Calculus 3 – Exam2 – Fall 2017 – Take Home

Instructor: Dr. Francesco Strazzullo **Name** KEY

I certify that I did not receive third party help in *completing* this test (sign)________________

Instructions. This is an open book test. Each exercise is worth 10 points. When approximating, use four decimal places. You cannot use a CAS to justify your answers, but only to perform computations.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Evaluate (if possible) or prove undefined the limit

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$$

$$f(x,x) = \frac{x^3}{2x^3} \xrightarrow[x\to 0]{H.R.} \frac{1}{2}$$

$$f(x,2x) = \frac{x(2x)^2}{x^3+(2x)^3} = \frac{4x^3}{9x^3} \xrightarrow[x\to 0]{H.R} \frac{4}{9}$$

DISTINCT $\Rightarrow$

LIMIT IS UNDEFINED

2. Simplify the expression of f_{yx} for the function $f(x,y) = 4xy^3 - e^{2x-3y}$.

$$f_{yx} = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}[f(x,y)]\right]$$

$$f_y = 12xy^2 + 3e^{2x-3y}$$

$$f_{yx} = 12y^2 + 6e^{2x-3y}$$

3. Find the tangent plane to the graph of the function $f(x, y) = y^3 + 2\sin(\pi xy)$ at the point $(2, -1, -1)$.

EXPLICIT FUNCTION, EQ. TAN. PLANE AT $P = (x_0, y_0, z_0)$:

$f_x(x_0, y_0) \cdot (x - x_0) + f_y(x_0, y_0) \cdot (y - y_0) - (z - z_0) = 0$, $P = (2, -1, -1)$

$f_x \big|_{(x_0, y_0)} = 2\pi y \cos(\pi xy) \big|_{(2,-1)} = -2\pi$

$f_y \big|_{(x_0, y_0)} = 3y^2 + 2\pi x \cos(2\pi xy) \big|_{(2,-1)} = 3 + 4\pi$

EQ: $-2\pi(x - 2) + (3 + 4\pi)(y + 1) - (z + 1) = 0$

$-2\pi x + (3 + 4\pi)y - z + 4\pi + 3 + 4\pi - 1 = 0$

$2\pi x - (3 + 4\pi)y + z = 8\pi + 2$

4. Express the (total) differential of $u = xt + \ln(yt^2)$.

$du = u_t\, dt + u_x\, dx + u_y\, dy$

$= \left(x + \frac{2}{t}\right) dt + t\, dx + \frac{1}{y}\, dy$

5. Find the linear approximation of the implicit function $y^3 + 2xy - yz = x^3$ at $(2,2,4)$.

$Z = L(X,Y)$ TAN. PLANE OF $F(X,Y,Z) = 0$ AT $P = (X_0, Y_0, Z_0)$,

$$F_X\big|_P (X - X_0) + F_Y\big|_P (Y - Y_0) + F_Z\big|_P (Z - Z_0) = 0$$

$F(X,Y,Z) = 0$ IS $Y^3 - 2XY - YZ - X^3 = 0$, AT $P = (2,2,4)$.

$F_X\big|_P = 2Y - 3X^2 \big|_{(2,2,4)} = -8$; $F_Z\big|_P = -Y\big|_{(2,2,4)} = -2$

$F_Y\big|_P = 3Y^2 + 2X - Z \big|_{(2,2,4)} = 12$

$$\overset{-4}{\cancel{-8}}(X-2) + \overset{+6}{\cancel{12}}(Y-2) \overset{-1}{\cancel{-2}}(Z-4) = 0$$

$$-4X + 6Y - Z + 8 - 12 + 4 = 0$$

$$\boxed{Z = -4X + 6Y}$$

6. Find the directional derivative of the function $f(x,y,z) = 2xz + yz^2$ at the point $(2,1,-1)$ in the direction of $\langle 1,2,1 \rangle$.

$\vec{u} = \dfrac{\vec{V}}{\|\vec{V}\|}$, THEN $D_{\vec{V}} f\big|_P = \vec{u} \cdot \nabla f\big|_P$.

$\vec{V} = \langle 1,2,1 \rangle \Rightarrow \vec{u} = \dfrac{1}{\sqrt{6}} \langle 1,2,1 \rangle$

$\nabla f\big|_P = \langle f_x, f_y, f_z \rangle\big|_P = \langle 2Z, Z^2, 2X + 2YZ \rangle\big|_{(2,1,-1)} = \langle -2, 1, 2 \rangle$ $\Rightarrow$

$$\Rightarrow D_{\vec{V}} f\big|_P = \frac{1}{\sqrt{6}} \langle 1,2,1 \rangle \cdot \langle -2,1,2 \rangle = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$$

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7. Find an equation of the tangent plane to parametric surface $\boldsymbol{r}(u,v) = \langle 2u, 3 - v^2, uv \rangle$ at $(2,2,-1)$.

$$(2,2,-1) = P = r(u,v) \Leftrightarrow \begin{cases} 2u = 2 \Rightarrow u = 1 \\ 3 - v^2 = 2 \ \checkmark \\ uv = -1 \Rightarrow v = -1 \end{cases} \Rightarrow (u_0, v_0) = (1,-1)$$

EQ. PLANE: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ FOR $P = (x_0, y_0, z_0)$

AND $\langle a, b, c \rangle = t \cdot \vec{r}_u \times \vec{r}_v \big|_P$

$$\vec{r}_u \times \vec{r}_v \Big|_{(u_0,v_0)} = \langle 2, 0, v \rangle \times \langle 0, -2v, u \rangle \Big|_{(1,-1)} =$$

$$= \langle 2, 0, -1 \rangle \times \langle 0, 2, 1 \rangle = \begin{vmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = \langle 2, -2, 4 \rangle$$

THEN $\langle a, b, c \rangle = \langle 1, -1, 2 \rangle$.

EQ. PLANE: $1 \cdot (x - 2) - 1(y - 2) + 2(z + 1) = 0$

$$\boxed{x - y + 2z = -2}$$

8. Classify (if any) the critical points of $f(x,y) = -x^2 - y + 2y^3$.

$$f_x = -2x \quad \overset{\text{CR.}}{\underset{\text{POINTS}}{\Rightarrow}} \begin{cases} -2x = 0 \Rightarrow x = 0 \\ -1 + 6y^2 = 0 \Rightarrow y = \pm \frac{1}{\sqrt{6}} \end{cases} \Rightarrow \left(0, \tfrac{1}{\sqrt{6}}\right), \left(0, -\tfrac{1}{\sqrt{6}}\right)$$

$$f_y = -1 + 6y^2$$

$$f_{xx} = -2; \quad f_{xy} = 0; \quad f_{yy} = 12y \Rightarrow D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = -24y.$$

1) $D\left(0, \frac{1}{\sqrt{6}}\right) < 0 \Rightarrow$ SADDLE POINT $\left(0, \frac{1}{\sqrt{6}}, f\left(0, \frac{1}{\sqrt{6}}\right)\right)$

$$-\frac{1}{\sqrt{6}}\left(1 - 2\left(\frac{1}{\sqrt{6}}\right)^2\right) = -\frac{2}{3\sqrt{6}} = -\frac{\sqrt{6}}{9} \Rightarrow = \left(0, \frac{1}{\sqrt{6}}, -\frac{\sqrt{6}}{9}\right)$$

2) $D\left(0, -\frac{1}{\sqrt{6}}\right) > 0$ AND $f_{xx} < 0$ $\Rightarrow$ MAX $\left(0, -\frac{1}{\sqrt{6}}, \frac{\sqrt{6}}{9}\right)$