Math 102 - Spring 2010 - Test 3

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Instructions. Only calculators are allowed on this examination. **Each problem is 10 points worth, unless otherwise specified.**

Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulas:

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left[(1+i)^n - 1\right], \quad A = \frac{R}{i}\left[1-(1+i)^{-n}\right].$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Use a calculator to find the following numbers.

   (a) $\log_5 72 =$ $\frac{\log 72}{\log 5} \approx 2.657$

   (b) $\log_{0.1} 2.4 =$ $\frac{\log 2.4}{\log 0.1} \approx -.38$

   (c) $5^{6/7} \approx 3.973$

2. Rewrite the following expression as a single logarithm:

   $\log_3(1-x) + 4\log_3 x$

   $= \log_3(1-x) + \log_3 x^4$

   $= \log_3\left((1-x)\cdot x^4\right)$

20

3. (15 points) Suppose the weekly cost for the production of x units of a product is given by

$$C(x) = 3452 + 50\ln(x+1) \text{ dollars.}$$

Estimate the number of units produced if the weekly cost is \$ 3556.

$C = 3556$ (USE GRAPHIC METHODS $\begin{cases} Y_1 = C(x) \\ Y_2 = 3556 \end{cases}$ OR ALGEBRA)

$3556 = 3452 + 50\ln(x+1)$ $\longrightarrow$ $\frac{104}{50} = \frac{50\ln(x+1)}{50}$ $\longrightarrow$

$\longrightarrow 2.08 = \ln(x+1)$ $\longrightarrow$ EXPONENTIATE BOTH SIDES, BASE e:

$e^{2.08} = e^{\ln(x+1)} = x+1 \longrightarrow x = e^{2.08} - 1 \approx 7.005$

ABOUT 7 UNITS WILL COST \$3556.

4. Find the future value of \$60,000 invested for 8 years at 7%, compounded annually.

THIS IS A SIMPLE INVESTMENT, FUTURE VALUE:

$S = P\left(1 + \frac{r}{k}\right)^{kt}$

$P = 60000,\ r = .07,\ t = 8,\ k = 1$ $\longrightarrow$

$\longrightarrow S = 60000\left(1 + \frac{.07}{1}\right)^{1\cdot 8} = \$103{,}091.17$

25

5. The composite SAT scores for entering freshman at GSU are shown in the table below.

Year	1995	1996	1997	1998	1999	2000
SAT score	943	967	973	983	987	1008
Year	2001	2002	2003	2004	2005	2006
SAT score	1028	1052	1056	1080	1098	1104

(a) Using your calculator, find the exponential model which is the best fit for the data. Consider x to be the number of years past 1995 and y to be the SAT score. Report your answer to 3 decimal places.

$Y = 943.055 \cdot (1.015)^X$

NOTE: $X = 0$ IN 1995 AND $X = 11$ IN 2006

(b) Use this model to predict the SAT score for this school in 2010.

IN 2010 WE HAVE $X = 15$.

$Y(15) = 1179$

(c) According to this model, during what year will the SAT score be above 1300?

USING THE TABLE ON TI-84, THE SAT SCORE IS 1300

FOR $21 < X < 22$, THAT IS DURING YEAR $1995 + 21 = 2016$

ALGEBRA: $Y = 1300 \rightarrow \frac{1300}{943.055} = \frac{943.055 (1.015)^X}{943.055} \rightarrow$

$\frac{1300}{943.055} = 1.015^X \rightarrow X = \log_{1.015}\left(\frac{1300}{943.055}\right) \approx 21.56$

10

6. (15 points) Find the future value in 18 years of an investment of \$3,000 at a 6.5% annual interest rate in the following cases.

(a) Interest continuously compounded.

FUTURE VALUE OF A STANDARD INVESTMENT CONTINUOUSLY COMPOUNDED: $S = Pe^{rt}$, WHERE $P = 3000$, $r = .065$, $t = 18$.

HERE $S = 3000\, e^{.065 \cdot 18} \approx \$9665.98$

(b) Interest compounded monthly.

IN THIS PART AND PART (C) WE COMPUTE THE FUTURE VALUE OF A STANDARD INVESTMENT: $S = P\left(1 + \frac{r}{k}\right)^{kt}$ FOR $P = 3000$, $r = .065$, $t = 18$

HERE $k = 12 \rightarrow S = 3000\left(1 + \frac{.065}{12}\right)^{12 \cdot 18} \approx \$9635.51$

(c) Interest compounded semiannually.

HERE $k = 2$: $S = 3000\left(1 + \frac{.065}{2}\right)^{2 \cdot 18} \approx \$9487.75$

13

7. (15 points) A couple wants to establish a fund that will provide $3000 for tuition at the end of each 6-month period for 4 years. If a lump sum can be placed in an account that pays 8% compounded semiannually, what lump sum is required?

"LUMP SUM REQUIRED" IS PRESENT VALUE OF AN ANNUITY:

$$A = \frac{R}{i}\left(1-(1+i)^{-n}\right), \quad R=3000,\ r=.08,\ k=2 \text{ (6 months)}$$

$$t=4 \rightarrow i=\frac{r}{k}=\frac{.08}{2}=.04,\quad n=k\cdot t=2\cdot 4=8$$

$$A=\frac{3000}{.04}\left(1-(1+.04)^{-8}\right)\approx 20198.23 \text{ DOLLARS}$$

8. (15 points) MrLawrence invests $600 at the end of each month in an account that pays 7% compounded monthly. How much will be in the account in 25 years?

"HOW MUCH WILL BE...?" FUTURE VALUE OF A STANDARD ANNUITY:

$$S=\frac{R}{i}\left((1+i)^{n}-1\right), \quad R=600,\ r=.07,\ k=12 \text{ (EACH MONTH)}$$

$$t=25 \rightarrow i=\frac{.07}{12},\quad n=12\cdot 25=300$$

$$S=\frac{600}{.07/12}\left(\left(1+\frac{.07}{12}\right)^{300}-1\right)\approx \$\,486,043.02$$

39