Math 102 - Fall 2009 - Test 4

Instructor: Dr. Francesco Strazzullo   Name KEY

Instructions. Only calculators are allowed on this examination. *Always use the appropriate wording and units of measure in your answers (when applicable).*

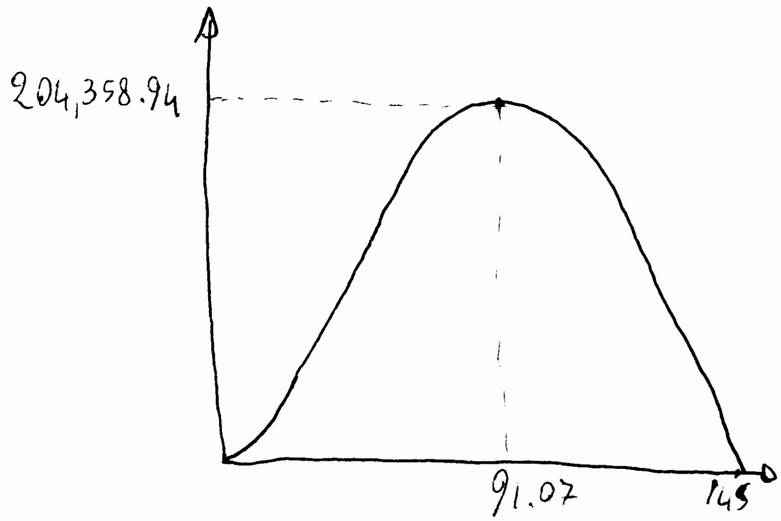
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. (15 points) A firm has total weekly revenue for its product given by $R(x) = 2000x + 30x^2 - 0.3x^3$ (in dollars), where x is the number of units sold.

   (a) Graph this function on the window $[-100, \quad 150]$ by $[-30,000, \quad 220,000]$. (DON'T GRAPH it HERE)

   (b) What restrictions should be placed on x and y in the context of the problem? Accordingly, determine a new window that makes sense for the problem and draw the graph of $R(x)$ here.

   BOTH X AND Y MUST BE NON-NEGATIVE: $X \geq 0$, $Y \geq 0$.
   A WINDOW SETTING LIKE $[0, 150]$ BY $[0, 220,000]$ MAKES SENSE.



   (c) What level of production will yield a maximum revenue?

   $R(91) = 204,359$ AND $R(92) = 204,314$, SO THAT THE MAXIMUM (POSSIBLE) REVENUE IS OBTAINED WHEN PRODUCING 91 UNITS

   (d) What is the revenue when 60 units are produced?

   $R(60) = 163,200$ DOLLARS

   USED THE TABLE ON TI-83

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2. For the following rational functions, use algebra to find (if any) the vertical asymptotes and use a calculator to find (if any) the horizontal asymptotes.

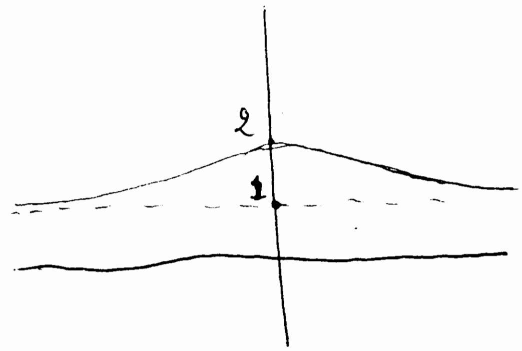
(a) (11 points) $f(x) = \dfrac{x^2+6}{x^2+3}$

DOMAIN: $X^2+3 \neq 0$.

$X^2+3=0 \rightarrow X^2=-3$ NOT POSSIBLE FOR REAL NUMBERS.

DOMAIN IS ALL REAL NUMBER THEN NONE VERTICAL ASYMPTOTE.

HORIZ. ASYMP.: $Y=1$



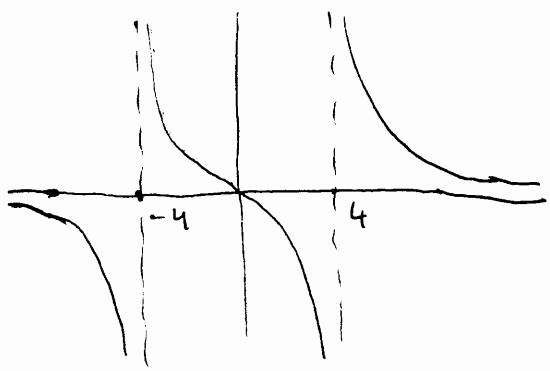
(b) (11 points) $f(x) = \dfrac{5x}{x^2-16}$

DOMAIN: $X^2-16 \neq 0$.

$X^2-16=0 \rightarrow (X-4)(X+4)=0 \rightarrow X=\pm 4$

VERTICAL ASYMPTOTES: $X=4$, $X=-4$

HORIZ. ASYMPTOTE: $Y=0$



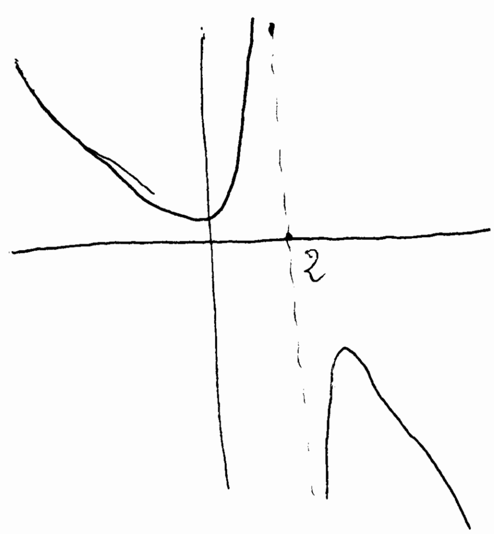
(c) (11 points) $f(x) = \dfrac{2x^2+1}{2-x}$

DOMAIN: $2-X \neq 0$.

$2-X=0 \rightarrow X=2$

VERTICAL ASYMP.: $X=2$

NONE - HORIZ. ASYMPT.



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3. (15 points) As an 8-hour shift progresses, the rate at which workers produce picture frames, in units per hour, changes during the day according to the equation

$$f(t) = \frac{100(t^2+3t)}{(t^2+3t+12)^2}, \qquad 0 \le t \le 8,$$

where t is the number of hours after the beginning of the shift.

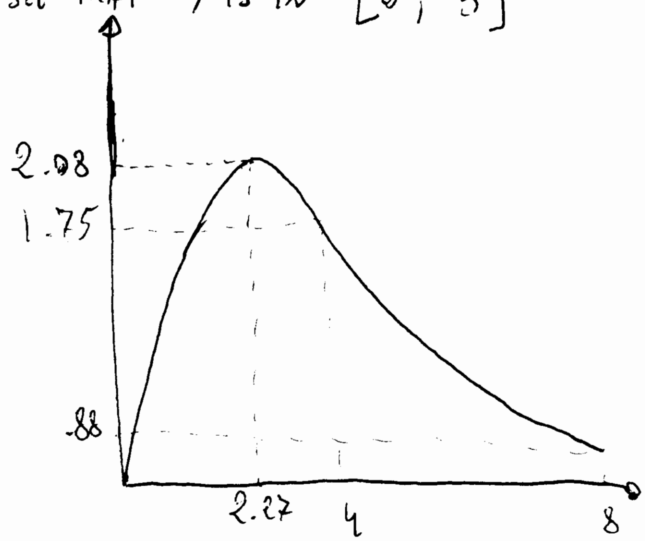
(a) Is the rate of productivity higher near lunch ($t = 4$) or near dinner ($t = 8$).

$f(4) = 1.75$, $f(8) = .88$. AT LUNCH THE RATE OF PRODUCTIVITY IS HIGHER, BEING 1.75 FRAMES PER HOUR

(b) Graph this model according to the context. When is the productivity at its higher rate and what is this rate?

WE KNOW THAT t IS IN $[0, 8]$, ACCORDINGLY $Y = f(t)$ IS POSITIVE. FROM THE TABLE WE SEE THAT Y IS IN $[0, 3]$

THE PRODUCTIVITY REACHES THE MAXIMUM RATE OF 2.08 FRAMES PER HOUR AFTER ABOUT 2.27 HOURS.



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4. (15 points) A bank loans \$285,000 to a development company to purchase three business properties. One of the properties costs \$45,000 more than the other and the third costs twice the sum of these two properties.

(a) Write the system of linear equations relating the costs of these properties.

SAY X, Y, AND Z ARE THESE COSTS IN THOUSAND DOLLARS, THEN:

$$\begin{array}{ll} \text{EQ1:} & \\ \text{EQ2:} & \\ \text{EQ3:} & \end{array} \begin{cases} X + Y + Z = 285 \\ X = Y + 45 \\ Z = 2(X + Y) \end{cases}$$

(b) Find the cost of each property.

I USE SUBSTITUTION:

PLUG EQ2 IN EQ3: $Z = 2(Y+45+Y) \to Z = 2(2Y+45)$

PLUG THIS AND EQ2 IN EQ1: $(Y+45) + Y + 2(2Y+45) = 285 \to$

$\to Y + \underbrace{45}_{} + Y + 4Y + \underset{-135}{90} = \underset{-135}{285} \to 6Y = 150 \to Y = \frac{150}{6} = 25$

NOW: $Z = 2(2Y+45) = 2(2 \cdot 25 + 45) = 2 \cdot 95 = 190$

$X = Y + 45 = 25 + 45 = 70$

THE PRICES ARE \$190,000, \$25,000, AND \$70,000.

NOTE: ONE CAN USE THE CALCULATOR AND RREF

AFTER REWRITING THE SYSTEM IN STANDARD FORM:

$$\begin{cases} X + Y + Z = 285 \\ X - Y = 45 \\ 2X + 2Y - Z = 0 \end{cases}$$

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5. Solve the following systems of linear equations.

(a) (11 points)
$$\begin{cases} x - 2y + z - 3w = 10 \\ 2x - 3y + 4z + w = 12 \\ 2x - 3y + z - 4w = 7 \\ x - y + z + w = 4 \end{cases}$$

MATRIX: $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 2 & -3 & 4 & 1 & 12 \\ 2 & -3 & 1 & -4 & 7 \\ 1 & -1 & 1 & 1 & 4 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -57 \\ 0 & 1 & 0 & 0 & -70 \\ 0 & 0 & 1 & 0 & -25 \\ 0 & 0 & 0 & 1 & 16 \end{array}\right]$

SOLUTION: $(x, y, z, w) = (-57, -70, -25, 16)$

(b) (11 points)
$$\begin{cases} 2x + 3y + 4z = 5 \\ x + y + z = 1 \\ 6x + 7y + 8z = 9 \end{cases}$$

$\text{RREF}\left(\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 6 & 7 & 8 & 9 \end{array}\right]\right) = \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \begin{cases} x - z = -2 \\ y + 2z = 3 \\ z = z \end{cases}$

$$\boxed{\begin{aligned} x &= z - 2 \\ y &= 3 - 2z \\ z &= z \end{aligned}}$$ SOLUTION

(INFINITELY MANY SOLUTIONS)

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