int_0^1 \sqrt{(3-3t^2)^2 + (6t)^2}\,dt$$

$$= \int_0^1 \sqrt{9 - 18t^2 + 9t^4 + 36t^2}\,dt = \int_0^1 3\sqrt{1 + 2t^2 + t^4}\,dt$$

$$= 3\int_0^1 \sqrt{(1+t^2)^2}\,dt = 3\int_0^1 1 + t^2\,dt = 3\left[t\left(1 + \frac{t^2}{3}\right)\right]_0^1 = 4$$

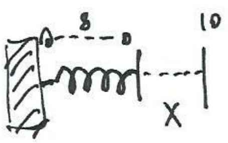
5. The temperature (in °F) in a certain city t hours after 9 A.M. is approximated by the function $T(t) = 50 + 14\sin(\frac{\pi}{12}t)$. Find the average temperature during the period from 9 A.M. to 9 P.M..

$$\text{"AVERAGE OF } T \text{ OVER } [0, 12]\text{"} = \frac{1}{12}\int_0^{12} T(t)\,dt$$

$$= \frac{1}{12}\int_0^{12} 50 + 14\sin\left(\frac{\pi}{12}t\right)dt = \frac{1}{12}\left[50t - 14\left(\frac{12}{\pi}\right)\cos\left(\frac{\pi}{12}t\right)\right]_0^{12}$$

$$= \frac{1}{12}\left(50(12) + \frac{28(12)}{\pi}\right) \approx 59°\,F$$

6. A force of 10 pounds is required to stretch a spring from its natural length of 8 inches to a length of 10 inches. How much work is done in stretching the spring to a length of 12 inches?



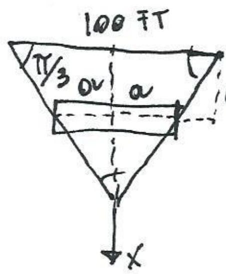
HOOKE'S : $10 = k \cdot 2 \Rightarrow k = 5 \Rightarrow F = 5x$

$x = 2 = 10 - 8$

STRETCH TO 12 $\Rightarrow x = 12 - 8 = 4$.

$$W = \int_0^4 F\,dx = \int_0^4 5x\,dx = \frac{5}{2}\left[x^2\right]_0^4 = 40 \text{ lb-in}$$

7. Find the total hydraulic force on a dam in the shape of an equilateral triangle with one vertex pointing down, if the side of the triangle is 100 feet and the water is even with the top.



MASS DENSITY OF WATER IS $\rho = 1000 \text{ kg}/\text{m}^3$

WEIGHT DENSITY OF WATER IS $\delta = 62.5 \text{ lb}/\text{FT}^3$

BY SIMILAR TRIANGLES: $\dfrac{a}{50} = \dfrac{100 - x_i^*}{50 \cdot \sqrt{3}} \Rightarrow a = \dfrac{100 - x_i^*}{\sqrt{3}}$

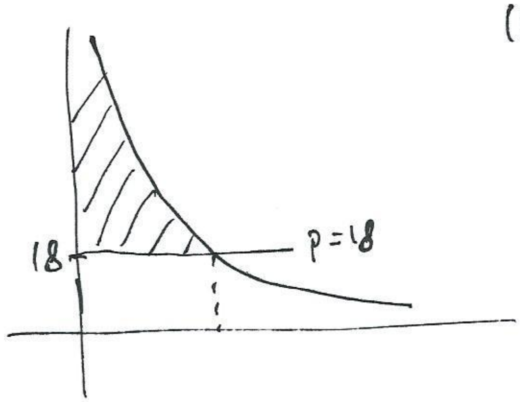
i-TH AREA ; $A_i = 2a \cdot \Delta x = \dfrac{2}{\sqrt{3}}(100 - x_i^*)\Delta x$;

$P_i = \delta d_i$ AND $F_i = P_i A_i \Rightarrow F_i = 62.5 \cdot x_i^* \cdot \dfrac{2}{\sqrt{3}}(100 - x_i^*)\Delta x \Rightarrow$

$\Rightarrow F = \lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} F_i = \displaystyle\int_0^{100} \frac{125}{\sqrt{3}}\, x(100 - x)\, dx = \frac{125}{\sqrt{3}}\left[x^2\left(50 - \frac{x}{3}\right)\right]_0^{100} =$

$= \dfrac{125}{\sqrt{3}}(100)^2 \cdot \dfrac{50}{3} \approx 12{,}028{,}131 \text{ lb}$

8. The demand function for a certain commodity is $p(x) = \dfrac{1800}{(x+5)^2}$. Find the consumer surplus when the selling price is \$18.



$18 = \dfrac{1800}{(x+5)^2} \Rightarrow (x+5)^2 = 100 \Rightarrow x = 5$

$CS = \displaystyle\int_0^5 p(x) - 18\, dx$

$= \displaystyle\int_0^5 \frac{1800}{(x+5)^2} - 18\, dx$

$= 18\left[-\dfrac{100}{x+5} - x\right]_0^5 = -18(15 - 20) = 90$ DOLLARS

9. Assume the daily consumption of electric power (in millions of kilowatt-hours) of a certain city

has the probability density $p(x) = \begin{cases} \frac{1}{9} x e^{-\frac{x}{3}} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

If the city's power plant has a daily capacity of 12 million kilowatt-hours, what is the probability that the available power supply will be inadequate on any given day?

INADEQUATE MEANS $X > 12$, THE NEED IS ABOVE THE CAPACITY

$$P(X>12) = \int_{12}^{\infty} p(x)\,dx = \int_{12}^{\infty} \frac{1}{9} x e^{-\frac{x}{3}}\,dx = \lim_{b\to\infty} \left(\frac{-3e^{-\frac{b}{3}}}{9}(3+b)\right) + 5e^{-4}$$

BY PARTS

$$\int x e^{-\frac{x}{3}}\,dx = -3xe^{-\frac{x}{3}} - 9e^{-\frac{x}{3}} = -3e^{-\frac{x}{3}}(3+x)$$

$u = x$; $dv = e^{-\frac{x}{3}}dx \Rightarrow v = -3e^{-\frac{x}{3}}$

$\to 0$

$= 5e^{-4} \approx 9\%$

NOTE: $P(X>12) = 1 - P(X<12)$

IMPROPER $\to$ INTEGRAL $\leftarrow$ DEFINITE

10. Let $f(t) = 0.2e^{-.2t}$, $t \geq 0$ be the probability density function of a random variable T, where t is the time that a customer spends in line at teller's window before being served. What is mean of the probability density function?

$$\mu = \int_{-\infty}^{\infty} t\,f(t)\,dt$$

$$\mu = \int_{0}^{\infty} .2t\,e^{-.2t} = \frac{.2}{.04} + \lim_{b\to\infty} -e^{-.2b}\left(\frac{1}{.2}+b\right) = 5 \text{ UNITS OF TIME}$$

$\to 0$

BY PARTS: $\int t e^{-.2t} = -\frac{1}{.2}e^{-.2t}\left(\frac{1}{.2}+t\right)$