Math 321- Spring 2013 - Exam1

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Instructions. Technology and notes (including the formula sheets from our book) are allowed on this exam. Each problem is worth 10 points. **If you use notes or formula sheets, make a reference. When using technology describe which commands (or keys typed) you used or print out your worksheet.**

1. Let $f(x)$ be a twice differentiable function on the interval $[-1,3]$ such that $f(-1) = 2$, $f'(-1) = -3$, $f(3) = 4$, and $f'(3) = 1$. Determine the value of $\int_{-1}^{3} xf''(x)\,dx$.

INTEGRATION BY PARTS: $u = x \to du = dx$

$dv = f''(x)\,dx \to v = \int dv = \int f''\,dx = f'$ $\Big] \to$

$\to \int_{-1}^{3} x f''(x)\,dx = [uv]_{-1}^{3} - \int_{-1}^{3} v\,du = [x \cdot f'(x)]_{-1}^{3} - \int_{-1}^{3} f'\,dx$

$= (3 \cdot f'(3) - (-1) f'(-1)) - (f(3) - f(-1))$

$= (3(1) + (-3)) - (4-2) = -2$

2. A bacteria population starts with 400 bacteria and grows at a rate of

$$r(t) = (450.268)e^{1.12567t}$$

bacteria per hour. How many bacteria will there be after three hours?

"NET CHANGE" $= \int_0^3 r(t)\,dt = \int_0^3 (450.268)\,e^{1.12567t}\,dt =$ $\boxed{2^{ND}} + \boxed{TRACE} + \boxed{7}$

$= 11313.23 \approx 11{,}313$ BACTERIA

"NET CHANGE" $= P(3) - P(0) \to 11{,}313 = P(3) - 400 \to$

$+400 \qquad +400$

$\to P(3) = 11{,}713$ BACTERIA AFTER 3 HOURS.

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3. Show the substitution method by using $u = \cos x$ to find the indefinite integral $\int \frac{\sin x}{\cos^2 x + 1}\, dx$.

PLUG IT IN GEOGEBRA: $\int \frac{\sin x}{(\cos(x))^2 + 1}\, dx = -\text{atan}(\cos x) + C$

$= -\arctan(\cos x) + C$

USE SUBS:

$u = \cos x \to dx = \frac{du}{u'} = \frac{du}{-\sin x} \to \int \frac{\sin x}{\cos^2 x + 1}\, dx =$

$= \int \frac{\cancel{\sin x}}{u^2+1} \frac{du}{-\cancel{\sin x}} = -\int \frac{1}{u^2+1}\, du =$ USE FORMULA 17 WITH $a = 1$

$= -\tan^{-1}(u) + C = -\tan^{-1}(\cos x) + C$ ✓

4. Someone punched a hole at the bottom of your soda cup. The volume of soda decreases at a rate of

$$V'(x) = -\frac{864(16x^2 - 32x - 793)}{125(4x+45)(4x+37)}$$

fluid ounces per seconds. How much soda did you have in your cup if none is left after 8 seconds?

"NET CHANGE" $= \int_0^8 V'(x)\, dx =$ $\boxed{2^{ND}}$ $\boxed{\text{TRACE}}$ $\boxed{7}$

$= -12.034 \approx$ LOST 12 FL OZ

THEREFORE THERE WERE ABOUT 12 FL OZ OF SODA.

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5. Show the integration by parts method by using $u = x$ and $dv = \ln x\, dx$ to find the indefinite integral $\int x \ln x\, dx$. Compare your result to Formula 101.

PLUG IN GEOGEBRA: $\int x \ln x\, dx = \frac{x^2}{4}(2\ln x - 1) + C$

BY PARTS: $u = x \to du = dx$

FORMULA 100 OR GGB

$dv = \ln x\, dx \to v = \int dv = \int \ln x\, dx = x\ln x - x \cancel{+C}$

$\to \int x \ln x\, dx = x(x\ln x - x) - \int (x\ln x - x)\, dx = x^2(\ln x - 1) - \int x\ln x\, dx + \int x\, dx$

$+ \int x \ln x\, dx \qquad + \int x\ln x\, dx$

$\frac{2\int x\ln x\, dx}{2} = x^2\ln x - x^2 + \frac{x^2}{2} + C = \frac{x^2\ln x - \frac{x^2}{2} + C}{2} \to$

$\to \int x \ln x\, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$ ✓

FORMULA 101 WITH $u = x$ AND $n = 1$: $\int x \ln x\, dx = \frac{x^{1+1}}{(1+1)^2}\left((1+1)\ln x - 1\right) + C$

$= \frac{x^2}{4}(2\ln x - 1) + C$ ✓

NOTE: $u = \ln x$, $dv = x\,dx$ IS AN EASIER CHOICE

6. Evaluate the integral $\int_1^3 \log_5 x\, dx$.

CHANGE OF BASE FORMULA (CALC I NOTES PAGE 8): $\log_5 x = \frac{\ln x}{\ln 5}$

YOU CAN JUST PLUG $\frac{\ln x}{\ln 5}$ IN YOUR TI

$\int_1^3 \log_5 x\, dx = \int_1^3 \frac{1}{\ln 5} \ln x\, dx = \frac{1}{\ln 5}\int_1^3 \ln x\, dx =$

$\overset{\text{FORMULA}}{=}{}_{100}\ \frac{1}{\ln 5}\left[u(\ln u - 1)\right]_1^3 =$

$= \frac{1}{\ln 5}\left[3(\ln 3 - 1) - 1(-1)\right] \approx .81$

$= \frac{3\ln 3 - 2}{\ln 5} = \frac{\ln 27 - 2}{\ln 5}$

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7. Compute the most general antiderivative $F(x)$ of $f(x) = x\sinh(2x)$, then the particular antiderivative satisfying the initial condition $F(1) = -1$

$$F(x) = \int x \sinh(2x)\,dx \overset{\text{GGB}}{=} \frac{2x\left(e^{4x}+1\right)-\left(e^{4x}-1\right)}{8e^{2x}} + C$$

NOTE: $\dfrac{e^{4x}+1}{2e^{2x}} = \dfrac{\left(e^{4x}+1\right)e^{-2x}}{2} = \dfrac{e^{2x}+e^{-2x}}{2} = \cosh(2x)$

$$\to F(x) = \frac{1}{4}\left(2x\cosh(2x) - \sinh(2x)\right) + C$$

(COMPARE TO FORMULAE 82 AND 103)

PARTICULAR (PLUG $x=1$): $-1 = F(1) = \frac{1}{4}\left(2\cosh(2) - \sinh(2)\right) + C$

$\to C = \frac{1}{4}\sinh(2) - \frac{1}{2}\cosh(2) - 1 \approx -1.97$

$$F(x) = \frac{x}{2}\cosh(2x) - \frac{1}{4}\sinh(2x) - 1.97$$

8. Integrate $x^2 e^{3x}$ with respect to x.

$$\int x^2 e^{3x}\,dx \overset{\text{GGB}}{=} \frac{e^{3x}}{27}\left(9x^2 - 6x + 2\right) + C$$

OR FORMULA 97 TWICE

THIS COULD BE FOUND BY PARTS (TWICE): $u = x^2$
$dv = e^{3x}\,dx$

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9. A particle moves with velocity function

$$v(t) = .3x^2 - 3.9x + 9$$

feet per second. Compute both the *displacement* and the *total distance covered* in 10 seconds.

$$\text{DISPLACEMENT} = \int_0^{10} v(t)\,dt \underset{\boxed{2^{ND}\ \text{TRACE}\ 7}}{=} -5 \rightarrow \text{MOVED "BACKWARD" OF 5 FT.}$$

$$\text{TOTAL DISTANCE} = \int_0^{10} |v(t)|\,dt \approx 29.3\ \text{FT.}$$

$$= \int_0^{3} v(t)\,dt - \int_3^{10} v(t)\,dt$$

NOTE: IN TI-84 TO USE THE
ABSOLUTE VALUE FUNCTION TYPE
$\boxed{\text{MATH}} \rightarrow \boxed{\text{NUM}}$: abs(_)
SAME SYNTAX IN BCB

10. Compute the partial fractions decomposition of

$$\frac{x-1}{x(x+1)^2}.$$

$$\frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} = \frac{(x+1)^2A + x(x+1)B + xC}{x(x+1)^2} = \frac{x-1}{x(x+1)^2} \rightarrow$$

$$\rightarrow (x+1)^2A + x(x+1)B + xC = x-1$$

$x=0 \rightarrow A=-1$

$x=-1 \rightarrow -C=-2 \rightarrow C=2$

$x=1,\ A=-1,\ C=2 \rightarrow 2^2(-1)+2B+2=0 \rightarrow$

$\rightarrow -4+2B+2=0 \rightarrow B=1$

$$\frac{x-1}{x(x+1)^2} = \frac{-1}{x} + \frac{1}{x+1} + \frac{2}{(x+1)^2}$$

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