Math 102 - Fall 2012 - Test 2

Instructor: Dr. Francesco Strazzullo Name KEY

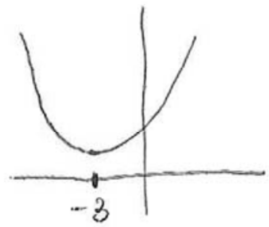
Instructions. Only calculators are allowed on this examination. Each problem is worth 10 points. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Solve the equation $x^2 + 6x + 10 = 0$.

QUADRATIC FORMULA: $X = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} = \dfrac{-6 \pm \sqrt{-4}}{2}$

$X = \dfrac{-6 \pm i\sqrt{4}}{2} = \dfrac{2(-3 \pm i)}{2} = -3 \pm i$ $\begin{cases} X = -3 - i \\ X = -3 + i \end{cases}$

WE CAN'T USE THE GRAPH



2. A bank loans \$390,000 to a development company to purchase three business properties. If one of the properties costs \$80,000 more than the other and the third costs half the sum of these two properties. Write a system of three linear equations in three variables modeling the costs of these properties, then find out each cost.

LET X, Y, AND Z BE THE COSTS OF THESE PROPERTIES, IN THOUSAND DOLLARS

"ONE COSTS 80 K\$ MORE THAN THE OTHER" $\to X = Y + 80 \to X - Y = 80$

"THIRD COSTS HALF THE SUM OF THESE TWO" $\to Z = \frac{1}{2}(X+Y) \to X + Y - 2Z = 0$

"ALLTOGETER 390 K\$" $\to X + Y + Z = 390$

$$\begin{cases} X - Y = 80 \\ X + Y - 2Z = 0 \\ X + Y + Z = 390 \end{cases} \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 80 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & 390 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 170 \\ 0 & 1 & 0 & 90 \\ 0 & 0 & 1 & 130 \end{array}\right] \longrightarrow$$

$$\longrightarrow \begin{cases} X = 170 \\ Y = 90 \\ Z = 130 \end{cases}$$

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3. For the extreme weather months, Canton Power charges \$121.97 plus 6.91 cents for all kilowatt-hours above 1600, while it charges \$15.50 plus 8.19 cents per kilowatt-hour (kWh) for the first 1600 kWh.

   (a) Write the function that gives the monthly charge in dollars as a function of the kilowatt-hours used.

   (b) What is the monthly charge if 1700 kWh are used?

   (c) What is the monthly charge if 700 kWh are used?

X = KWh CONSUMED ; Y = CHARGE IN DOLLARS ; RATES MUST BE IN \$/KWh

SO $6.91\ ¢/\text{KWh} = \frac{6.91}{100}\ \$/\text{KWh} = .0691$ AND $8.19\ ¢/\text{KWh} = .0819\ \$/\text{KWh}$

a)
$$Y = \begin{cases} 15.5 + .0819X & , 0 \le X \le 1600 \\ 121.97 + .0691(X - 1600) & , X > 1600 \end{cases}$$

b) $X = 1700 > 1600 \rightarrow Y = 121.97 + .0691(100) = 128.88$ DOLLARS

c) $X = 700 < 1600 \rightarrow Y = 15.5 + .0819(700) = 72.83$ DOLLARS

4. The 2wheel Company produces two models of bikes, the ladybug and the yellowjacket. It takes 45 minutes to make the ladybugs and 1.5 hours to make the yellowjacket. This company can make at most 210 bikes per day and it has at least 100 labor-hours available per day.

   (a) Set up a system of linear inequality describing this application.

   (b) List and check three possible pairs of production levels.

AT LEAST $\rightarrow \ge$

AT MOST $\rightarrow \le$

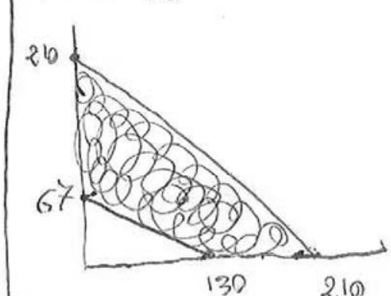
X = # OF LADYBUGS ; Y = # YELLOWJACKETS ; DAILY JOB-RATES IN h/BIKE

SO $45\ \text{MIN}/\text{BIKE} = \frac{45}{60}\ \text{h}/\text{BIKE} = .75\ \text{h}/\text{BIKE}$ (WE COULD USE min/BIKE).

BIKE	Q	R JOB-RATE	QR HOURS
LADYBUG	X	.75	.75X
YELLOWJACK.	Y	1.5	1.5Y
TOTAL	X+Y ≤ 210		.75X+1.5Y ≥ 100

(a)
$$\rightarrow \begin{cases} X + Y \le 210 \\ .75X + 1.5Y \ge 100 \end{cases}$$

OPTIONAL



(b)

	I) X = 130, Y = 67	II) X = 0, Y = 210	III) X = 210, Y = 0
CHECK	$130 + 67 = 197 \le 210$	$0 + 210 = 210 \le 210$	$210 + 0 = 210 \le 210$
	$.75(130) + 1.5(67) = 198 \ge 100$	$.75(0) + 1.5(210) = 315 \ge 100$	$.75(210) + 1.5(0) = 157.5 \ge 100$

5. Solve the following systems of linear equations (10 points for each).

(a) $\begin{cases} x + 2y - z + 4w = 5 \\ x - 6y - 3z - 2w = -1 \\ 3x - y + z + w = 1 \\ x + 5y \quad\quad - w = 3 \end{cases} \longrightarrow \left[\begin{array}{cccc|c} 1 & 2 & -1 & 4 & 5 \\ 1 & -6 & -3 & -2 & -1 \\ 3 & -1 & 1 & 1 & 1 \\ 1 & 5 & 0 & -1 & 3 \end{array}\right] \longrightarrow$ RREF

$\longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 107/161 \\ 0 & 1 & 0 & 0 & 93/161 \\ 0 & 0 & 1 & 0 & -156/161 \\ 0 & 0 & 0 & 1 & 89/161 \end{array}\right] \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & .665 \\ 0 & 1 & 0 & 0 & .578 \\ 0 & 0 & 1 & 0 & -.969 \\ 0 & 0 & 0 & 1 & .553 \end{array}\right]$

$\longrightarrow \begin{cases} X = .665 \\ Y = .578 \\ Z = -.969 \\ W = .553 \end{cases}$ OR PRECISELY $\begin{cases} X = 107/161 \\ Y = 93/161 \\ Z = -156/161 \\ W = 89/161 \end{cases}$

SOLUTION

(b) $\begin{cases} x + y - z = -2 \\ 5x - 5y + 4z = 8 \end{cases} \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 5 & -5 & 4 & 8 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & -.1 & -.2 \\ 0 & 1 & -.9 & -1.8 \end{array}\right]$

Z IS FREE

$\longrightarrow \begin{cases} X - .1Z = -.2 \longrightarrow X = .1Z - .2 \\ Y - .9Z = -1.8 \longrightarrow Y = .9Z - 1.8 \\ Z = Z \end{cases}$

SOLUTION: $\begin{cases} X = .1Z - .2 \\ Y = .9Z - 1.8 \\ Z = Z \end{cases}$

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6. The cost (in dollars) for a product can be described by the function $C(x) = -30x + 2500 + 0.1x^2$, where x is the number of units produced and sold.

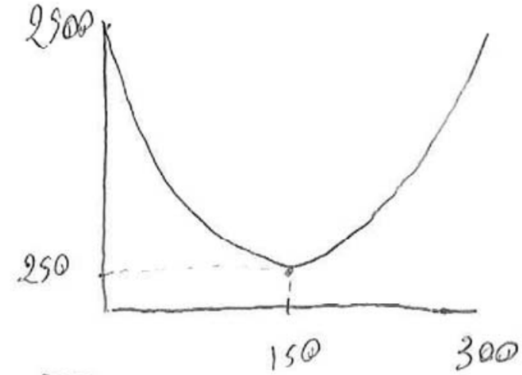
(a) To minimize costs, how many units must be produced and sold? What is the minimum possible cost?

(b) What levels of production will cost about \$1,500?

(a) $Y = C(x)$ IS AN UPWARD PARABOLA, THEN THE VERTEX IS A MINIMUM:

$h = -\frac{b}{2a} = -\frac{-30}{2(.1)} = 150$ UNITS

$C(150) = 250$ DOLLARS IS THE MINIMUM COST.



$\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{3}$

(b) $C(x) = 1500$

$$\downarrow$$

$$.1x^2 - 30x + 2500 = 1500$$

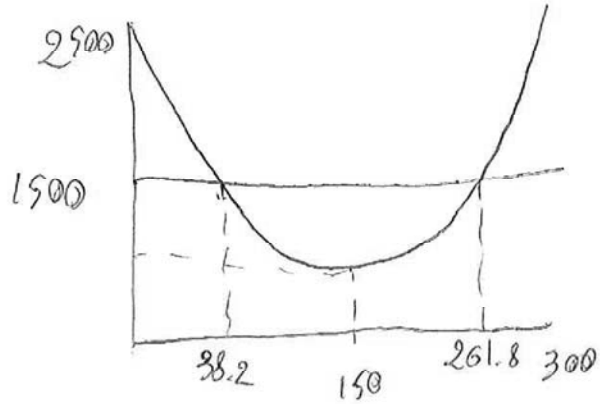
$$\downarrow \; -1500 \quad -1500$$

$$.1x^2 - 30x + 1000 = 0$$

$$\downarrow \text{DIVIDE BY } .1$$

$$x^2 - 300x + 10000 = 0 \xrightarrow{\text{QUAD. FORMULA}} x = \frac{300 \pm \sqrt{300^2 - 4 \cdot 10{,}000}}{2} =$$

$$= \frac{300 \pm 100\sqrt{5}}{2} = 150 \pm 50\sqrt{5} \begin{cases} x = 38.2 \\ x = 261.8 \end{cases}$$



$\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{5}$

X	38	39	261	262
C(X)	1504	1482	1482	1504
	✓			✓

$\rightarrow$ IN THIS CONTEXT $X = 38$, $X = 262$

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7. Graph the solution set of the following system of inequalities, labeling every corner point and highlighting the contour.

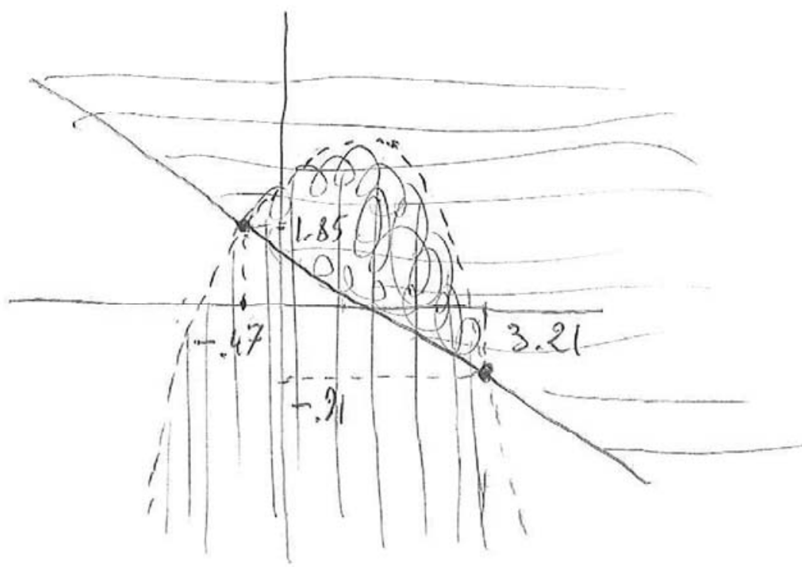
$$\begin{cases} y < -x^2 + 2x + 3 \\ 3x + 4y \geq 6 \end{cases}$$

$3x + 4y \geq 6 \to \frac{4Y}{4} \geq \frac{-3X}{4} + \frac{6}{4} \to Y \geq -\frac{3}{4}X + \frac{3}{2}$

BOUNDARY LINES:

$Y_1 = -X^2 + 2X + 3$, DASHED, REGION BELOW

$Y_2 = -\frac{3}{4}X + \frac{3}{2}$, SOLID, REGION ABOVE



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