

Instructor: Dr. Francesco Strazzullo

Name

KEY

**Instructions.** Only calculators are allowed on this examination. Point values of each problem are indicated. Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. (20pts) What is the volume of an athletic shoe box  $5\frac{1}{2}$  inches in height, 6 inches wide, and  $15\frac{1}{2}$  inches long? How many cubic feet would it take to store 2000 such boxes?



$$\text{Volume} = H \cdot W \cdot L \rightarrow V = 5\frac{1}{2} \text{ in} \times 6 \text{ in} \times 15\frac{1}{2} \text{ in}$$

$$= 5.5 \cdot 6 \cdot 15.5 \text{ in}^3 = 511.5 \text{ in}^3$$

THE VOLUME OF EACH BOX IS  $511.5 \text{ in}^3$ . SINCE  $1 \text{ ft} = 12 \text{ in}$ ,

$$\text{THEN } 1 \text{ ft}^3 = 12^3 \text{ in}^3. \text{ THUS: } V = 511.5 \text{ in}^3 = 511.5 \text{ in}^3 \cdot \frac{1 \text{ ft}^3}{12^3 \text{ in}^3} = \frac{511.5}{12^3} \text{ ft}^3$$

THEREFORE TO STORE 2000 SUCH BOXES WE NEED A VOLUME OF  $\frac{511.5}{12^3} \text{ ft}^3$ .

$$2000 \cdot \frac{511.5}{12^3} \text{ ft}^3 = 592.014 \text{ ft}^3$$

2. (20pts) The total cost for a credit card call is \$ 0.13 per minute for the first two minutes and \$ 0.18 for each additional minute. Write two equations with inputs for each described with an inequality. Define your variables so  $y$  depends on  $x$ .

$X$  IS THE NUMBER OF MINUTES USED AND  $Y$  IS THE TOTAL COST IN DOLLARS.

I) WHEN  $0 \leq x \leq 2$ , WE HAVE  $Y = .13x$

II) WHEN  $x > 2$ , WE HAVE  $Y = .13 \cdot 2 + .18(x-2)$ , THAT IS  
 $Y = .26 + .18(x-2)$

3. (20pts) A pressure washer costs \$25 for the first 3 hours and \$12 per additional hour. Write a piecewise-defined model for the total cost. What is the total hour rented,  $x$ , if the total cost is \$73?

$X$  IS THE HOURS OF RENTAL AND  $Y$  IS THE TOTAL COST IN DOLLARS

$$Y = \begin{cases} 25 & , 0 \leq x \leq 3 \\ 25 + 12(x-3) & , x > 3 \end{cases}$$

SINCE  $73 > 25$ , THE RENTAL HAS BEEN FOR MORE THAN 3 HOURS. WE MUST USE THE MODEL FOR  $x > 3$ .

$$73 = 25 + 12(x-3) \Rightarrow 48 = 12x - 36 \Rightarrow \frac{84}{12} = \frac{12x}{12} \Rightarrow x = 7$$

THE PRESSURE WASHER HAS BEEN RENTED FOR 7 HOURS.

4. (15pts) Solve the equation  $5x - 3 = 4(2 + x) - 3$ .

$$\begin{array}{ccccccc}
 5x & - & 3 & = & 4(2 + x) & - & 3 \\
 +3 & & & & +3 & & \\
 \hline
 5x & & & = & 4 \cdot 2 + 4 \cdot x & & \\
 & & & & -4x & & -4x
 \end{array}
 \Rightarrow$$

DISTRIBUTE

$$\Rightarrow \boxed{x = 8}$$

5. In New-York City, a U-Haul 24-foot moving truck costs \$19.95 plus \$1.39 per mile for 6 hours. The same truck from Budget costs \$39.95 plus \$.69 per mile

- (a) (10pts) For each agency, write an equation described the total cost.

$x$  IS THE MILES TRAVELED IN 6 HOURS AND  $y$  THE TOTAL COST IN DOLLARS:

U-HAUL:  $y = 19.95 + 1.39x$

BUDGET:  $y = 39.95 + .69x$

WE COULD USE  $y_1$  FOR THE COST WITH U-HAUL AND  $y_2$  FOR THAT WITH BUDGET.

- (b) (10pts) How many miles does one need to drive so that the two costs are the same?

$$\begin{array}{ccccccc}
 y_1 & = & y_2 & \text{WHEN} & 19.95 + 1.39x & = & 39.95 + .69x & (\text{WE USE ALGEBRA}) \\
 & & & & -19.95 & - & .69x & -19.95 & - & .69x
 \end{array}$$

$$\Rightarrow .7x = 20 \Rightarrow x = \frac{20}{.7} \approx 28.57$$

ONE NEEDS TO DRIVE FOR ABOUT 28.57 mi FOR THE TWO COSTS TO BE THE SAME (SEE GRAPH NEXT PAGE)

- (c) (10pts) How many miles does one need to drive in order for the Budget truck to be less expensive than the U-Haul?

THE COST FOR RENTING THE BUDGET TRUCK IS

$$Y_0 = 39.95 + .69X$$

FOR THIS TO BE

LESS EXPENSIVE THAN THE WHOLE  $Y_L = 19.95 + 1.39X$ ,

WE MUST SET  $Y_2 < Y_1$ , THAT IS,

$$\begin{array}{r} 39.75 + .69x < 19.75 + 1.39x \\ -39.75 \quad -1.39x \quad \underline{-39.75 \quad -1.39x} \end{array} \quad (\text{ALGEBRAICALLY})$$

$$\begin{array}{r} -39.95 \quad -1.39x \quad -39.95 \quad -1.39x \\ \hline \end{array}$$

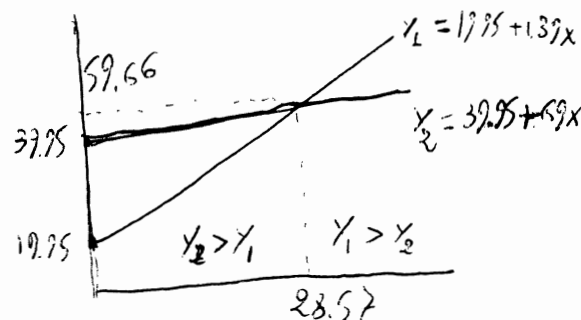
$$\frac{-7x < -20}{-7} \Rightarrow x > 28.57$$

SWITCH

SWITCH

SWITCH

$$\Rightarrow X > 28.57$$



FROM THIS GRAPH, WE SEE  
THAT THE COST  $Y_2$  IS BELOW  
THE COST  $Y_1$  WHEN  $X > 28.57$

ONE NEEDS TO DRIVE FOR MORE THAN 28.57 MILES IN ORDER FOR THE  
BUDDGE RENTAL TO BE LESS EXPENSIVE THAN THE U-HAUL.

(NOTICE,  $X = 28.57$  IS ACHIEVABLE FOR A 6 HOURS RENTAL)

6. (15pts) Give the interval which is the solution set for the inequality  $\frac{1}{3}x > 5 + 2x$ .

$$\frac{1}{3}x > 5 + 2x \quad | -2x \quad \Rightarrow \quad -\frac{5}{3}x > 5 \quad \Rightarrow \quad x < 5 \cdot \left(-\frac{3}{5}\right) \Rightarrow$$
  

$$\frac{1}{3} - 2 = \frac{1-6}{3} = -\frac{5}{3} \quad \left| \frac{-5}{3} \right| \quad \frac{-5}{3} \quad \left| \frac{-5}{3} \right| \quad \text{Switch}$$

$$\begin{array}{r} \frac{1}{3}x > 5 + 2x \\ -2x & \quad -2x \end{array}$$

$$\frac{1}{3} - 2 = \frac{1-6}{3}$$

$$\begin{array}{r} -\frac{5}{3} \times > 5 \\ \frac{3}{-5} \end{array} \quad \begin{array}{r} \uparrow \\ -\frac{5}{3} \end{array}$$

SWITCH

$$\Rightarrow x < 5 \cdot \left(-\frac{3}{5}\right) \Rightarrow$$

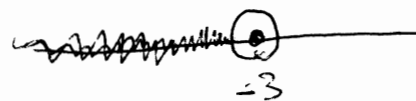
$$\begin{array}{r} 3 \\ -5 \\ \hline 3 \end{array} \quad \begin{array}{r} 25 \\ -5 \\ \hline 3 \end{array}$$

SWITCH

SWITCH

$$\Rightarrow x < -3$$

GRAPH:



INTERVAL  $(-\infty, -3)$