Math 221- Fall 2014 - Test 4

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. You **can not** use a graph to justify your answer. Each exercise is worth 10 points.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the most general antiderivative of the function $f(x) = \cos(2x) - 2e^{5x}$, then the particular antiderivative $F(x)$ that satisfies the condition $F(0) = -2$.

$\int f(kx)\,dx = \frac{1}{k}\int f(t)\,dt$ AT $t = kx$.

$\int \cos(2x) - 2e^{5x} = \frac{1}{2}\sin(2x) - 2\,\frac{1}{5}e^{5x} + C$ GEN. ANTIDERIVATIVE

PLUG CONDITION:

$-2 = F(0) = \frac{1}{2}\sin(0) - \frac{2}{5}e^{0} + C \Rightarrow C = -2 + \frac{2}{5} = -\frac{8}{5}$

PARTICULAR ANTIDERIVATIVE: $F(x) = \frac{1}{2}\sin(2x) - \frac{2}{5}e^{5x} - \frac{8}{5}$

2. Check if the function $y = 2x^3 + \cos(4x)$ is a solution of the second order ODE $x^2y'' - xy' - 3y = -3\cos(4x) + 4x\sin(4x) - 16x^2\cos(4x)$.

$y' = 6x^2 - 4\sin(4x)$; $y'' = 12x - 16\cos(4x)$

L.H.S. $= x^2(12x - 16\cos(4x)) - x(6x^2 - 4\sin(4x)) - 3(2x^3 + \cos(4x))$

$= 12x^3 - 16x^2\cos(4x) - 6x^3 + 4x\sin(4x) - 6x^3 - 3\cos(4x)$

$= -16x^2\cos(4x) + 4x\sin(4x) - 3\cos(4x) =$ R.H.S ✓

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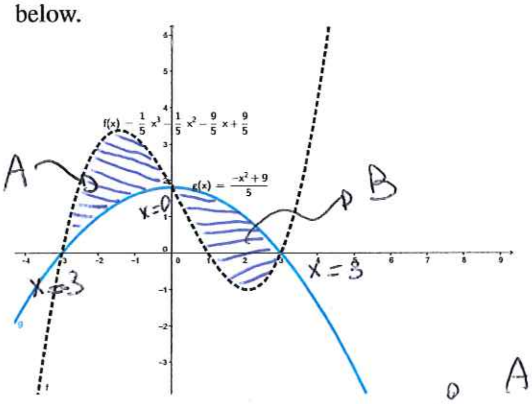
3. Compute the area of the region enclosed by the graphs of $f(x) = \frac{1}{5}(x^3 - x^2 - 9x + 9)$ and $g(x) = \frac{1}{5}(9 - x^2)$ below.



$f(x) = g(x) \Rightarrow$

$\Rightarrow x^3 - x^2 - 9x + 9 = 9 - x^2$

$\Leftrightarrow x^3 - 9x = 0 \Leftrightarrow x(x^2 - 9) = 0$

$\Leftrightarrow x = 0, \pm 3$

AREA $= A + B$, on A $f > g$

on B $g > f$ $\Rightarrow$ $A = \int_{-3}^{0} f - g \, dx$ AND $B = \int_{0}^{3} g - f \, dx = \int_{3}^{0} f - g \, dx$

$\int f - g \, dx = \int \frac{1}{5}(x^3 - 9x) \, dx = \frac{1}{5} \int x^3 - 9x \, dx = \frac{1}{5}\left(\frac{x^4}{4} - 9\frac{x^2}{2}\right) + C$

$A = \frac{1}{5}\left[\frac{1}{2}\left(\frac{x^4}{2} - 9x^2\right)\right]_{-3}^{0} = \frac{1}{10}\left(0 - (-3)^2\left(\frac{(-3)^2}{2} - 9\right)\right) = \frac{81}{20}$

$B = \frac{1}{5}\left[\frac{1}{2} x^2 \left(\frac{x^2}{2} - 9\right)\right]_{3}^{0} = A = \frac{81}{20}$

Even function

AREA $= 2 \cdot \frac{81}{20} = \frac{81}{10} = 8{,}1$

4. $\int_{-2}^{3} x - \sin(2x) \, dx = \left[\frac{x^2}{2} + \frac{1}{2}\cos(2x)\right]_{-2}^{3} = \frac{1}{2}\left(3^2 + \cos 6 - (-2)^2 \cos(-4)\right)$

$\int f(kx) \, dx = \frac{1}{k} \int f(t) \, dt$ AT $t = kx$

$= \frac{1}{2}(5 + \cos 6 - \cos(-4)) \approx 3{,}3069$

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5. A rocket is lunched from a vertical position while at rest at the sea level. Immediately after lunch the rocket's acceleration remains proportional to the seconds after it is lunched, so that after 10 seconds its acceleration is 560 ft^2 per second. Ten seconds after lunch the rocket is only subject to the gravitational force and it starts falling as a free object.

   (a) Express the hight of the rocket as a function of the time (in seconds).

   (b) How long does it take the rocket to hit the ground?

(a) $\boxed{0 \le t \le 10}$ DENOTE HIGHT $s = s(t)$, $v = s'(t)$, $a = v'(t) = s''(t)$

$t=0$: $s_0 = s(0) = 0$, $v_0 = v(0) = 0$ (AT REST)

$a = k \cdot t$ (PROPORTIONAL) WITH $560 = k \cdot 10 \Rightarrow k = \frac{560}{10} = 56.$

$a = 56t \Rightarrow v = 56\frac{t^2}{2} + v_0 = 28t^2 \Rightarrow s = 28\frac{t^3}{3} + s_0$

$\Rightarrow \boxed{s = \frac{28}{3}t^3}$

$\boxed{t \ge 10}$: $a = -32 \Rightarrow v = -32t + C \Rightarrow 28 \cdot 10^2 = -32 \cdot 10 + C$

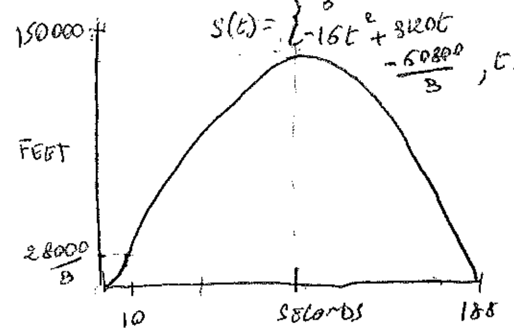
$\Rightarrow C = 2800 + 320 = 3120 \Rightarrow v = -32t + 3120 \Rightarrow$

$\Rightarrow s = -32\frac{t^2}{2} + 3120t + C \Rightarrow \frac{28}{3}10^3 = -16 \cdot 10^2 + 3120 \cdot 10 + C$

$\Rightarrow C = \frac{28000}{3} + 1600 - 31200 = -20266\frac{2}{3} = -\frac{60800}{3}$

$\Rightarrow \boxed{s = -16t^2 + 3120t - \frac{60800}{3}}$

$s(t) = \begin{cases} \frac{28}{3}t^3, & 0 \le t \le 10 \\ -16t^2 + 3120t - \frac{60800}{3}, & t > 10 \end{cases}$



(b)

$\Rightarrow -16t^2 + 3120t - \frac{60800}{3} = 0$

$\Rightarrow 48t^2 - 9360t + 60800 = 0$

$t = \frac{9360 \pm \sqrt{75936000}}{96} \approx 188.27$ SECONDS

REJECT (−) $t < 10$.

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