Math 221 - Test 4 - Part 1/2 - Fall 2017

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I certify that I did not receive third party help in *completing* this test.   (*sign*)________________

Instructions. You **can not** use a graph to justify your answer. Each exercise is worth 10 points.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the most general antiderivative of the function $f(x) = x\sqrt{4+x^2} + x^3$, then the particular antiderivative $F(x)$ that satisfies the condition $F(3) = -1$.

$$F(x) = \int x\sqrt{4+x^2} + x^3\,dx = \frac{1}{2}\int 2x\sqrt{4+x^2}\,dx + \int x^3\,dx =$$

$$u = 4+x^2$$
$$u' = 2x$$

$$= \frac{1}{2}\int u^{\frac{1}{2}}\,du + \frac{x^4}{4} = \frac{1}{2}\frac{(u)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^4}{4} + C$$

$$= \frac{1}{3}(4+x^2)^{\frac{3}{2}} + \frac{x^4}{4} + C$$ GEN. ANTID.

$$-1 = F(3) = \frac{1}{3}\left(\sqrt{4+3^2}\right)^3 + \frac{3^4}{4} + C = \frac{13}{3}\sqrt{13} + \frac{81}{4} + C \Rightarrow$$

$$\Rightarrow C = -\frac{85}{4} - \frac{13}{3}\sqrt{13} \approx 36.87$$ THEN

$$\boxed{F(x) = \frac{1}{3}(4+x^2)\sqrt{4+x^2} + \frac{x^4}{4} - \frac{85}{4} - \frac{13}{3}\sqrt{13}}$$

2. Solve the second order ODE

$$f''(x) = 2x + 3\cos x,$$

subject to the conditions $f(0) = 0$ and $f\left(\frac{\pi}{4}\right) = 1$.

$$f'(x) = \int f''(x)\,dx = \int 2x + 3\cos x \, dx = x^2 + 3\sin x + C_1$$

$$f(x) = \int f'(x)\,dx = \int x^2 + 3\sin x + C_1 \, dx$$

$$= \frac{x^3}{3} - 3\cos x + C_1 x + C_2$$

$$0 = f(0) = \frac{0^3}{3} - 3\cos(0) + C_1(0) + C_2 = -3 + C_2 \Rightarrow C_2 = 3$$

$$1 = f\left(\frac{\pi}{4}\right) = \frac{1}{3}\left(\frac{\pi}{4}\right)^3 - 3\cos\left(\frac{\pi}{4}\right) + C_1\left(\frac{\pi}{4}\right) + C_2 =$$

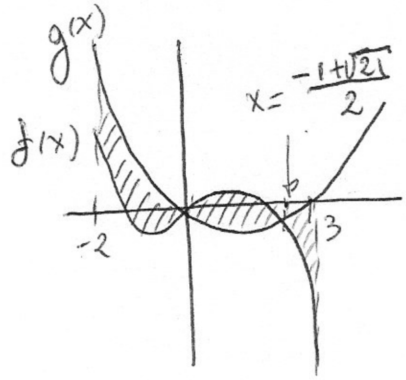
$$= \frac{\pi^3}{192} - 3\frac{\sqrt{2}}{2} + \frac{\pi}{4}C_1 + 3 \Rightarrow C_1 = \frac{4}{\pi}\left(\frac{3}{2}\sqrt{2} - 2 - \frac{\pi^3}{192}\right) \Rightarrow$$

$$\Rightarrow C_1 = \frac{6\sqrt{2}}{\pi} - \frac{8}{\pi} - \frac{\pi^2}{48} \approx -.0511$$

THEN

$$\boxed{f(x) = \frac{x^3}{3} + \left(\frac{6\sqrt{2} - 8}{\pi} - \frac{\pi^2}{48}\right)x - 3\cos x + 3}$$

3. Compute the area of the region enclosed by the graphs of $f(x) = -x^3 + 2x$ and $g(x) = x^2 - 3x$ over the interval $[-2, 3]$.



INTERSECTION POINTS

$$f(x) = g(x)$$

$$f(x) - g(x) = 0$$

$$-x^3 + 2x - (x^2 - 3x) = 0$$

$$-x^3 - x^2 + 5x = 0$$

$$-x(x^2 + x - 5) = 0 \quad \begin{cases} x = 0 \\ x = \dfrac{-1 \pm \sqrt{21}}{2} \approx \begin{matrix} + & 1.8 \\ - & \cancel{-2.8} < -2 \end{matrix} \end{cases}$$

$$\text{AREA} = \int_{-2}^{3} |f-g|\,dx = \int_{-2}^{0} g-f\,dx + \int_{0}^{\frac{-1+\sqrt{21}}{2}} f-g\,dx + \int_{\frac{-1+\sqrt{21}}{2}}^{3} g-f\,dx$$

$$\int g-f\,dx = \int x^3 + x^2 - 5x\,dx = \frac{x^4}{4} + \frac{x^3}{3} - \frac{5}{2}x^2 + C = F(x) + C$$

$$\to \text{AREA} = \Big[F(x)\Big]_{-2}^{0} + \Big[-F(x)\Big]_{0}^{\frac{-1+\sqrt{21}}{2}} + \Big[F(x)\Big]_{\frac{-1+\sqrt{21}}{2}}^{3} =$$

$$= F(0) - F(-2) - F\left(\frac{-1+\sqrt{21}}{2}\right) + F(0) + F(3) - F\left(\frac{-1+\sqrt{21}}{2}\right)$$

$$= F(3) + 2F(0) - 2F\left(\frac{-1+\sqrt{21}}{2}\right) - F(-2) = \frac{122 - 7\sqrt{21}}{4} \approx$$

$$\approx 6.75 + 0 + 7.0638 + 8.6667 \approx 22.4805$$

4. $\int_0^3 x^2 - 2x + 3\,dx = \left[\frac{x^3}{3} - x^2 + 3x\right]_0^3 = \frac{3^3}{3} - 3^2 + 3(3) - 0 = 9$

5. A bacteria population is growing at a rate modeled by $P'(t) = 200e^{.05t}$ individuals per hour, where $P(t)$ is the number of bacteria t hours after the first count. What is the net-change of population between hour 1 and hour 4?

NET-CHANGE $= P(4) - P(1) = \int_1^4 P'(t)\,dt = \int_1^4 200\,e^{.05t}\,dt$

$= \frac{200}{.05}\int_1^4 (.05)e^{.05t}\,dt = 4000\left[e^{.05t}\right]_1^4 =$

$u = .05t \to u' = .05$

$= 4000\left(e^{.2} - e^{.05}\right) \approx 680.53$ TRUNCATE.

≈ 680 BACTERIA.

6. A rocket is launched from a vertical position 20 feet above the sea level, with an initial velocity of 200 ft/sec. Immediately after launch the rocket's acceleration is modeled by $a(t) = 3t^2 + 1$, where t is the time in seconds after the rocket is launched. After 20 seconds the rocket is only subject to the gravitational force and it starts falling as a free object.

(a) Express the height of the rocket as a function of the time (in seconds).

(b) Will the rocket reach the 6,000 feet altitude? If it does, how long does it take?

$0 \le t \le 20$ $\quad h(0) = 20 \ ; \ v(0) = 200 \ ; \ a(t) = 3t^2+1$

$v(t) = \int a(t)\,dt = \int 3t^2+1\,dt = t^3+t+c.$ THEN

$200 = v(0) = 0^3+0+c \Rightarrow c = 200 \Rightarrow v(t) = t^3+t+200.$

$h(t) = \int v(t)\,dt = \int t^3+t+200\,dt = \frac{1}{4}t^4 + \frac{1}{2}t^2 + 200t + c$

THEN $\quad 20 = h(0) = 0 + c \Rightarrow c = 20 \Rightarrow \boxed{h(t) = \frac{t^4}{4} + \frac{t^2}{2} + 200t + 20.}$

$t > 20$ $\quad h(20) = \frac{20^4}{4} + \frac{20^3}{2} + 200(20) + 20 = 44220$ FT. ;

$v(20) = 20^3 + 20 + 200 = 8220$ FT/SEC. ; $\quad a(t) = -32$;

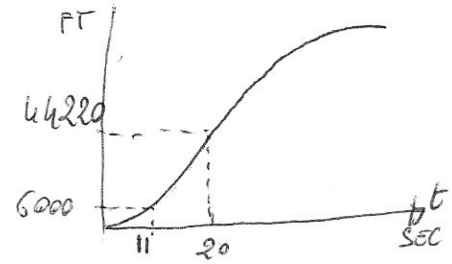
$v(t) = \int -32\,dt = -32t + c \Rightarrow 8220 = v(20) = -32(20) + c \Rightarrow$

$\Rightarrow c = 8860 \Rightarrow v(t) = -32t + 8860 \Rightarrow h(t) = \int v(t)\,dt =$

$= \int -32t + 8860\,dt = -16t^2 + 8860t + c \Rightarrow 44220 = h(20) =$

$= -16(20)^2 + 8860(20) + c \Rightarrow c = -126,580 \Rightarrow$

$\Rightarrow \boxed{h(t) = -16t^2 + 8860t - 126580.}$



(a) $h(t) = \begin{cases} \frac{1}{4}t^4 + \frac{1}{2}t^2 + 200t + 20, & 0 \le t \le 20 \\ -16t^2 + 8860t - 126580, & t > 20 \end{cases}$

(b) $h(t) = 6000$ DURING THE FIRST TIME-FRAME: $\underline{\frac{1}{4}t^4 + \frac{1}{2}t^2 + 200t - 5980 = 0}$

$F(t) = 0$

NEWTON'S METHOD: $t_0 = 10$, $t_{n+1} = t_n - \frac{F(t_n)}{F'(t_n)} \rightarrow t \approx t_4 = 11.0383$

THEN AFTER 11.0383 SECONDS.

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7. $\int_2^4 x^3 - 3x + \frac{1}{x}\,dx = \left[\frac{x^4}{4} - \frac{3}{2}x^2 + \ln|x|\right]_2^4 =$

$$= \frac{4^4}{4} - \frac{3}{2}(4)^2 + \ln 4 - \frac{2^4}{4} + \frac{3}{2}(2)^2 - \ln 2$$

$$= 64 - 24 + \ln 2^2 - 4 + 6 - \ln 2$$

$$= 42 + 2\ln 2 - \ln 2$$

$$= 42 + \ln 2 \approx 42.6931$$