MAT 320 – Spring 2019 – Exam4

Instructor: Dr. Francesco Strazzullo **Name** MEX

I certify that I did not receive third party help in *completing* this test (sign) ____________________

Instructions. Technology is allowed on this exam. Each problem is worth 10 points. If you use formulas or properties from our book, make a reference. You are expected to use a CAS for some computations, then upload your files in Eagleweb.

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1) Let $\mathcal{C} = \left\{C_1 = \begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix}, C_2 = \begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}, C_3 = \begin{bmatrix}0 & 2\\ 0 & 2\end{bmatrix}, C_4 = \begin{bmatrix}0 & 1\\ 0 & 2\end{bmatrix}\right\}$ be a subset of the real vector space of the 2-by-2 matrices $M_{2,2}$.

(a) Prove that $\mathcal{C}$ is a basis of $M_{2,2}$.

(b) Given $A = \begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$, compute the coordinate vector $[A]_{\mathcal{C}}$.

IF $\mathcal{E} = \left\{E_{ij} = [\delta_{hk}(i,j)],\ \delta_{hk}(i,j) = \begin{cases}1, & (i,j)=(h,k)\\ 0 & \text{OTHERWISE}\end{cases}\right\}$ STANDARD BASIS.

(a) $C = \left[[C_1]_{\mathcal{E}}\ [C_2]_{\mathcal{E}}\ [C_3]_{\mathcal{E}}\ [C_4]_{\mathcal{E}}\right] = \begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 1 & 2 & 1\\ 1 & -1 & 0 & 0\\ -1 & 0 & 2 & 2\end{bmatrix} \xrightarrow{\text{RREF}} I_4 \Rightarrow$

THEN $\mathcal{C}$ IS A MAXIMAL SET OF LIN. IND. VECTORS $\Rightarrow$ BASIS.

(b) $[A]_{\mathcal{C}}$ IS THE SOLUTION OF THE SYSTEM $C\vec{x} = [A]_{\mathcal{E}} = \begin{bmatrix}2\\1\\1\\1\end{bmatrix}$

$\text{RREF}([C\ [A]_{\mathcal{E}}]) = \left[\begin{array}{c|c} I_4 & \begin{matrix}3/2\\ 1/2\\ -3/4\\ 2\end{matrix}\end{array}\right] \Rightarrow [A]_{\mathcal{C}} = \begin{bmatrix}3/2\\ 1/2\\ -3/4\\ 2\end{bmatrix}$

2) Consider $\mathcal{C} = \{p_1 = 1 - x^2,\ p_2 = 3 + x, p_3 = x + x^3, p_4 = 2 + x^3\}$ in P_3 , the real vector space of polynomials of degree at most 3.

(a) Prove that $\mathcal{C}$ is a basis for P_3.

(b) Compute $[1 - 3x + x^2 - 3x^3]_{\mathcal{C}}$.

(a) USE $\mathcal{E}_3 = \{1, x, x^2, x^3\}$ AND $C = \left[[\vec{p}_i]_{\mathcal{E}} \right] = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ THEN

$RREF(C) = I_4 \Rightarrow$ MAX. SET OF IND. VECTORS $\Rightarrow$ BASIS.

(b) $[1-3x+x^2-3x^3]_{\mathcal{C}}$ IS THE SOLUTION OF $C\vec{x} = [1-3x+x^2-3x^3]_{\mathcal{E}_3} = \begin{bmatrix} 1 \\ -3 \\ 1 \\ -3 \end{bmatrix}$

$RREF\left(\left[C \,\middle|\, \begin{matrix} 1 \\ -3 \\ 1 \\ -3 \end{matrix} \right] \right) = \left[I_4 \,\middle|\, \begin{matrix} -1 \\ 2/5 \\ -17/5 \\ 2/5 \end{matrix} \right] \Rightarrow [1-3x+x^2-3x^3]_{\mathcal{C}} = \begin{bmatrix} -1 \\ 2/5 \\ -17/5 \\ 2/5 \end{bmatrix}$

3) Consider two bases $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$ of $\mathbb{R}^3$.

(a) Find $P_{\mathcal{C}}^{\mathcal{B}}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

(b) Compute $[x]_{\mathcal{B}}$ for $x = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

(c) Use part (a) and (b) to compute $[x]_{\mathcal{C}}$.

$[\vec{x}]_{\mathcal{E}} = P_{\mathcal{E}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}}$ AND $[\vec{x}]_{\mathcal{C}} = P_{\mathcal{C}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}} \Rightarrow$

$P_{\mathcal{C}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}} = (P_{\mathcal{E}}^{\mathcal{C}})^{-1} [\vec{x}]_{\mathcal{E}} = (P_{\mathcal{E}}^{\mathcal{C}})^{-1} P_{\mathcal{E}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}} \Rightarrow$

$\Rightarrow P_{\mathcal{C}}^{\mathcal{B}} = (P_{\mathcal{E}}^{\mathcal{C}})^{-1} P_{\mathcal{E}}^{\mathcal{B}}$ (OR $P_{\mathcal{C}}^{\mathcal{E}} P_{\mathcal{E}}^{\mathcal{B}}$)

(a) $P_{\mathcal{E}}^{\mathcal{B}} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $P_{\mathcal{E}}^{\mathcal{C}} = \begin{bmatrix} 0 & 2 & 0 \\ -1 & -1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \Rightarrow P_{\mathcal{C}}^{\mathcal{B}} = \frac{1}{6} \begin{bmatrix} -8 & -1 & 2 \\ 6 & 3 & 0 \\ 2 & 1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}_{\mathcal{B}} = (P_{\mathcal{E}}^{\mathcal{B}})^{-1} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ -2 \\ -.5 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}_{\mathcal{C}} = P_{\mathcal{C}}^{\mathcal{B}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}_{\mathcal{B}} = \frac{1}{6} \begin{bmatrix} -11 \\ 3 \\ -1 \end{bmatrix}$

4) Consider two bases $\mathcal{B} = \{\boldsymbol{b}_1, \boldsymbol{b}_2\}$ and $\mathcal{C} = \{\boldsymbol{c}_1, \boldsymbol{c}_2\}$ of a real vector space $\mathbb{V}$ such that

$$\boldsymbol{b}_1 = \boldsymbol{c}_1 + 3\boldsymbol{c}_2 \text{ and } \boldsymbol{b}_2 = -2\boldsymbol{c}_1 + 5\boldsymbol{c}_2.$$

Suppose that $\boldsymbol{x}$ is a vector in $\mathbb{V}$ such that $\boldsymbol{x} = 2\boldsymbol{b}_1 - 3\boldsymbol{b}_2$, that is $[\boldsymbol{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

(a) Find $P_{\mathcal{C}}^{\mathcal{B}}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

(b) Compute $[\boldsymbol{x}]_{\mathcal{C}}$.

(a) $P_{\mathcal{C}}^{\mathcal{B}} = \left[[\vec{b}_1]_{\mathcal{C}} \; [\vec{b}_2]_{\mathcal{C}} \right] = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$

(b) $[\vec{x}]_{\mathcal{C}} = P_{\mathcal{C}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -9 \end{bmatrix}$

5) Let $L\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear mapping defined by

$$L(\boldsymbol{b}_1) = 3\boldsymbol{b}_1 + \boldsymbol{b}_3, \qquad L(\boldsymbol{b}_2) = \boldsymbol{b}_1 + 3\boldsymbol{b}_2 - 2\boldsymbol{b}_3, \text{ and } L(\boldsymbol{b}_3) = \boldsymbol{b}_1 - \boldsymbol{b}_2 - \boldsymbol{b}_3,$$

where $\mathcal{B} = \{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$ is a basis of $\mathbb{R}^3$. Find $[L]_{\mathcal{B}}$, the matrix of the linear operator L relative to $\mathcal{B}$.

$[L]_{\mathcal{B}} = \left[[L(\vec{b}_1)]_{\mathcal{B}} \cdots [L(\vec{b}_n)]_{\mathcal{B}} \right] = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 3 & -1 \\ 1 & -2 & -1 \end{bmatrix}$

6) Let $L\colon P_2 \longrightarrow P_2$ be the linear mapping defined by

$$L(1-2x) = 1+x^2, \qquad L(1+x+x^2) = 1+x\,, \text{and } L(x+x^2) = 1-x+2x^2\,,$$

where $\mathcal{B} = \{\mathbf{p}_1 = 1-2x, \mathbf{p}_2 = 1+x+x^2, \mathbf{p}_3 = x+x^2\}$ is a basis of P_2, the real vector space of polynomials of degree at most 2. Let $\mathcal{E} = \{1, x, x^2\}$ be the standard basis of P_2.

(HONOR: consider $L\colon P_3 \longrightarrow P_3$ with $L(1+x^3) = 2-x$, $\mathcal{B} = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4 = 1+x^3\}$, and $\mathcal{E} = \{1, x, x^2, x^3\}$, over P_3, the real vector space of polynomials of degree at most 3.)

(a) Find $P_{\mathcal{E}}^{\mathcal{B}}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{E}$.

(b) Find $[L]_{\mathcal{E}}^{\mathcal{B}}$, the matrix of the linear operator L relative to $\mathcal{B}$ and $\mathcal{E}$.

(c) Find $[L]_{\mathcal{B}}$, the matrix of the linear operator L relative to $\mathcal{B}$.

(d) Find $L(2+x)$.

(a) $P_{\mathcal{E}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (HONOR $P_{\mathcal{E}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ -2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$)

(b) $[L]_{\mathcal{E}}^{\mathcal{B}} = \left[[L(\vec{b}_1)]_{\mathcal{E}} \cdots [L(\vec{b}_n)]_{\mathcal{E}} \right] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ (HONOR $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$)

(c) $[L]_{\mathcal{B}} = \left(P_{\mathcal{E}}^{\mathcal{B}}\right)^{-1} [L]_{\mathcal{E}}^{\mathcal{B}} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 3 \\ 1 & 3 & -1 \\ 1 & -3 & 5 \end{bmatrix}$ (HONOR $\frac{1}{2}\begin{bmatrix} 1 & -1 & 3 & 1 \\ 1 & 3 & -1 & 3 \\ 1 & -3 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$)

(d) $[L(2+x)]_{\mathcal{E}} = [L]_{\mathcal{E}}^{\mathcal{B}} [2+x]_{\mathcal{B}} = [L]_{\mathcal{E}}^{\mathcal{B}} \left(P_{\mathcal{E}}^{\mathcal{B}}\right)^{-1} [2+x]_{\mathcal{E}}$

$= \frac{1}{2}\begin{bmatrix} 0 & -1 & 3 \\ 4 & 2 & -4 \\ -4 & -3 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 \\ 10 \\ -11 \end{bmatrix} \Rightarrow L(2+x) = -\frac{1}{2} + 5x - \frac{11}{2}x^2$

(HONOR: $[L(2+x)]_{\mathcal{E}} = \frac{1}{2}\begin{bmatrix} 0 & -1 & 3 & 4 \\ 4 & 2 & -4 & -6 \\ -4 & -3 & 7 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 \\ 10 \\ -11 \end{bmatrix} \Rightarrow$)

7) Let $L: P_4 \longrightarrow \mathbb{R}^3$ be the linear mapping defined by

$L(1) = (1,-1,1)^T$, $L(x) = (1,1,2)^T$, $L(x^2) = (-1,3,0)^T$, $L(x^3) = (1,-1,2)^T$, and $L(x^4) = (-1,5,2)^T$.

(a) Find a basis for Ker(L).

(b) Find a basis for Range(L).

(c) Use part (a) and (b) to check the Rank-Nullity Theorem.

(d) Specify why L is or is not injective or surjective.

IN THE STANDARD BASES $\mathcal{E}_4 = \{1, x, x^2, x^3, x^4\}$ AND $\mathcal{E}^3 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$

$$[L] = [L]_{\mathcal{E}^3}^{\mathcal{E}^4} = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & 3 & -1 & 5 \\ 1 & 2 & 0 & 2 & 2 \end{bmatrix} \xrightarrow{\text{RREF}} \underbrace{\begin{bmatrix} 1 & 0 & -2 & 0 & -4 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}}_{\text{PIVOTS}}$$

(a) Ker(L) IS GENERATED BY VECTORS WHOSE COMPONENTS IN $\mathcal{E}_4$ ARE A BASIS OF $Null([L]) = \text{SPAN}\left(\begin{bmatrix} 2\\-1\\1\\0\\0 \end{bmatrix}, \begin{bmatrix} 4\\-2\\0\\-1\\1 \end{bmatrix} \right) \Rightarrow$

$\Rightarrow \text{Ker}(L) = \text{SPAN}(2 - x + x^2,\ 4 - 2x - x^2 + x^3)$

(b) RANGE(L) IS GENERATED BY VECT. WHOSE COMPONENTS IN $\mathcal{E}^3$ ARE A BASIS OF $\text{COL}([L]) = \text{SPAN}\left(\begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}1\\-1\\2\end{bmatrix} \right) = \mathbb{R}^3$

(e) RANK = dim(RANGE(L)) = 3 ; NULLITY = dim(Ker(L)) = 2 ;

TH: RANK + NULLITY = dim (DOMAIN L) = dim(P_4) = 5 ✓

(d) • L IS NOT INJECTIVE BECAUSE $\text{Ker}(L) \neq \{\vec{0}\}$

• L IS SURJECTIVE BECAUSE RANGE(L) = CODOMAIN = $\mathbb{R}^3$.

MAT 320 – Spring 2019 – Exam 4 – In Class

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SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

8) Let $L: P_1 \longrightarrow P_1$ be the linear mapping defined by

$$L(1+3x) = 1 + x, \qquad \text{and } L(2-x) = 3 + 3x,$$

where $\mathcal{B} = \{\boldsymbol{p}_1 = 1 + 3x, \boldsymbol{p}_2 = 2 - x\}$ is a basis of P_1, the real vector space of polynomials of degree at most 1. Let $\mathcal{E} = \{1, x\}$ be the standard basis of P_1.

(HONOR: consider $L: P_2 \longrightarrow P_2$ with $L(x + x^2) = x^2$, $\mathcal{B} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3 = x + x^2\}$, and $\mathcal{E} = \{1, x, x^2\}$, over P_2, the real vector space of polynomials of degree at most 2.)

(a) Find $P_{\mathcal{E}}^{\mathcal{B}}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{E}$.

(b) Find $[L]_{\mathcal{E}}^{\mathcal{B}}$, the matrix of the linear operator L relative to $\mathcal{B}$ and $\mathcal{E}$.

(c) Find $[L]_{\mathcal{B}}$, the matrix of the linear operator L relative to $\mathcal{B}$.

(d) Find $L(2 + x)$.

a) $\mathcal{B} = P_{\mathcal{E}}^{\mathcal{B}} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ (HONOR: $P_{\mathcal{E}}^{\mathcal{B}} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$)

b) $[L]_{\mathcal{E}}^{\mathcal{B}} = [[L(\mathcal{B})]_{\mathcal{E}}] = [[L(\vec{b}_1)]_{\mathcal{E}} \cdots [L(\vec{b}_n)]_{\mathcal{E}}] = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$ (HONOR $[L]_{\mathcal{E}}^{\mathcal{B}} = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$)

c) $[L(\vec{x})]_{\mathcal{B}} = [L]_{\mathcal{B}} [\vec{x}]_{\mathcal{B}}$ THEN $[L]_{\mathcal{B}} = (P_{\mathcal{E}}^{\mathcal{B}})^{-1} [L]_{\mathcal{E}}^{\mathcal{B}} =$

$= \frac{1}{7}\begin{bmatrix} 3 & 9 \\ 2 & 6 \end{bmatrix}$ (HONOR: $[L]_{\mathcal{B}} = \frac{1}{7}\begin{bmatrix} 3 & 9 & -2 \\ 2 & 6 & 1 \\ 0 & 0 & 7 \end{bmatrix}$)

d) $[\vec{x}]_{\mathcal{E}} = P_{\mathcal{E}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}} \Rightarrow [\vec{x}]_{\mathcal{B}} = (P_{\mathcal{E}}^{\mathcal{B}})^{-1} [\vec{x}]_{\mathcal{E}} \Rightarrow [2+x]_{\mathcal{B}} = (P_{\mathcal{E}}^{\mathcal{B}})^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$

$= \frac{1}{7}\begin{bmatrix} 4 \\ 5 \end{bmatrix} \Rightarrow [L(2+x)]_{\mathcal{E}} = [L]_{\mathcal{E}}^{\mathcal{B}} [\vec{x}]_{\mathcal{B}} = \frac{1}{7}\begin{bmatrix} 19 \\ 19 \end{bmatrix} \Rightarrow L(2+x) = \frac{19}{7} + \frac{19}{7}x$

(HONOR: $[2+x]_{\mathcal{B}} = \frac{1}{7}\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} \Rightarrow [L(2+x)]_{\mathcal{E}} = \frac{1}{7}\begin{bmatrix} 19 \\ 19 \\ 0 \end{bmatrix} \Rightarrow L(2+x) = \frac{19}{7} + \frac{19}{7}x$)

9) Let $L\colon P_2 \longrightarrow \mathbb{R}^2$ be the linear mapping defined by

$$L(1) = (1,-2)^T, \qquad L(x) = (-1,2)^T, \qquad \text{and } L(x^2) = (2,3)^T.$$

(a) Find a basis for Ker(L).

(b) Find a basis for Range(L).

(c) Use part (a) and (b) to check the Rank-Nullity Theorem.

(d) Specify why L is or is not injective or surjective.

IN THE STANDARD BASES $\mathcal{E}_2 = \{1, x, x^2\}$ AND $\mathcal{E}^2 = \left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$

THE MATRIX ASSOCIATED TO L IS $[L] = [L]^{\mathcal{E}_2}_{\mathcal{E}^2} = [L(\mathcal{E}_2)]_{\mathcal{E}^2} =$

$= \begin{bmatrix} L(1) & L(x) & L(x^2) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 2 & 3 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a) A BASIS OF $Ker(L)$ HAS VECTORS OF COMPONENTS W.R.T. $\mathcal{E}_2$ GIVEN BY A BASIS OF $Null([L]) \overset{RREF}{=} SPAN\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) \Rightarrow Ker(L) = SPAN(\vec{p})$

WHERE $[\vec{p}]_{\mathcal{E}_2} = \begin{bmatrix}1\\1\\0\end{bmatrix} \Rightarrow \vec{p} = 1 + x$. (BASIS OF $Ker(L)$)

(b) A BASIS OF RANGE(L) HAS VECT. OF COMP. W.RT $\mathcal{E}^2$ GIVEN BY A BASIS OF $COL([L]) \overset{RREF}{=} SPAN\left(\begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix}\right) \Rightarrow RANGE(L) =$

$= SPAN\left(\begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix}\right)$

(c) $RANK(L) = \dim(RANGE(L)) = 2$; $NULLITY(L) = \dim(Ker(L)) = 1$.

TH: RANK + NULLITY = "DIMENSION OF DOMAIN"

$2 + 1 = 3 = \dim P_2$ ✓

(d) • L IS NOT INJECTIVE, BECAUSE $Ker(L) \neq \{\vec{0}\}$.

• L IS SURJECTIVE, BECAUSE $RANGE(L) = \mathbb{R}^2$ CODOMAIN.