Mat 320 – Spring 2019 –Exam1 – Take Home

Instructor: Dr. Francesco Strazzullo **Name**______________________________

I certify that I did not receive third party help in *completing* this test (sign) ______________________________

Instructions. Technology is allowed on this exam, unless otherwise specified. Each problem is worth 10 points and all exercises must be completed. If you use formulas or properties from our book, make a reference. When using technology describe which commands (or keys typed) you used or print out and attach your worksheet.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1) Let $\boldsymbol{a_1} = \begin{bmatrix} 3 \\ -3 \\ 1 \\ 1 \end{bmatrix}$, $\boldsymbol{a_2} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, $\boldsymbol{a_3} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$, and $\boldsymbol{b} = \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix}$. Determine whether $\boldsymbol{b}$ can be written as a linear combination of $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, and $\boldsymbol{a_3}$, if possible find such a linear combination otherwise state why it is not possible.

IF $A = [\vec{a}_1 \; \vec{a}_2 \; \vec{a}_3]$, THEN WE JUST CHECK IF $A\vec{x} = \vec{b}$ IS CONSISTENT.

$RREF\,[A\,\vec{b}] = I_4 \Rightarrow RANK\,[A\,\vec{b}] = 4 \neq RANK(A) = 3 \Rightarrow$

$\Rightarrow$ SYST. IS INCONSISTENT, THEREFORE $\vec{b}$ IS NOT A LINEAR COMBINATION OF $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

2) Determine the dimension of $Span\left\{\begin{bmatrix}1\\1\\0\\1\\1\end{bmatrix},\begin{bmatrix}-1\\2\\3\\1\\1\end{bmatrix},\begin{bmatrix}0\\1\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\\1\\1\end{bmatrix}\right\}. = H$

$\vec{V_1} \quad \vec{V_2} \quad \vec{V_3} \quad \vec{V_4}$

$$RREF([\vec{V_1}\ \vec{V_2}\ \vec{V_3}\ \vec{V_4}]) = \begin{bmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{RANK IS } 3$$

PIVOT

THEN DIMENSION $H = 3$.

4) Determine if $H = \left\{\begin{bmatrix}3x_3 - 2x_4\\1 - 4x_3\\4x_3 - 2x_4\\4 - x_4\end{bmatrix} \in \mathbb{R}^4 : x_3 \in \mathbb{R}, x_4 \in \mathbb{R}\right\}$ is a vector subspace $\mathbb{R}^4$, by writing it as the span of one of its basis, or if H is not a subspace, by indicating which vector subspace property it does not satisfy.

H IS NOT A SUBSPACE BECAUSE $\vec{X} \in H$ IF AND ONLY IF

$$\vec{X} = \begin{bmatrix}0\\1\\0\\4\end{bmatrix} + X_3\begin{bmatrix}3\\-4\\4\\0\end{bmatrix} + X_4\begin{bmatrix}-2\\0\\-2\\-1\end{bmatrix} \quad \text{THUS} \quad \vec{0} \notin H.$$

NOTE: $\begin{cases} 4 - X_4 = 0 \Rightarrow X_4 = 4 \\ 1 - 4X_3 = 0 \Rightarrow X_3 = \frac{1}{4} \end{cases} \Rightarrow 3X_3 - 2X_4 = \frac{3}{4} - 4(2) \neq 0.$

5) The augmented matrix is given for a system of equations. If the system is consistent, state which variables (if any) are free and find the general solution, otherwise state that there is no solution.

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 1 & 4 & 0 \\ 1 & 1 & 0 & 2 & 1 & 1 \\ -1 & 0 & 1 & 3 & 0 & -1 \\ 0 & 3 & 1 & 0 & -2 & 2 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 5/2 & 3/4 \\ 0 & 1 & 0 & 0 & -3/2 & 7/12 \\ 0 & 0 & 1 & 0 & 5/2 & 1/4 \\ 0 & 0 & 0 & 1 & 0 & -1/6 \end{array}\right]$$

PIVOTS

CONSISTENT

X_5 FREE.

X_1, X_2, X_3, X_4 ARE NOT FREE

$$\begin{cases} X_1 + 5/2\, X_5 = 3/4 \\ X_2 - 3/2\, X_5 = 7/12 \\ X_3 + 5/2\, X_5 = 1/4 \\ X_4 = -1/6 \end{cases} \Rightarrow \text{GEN. SOL. } \vec{X} = \begin{bmatrix} 3/4 - 5/2\, X_5 \\ 7/12 + 3/2\, X_5 \\ 1/4 - 5/2\, X_5 \\ -1/6 \\ X_5 \end{bmatrix}$$

6) Linda invests about \$45,000 for one year. Part is invested at 3%, another part at 5.5%, and the rest at 3.75%. The total income from all 3 investments is about \$2300. The total income from the 3% and 3.75% investments is equal to the income from the 5.5% investment. Find the amount invested at each rate. (*You must set up a system of linear equations and solve it.*)

X_1 @ .03 ; X_2 @ .055 ; X_3 @ .0375 . X_i DOLLARS INVESTED.

$$\begin{cases} X_1 + X_2 + X_3 = 45000 \\ .03X_1 + .055X_2 + .0375X_3 = 2300 \\ .03X_1 + .0375X_3 = .055X_2 . \end{cases} \xrightarrow{\text{AUGM.}} [A\ \vec{b}] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 45000 \\ .03 & .055 & .0375 & 2300 \\ .03 & -.055 & .0375 & 0 \end{array}\right] \rightarrow$$

$$\xrightarrow[\text{TO THE CENTS}]{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -32878.79 \\ 0 & 1 & 0 & 20909.09 \\ 0 & 0 & 1 & 56969.70 \end{array}\right]$$

BECAUSE $X_1 < 0$, THIS FINANCIAL PLAN IS INCONSISTENT.

7) Use row reduction to compute the rank of A, without using technology and indicating the row operations you are applying, like $R_5 - 3R_2$.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & 5 & 7 \\ 0 & 0 & 3 & 2 \\ -1 & 1 & -4 & 1 \end{bmatrix} \xrightarrow[R_4 + R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -3 & 7 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 3 & -5 & 4 \end{bmatrix} \xrightarrow[R_4 + R_2]{}$$

$$\longrightarrow \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -3 & 7 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \xrightarrow[R_4 - \frac{2}{3}R_3]{} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -3 & 7 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 11/3 \end{bmatrix}$$

FOUR PIVOTS $\Rightarrow$

$\Rightarrow$ RANK$(A) = 4$

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8) Find a unit vector in the direction of the vector $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \\ -1 \end{bmatrix}$. (*Do not approximate, but simplify your answer*)

$$\vec{V} = \frac{\vec{u}}{\|\vec{w}\|}$$ ⇒ UNIT VECTOR

$$\|\vec{w}\| = \sqrt{1^2+2^2+1^2+1^2+(-1)^2} = \sqrt{8} = 2\sqrt{2}$$

IS $\vec{V} = \dfrac{1}{2\sqrt{2}}\vec{u} = \dfrac{1}{2\sqrt{2}}\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \\ -1 \end{bmatrix}$

9) Compute the reduced row echelon form of the following matrix, without using technology and indicating the row operations you are applying, like $R_5 - 3R_2$.

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ -1 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{\substack{R_2 - R_1 \\ R_3 + R_1}} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 \leftrightarrow R_3 \\ R_2 - 2R_3}} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -2 & -1 \end{bmatrix}$$

$$R_3 \to -\frac{1}{2}R_3 \quad \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \quad \begin{matrix} R_1 - R_3 \\ R_2 - 2R_3 \end{matrix} \quad \begin{bmatrix} 1 & -1 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

$$R_1 + R_2 \quad \begin{bmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

10) Determine a system of homogeneous linear equations representing

Sec 010: $H = Span\left\{\begin{bmatrix}1\\0\\1\\1\end{bmatrix},\begin{bmatrix}2\\2\\0\\0\end{bmatrix},\begin{bmatrix}0\\-1\\1\\1\end{bmatrix},\begin{bmatrix}1\\1\\2\\1\end{bmatrix}\right\}$

Sec 02H: $H = Span\left\{\begin{bmatrix}1\\0\\1\\1\\-1\end{bmatrix},\begin{bmatrix}2\\2\\0\\0\\1\end{bmatrix},\begin{bmatrix}0\\-1\\1\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\2\\1\\-1\end{bmatrix},\begin{bmatrix}3\\2\\1\\1\\0\end{bmatrix}\right\}$.

$\to$ SAY $H = SPAN\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m\}$

SAY $A = [\vec{v}_1 \dots \vec{v}_m]$, THEN $H^{\perp} = NULL(A^T) = SPAN\{w_1, \dots, w_k\}$

THUS IF $B = [\vec{w}_1\ \vec{w}_2 \dots \vec{w}_k]$, THEN $H = NULL(B) \equiv B\vec{x} = \vec{0}$

- SEC. 010

$$A^T = \begin{bmatrix}1&0&1&1\\2&2&0&0\\0&-1&1&1\\1&1&2&1\end{bmatrix} \xrightarrow{RREF} \begin{bmatrix}1&0&0&1/2\\0&1&0&-1/2\\0&0&1&1/2\\0&0&0&0\end{bmatrix} \Rightarrow NULL(A^T) = \left\{\begin{bmatrix}-x_4/2\\x_4/2\\-x_4/2\\x_4\end{bmatrix}, x_4 \in \mathbb{R}\right\}$$

$$\Rightarrow H^{\perp} = SPAN\left\{\begin{bmatrix}-1\\1\\-1\\2\end{bmatrix}\right\} \Rightarrow H \equiv -x_1 + x_2 - x_3 + 2x_4 = 0$$

- SEC. 02H

$$A^T = \begin{bmatrix}1&0&1&1&-1\\2&2&0&0&1\\0&-1&1&1&0\\1&1&2&1&-1\\3&2&1&1&0\end{bmatrix} \xrightarrow{RREF} \begin{bmatrix}1&0&0&1/2&0\\0&1&0&-1/2&0\\0&0&1&1/2&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix} \Rightarrow NULL(A^T) = \left\{\begin{bmatrix}-x_5/2\\x_5/2\\-x_5/2\\0\\x_5\end{bmatrix} : x_5 \in \mathbb{R}\right\}$$

$$\Rightarrow H^{\perp} = SPAN\left\{\begin{bmatrix}-1\\1\\-1\\0\\2\end{bmatrix}\right\} \Rightarrow H \equiv -x_1 + x_2 - x_3 + 2x_5 = 0$$