MAT 102 - Exam4 - Fall 2014

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Instructions. Each problem is worth 10 points. If you solve a problem graphically then draw the graph you used. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded at least to two decimal places, unless otherwise specified. You might need some of the following formulas:

$\log_a(MN) = \log_a(M) + \log_a(N)$; $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$; $A = P\left(1+\frac{r}{n}\right)^{nt}$; $A = Pe^{rt}$; $R = \log\left(\frac{I}{I_0}\right)$; $D = 10\log\left(\frac{I}{I_0}\right)$; and $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Evaluate the following expressions rounding to four decimal places.

(a) $\log_{1.4}\pi = \log(\pi)/\log(1.4) = 3.4022$

(b) $(2.3)(3.4)^{-\pi/2} = .3364$

2. Adam can invest \$3700 in a mutual fund for 4 years. His broker finds two plans, one offering an annual interest rate of 3.5% compounded annually (Plan 1), and another offering an annual interest rate of 3.48% compounded weekly (Plan 2). Which plan is more profitable? (Round to two decimal places.)

BOTH USE SIMPLE INVESTMENT FORMULA: $A = P\left(1+\frac{r}{n}\right)^{nt}$ WITH $t=4$.

PLAN 1: $A = 3700\left(1+\frac{.035}{1}\right)^{1\cdot 4} = \$4245.84$

ANNUALLY → $n = 1$, $r = \frac{3.5}{100} = 0.035$

PLAN 2: WEEKLY → $n = 52$; $r = \frac{3.48}{100} = .0348$

$A = 3700\left(1+\frac{.0348}{52}\right)^{52\cdot 4} = \$4252.41$

PLAN 2 IS MORE PROFITABLE.

3. Solve the following exponential equation. If you don't write your answers as an exact expression then round it to four decimal places.

$$2^{5+4x} = 210$$

$\log\left(2^{5+4x}\right) = \log(210) \Rightarrow (5+4x)\cdot\log 2 = \log 210 \Rightarrow$

$\Rightarrow 5+4x = \dfrac{\log 210}{\log 2} \Rightarrow 4x = \dfrac{\log 210}{\log 2} - 5 \Rightarrow$

$\Rightarrow x = \dfrac{\log 210 - 5\log 2}{4\log 2} = \dfrac{\log 210 - \log 2^5}{4\log 2} = \dfrac{1}{4}\,\dfrac{\log \frac{210}{32}}{\log 2} \Rightarrow$

$\Rightarrow x = \frac{1}{4}\log_2 \frac{105}{16} \approx .6786$

4. Solve the following logarithmic equation. If you don't write your answers as an exact expression then round it off to four decimal places. **Check for extraneous solutions.**

$$\log_5(2x-1) + \log_5(x+4) = 3$$

$\log_5\left((2x-1)(x+4)\right) = 3 \Rightarrow (2x-1)(x+4) = 5^3 \Rightarrow$

$\Rightarrow 2x^2 + 7x - 4 - 125 = 0 \Rightarrow 2x^2 + 7x - 129 = 0$

$\Rightarrow x = \dfrac{-7 \pm \sqrt{49 + 8\cdot 129}}{4} = \dfrac{-7 \pm \sqrt{3433}}{4} \approx \begin{cases} 12.898 \\ -16.398 \end{cases}$

CHECK:

- PLUG $x \approx 12.898 \to$ LOGS ARE DEFINED ✓
- PLUG $x \approx -16.398 \to \log_5(2(-16.398)-1)$ IS NOT DEFINED $\to$ REJECT.

5. Brandon, Tayler, and Ryan have dinner together at a fast food. They all order the same items, chips (in 1 oz. bags), hotdogs, and cookies, the sodas were free, but they place different orders and each pays his own check. They respectively order: 2 chips, 3 hotdogs, and 2 cookies; 1 bag of chips, 2 hotdogs, and 1 cookies; and 3 chips, 3 hotdogs, and 3 cookies. Before taxes and gratuity, their checks were in the following amounts: \$11.7 for Brandon, \$7.1 for Tayler, and \$13.8 for Ryan. Set up a system of linear equations modeling the prices of the items purchased then find each price.

PRICES	X	Y	Z	
ORDERS	CHIPS	HOTD.	COOKIES	TOT \$
BRANDON	2	3	2	11.7
TAYLER	1	2	1	7.1
RYAN	3	3	3	13.8

$$\begin{cases} 2X + 3Y + 2Z = 11.7 \\ X + 2Y + Z = 7.1 \\ 3X + 3Y + 3Z = 13.8 \end{cases}$$

AUGMENTED MATRIX $= \left[\begin{array}{ccc|c} 2 & 3 & 2 & 11.7 \\ 1 & 2 & 1 & 7.1 \\ 3 & 3 & 3 & 13.8 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2.1 \\ 0 & 1 & 0 & 2.5 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$

$\rightarrow \begin{cases} X + Z = 2.1 \\ Y = 2.5 \end{cases}$

$X + Z = 2.1 \Rightarrow$ CHIPS AND COOKIES ARE SOLD TOGETHER @ \$2.10 PER PAIR (1 BAG AND 1 COOKY)

$Y = 2.5 \Rightarrow$ HOTDOGS COST \$2.50 EACH.

6. Write the augmented matrix associated to the following system of linear equations then express the solution as a quadruple (x, y, z, w).

$$\begin{cases} 3x + 2y - z + 2w = 1 \\ 6x - y + 5z - w = -2 \\ x + y - 3w = -1 \end{cases}$$

A. MATRIX $= \left[\begin{array}{cccc|c} 3 & 2 & -1 & 2 & 1 \\ 6 & -1 & 5 & -1 & -2 \\ 1 & 1 & 0 & -3 & -1 \end{array}\right] \xrightarrow{RREF}$

$\xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & -6 & -2 \\ 0 & 0 & 1 & -5 & -2 \end{array}\right] \rightarrow \begin{cases} X + 3W = 1 \rightarrow X = 1 - 3W \\ Y - 6W = -2 \rightarrow Y = 6W - 2 \\ Z - 5W = -2 \rightarrow Z = 5W - 2 \end{cases}$

SOLUTION $(1 - 3W, 6W - 2, 5W - 2, W)$

7. The following are the winning times for the Olympic Men's 110 Meter Hurdles, for selected years:

X	**Year**	**Time (sec)**	X	**Year**	**Time (sec)**
36	1936	14.2	76	1976	13.3
	1948	13.9		1980	13.39
	1952	13.7		1984	13.2
	1956	13.5		1988	12.98
60	1960	13.8	92	1992	13.12
	1964	13.6		1996	12.95
	1968	13.3		2000	13
	1972	13.24	104	2004	12.91

Consider x to be the number of years after 1900, and y to be the winning time. Use technology to answer to the following questions.

(a) What are the quadratic and the exponential models that are the best fit for these data? (Round your answer to five decimal places).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate the winning times in 1980, in 1996, and 2012. Round to the hundredth of second.

(a) QUADRATIC: $Y = 0.00014X^2 - .03794X + 15.36309$, $R^2 = .91754$

EXPO: $Y = 14.746(.99867)^X$, $R^2 = .9001$

(b) QUADRATIC IS BETTER, R^2 IS BIGGER.

(c) $1980 \to X = 80 \to Y = 13.22$ SECONDS

$1996 \to X = 96 \to Y = 13.01$ SECONDS

$2012 \to X = 112 \to Y = 12.87$ SECONDS