Math 102 - Fall 2011 - Test 3

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. If you use graphic methods, sketch the graphs and label significant points, like intersection points or intercepts. Each exercise is worth 10 points, unless otherwise specified. *Always use the appropriate wording and units of measure in your answers (when applicable).*

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the integral root of the cubic polynomial $6x^3 - 13x^2 - 41x - 12$. $= f(x)$

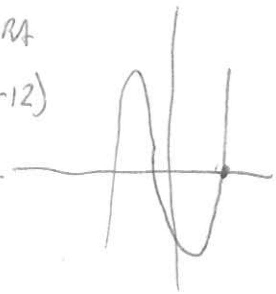
"INTEGRAL ROOT" = "ROOT THAT IS AN INTEGER" : GRAPH OR ALGEBRA

ALGEBRA: IF THERE IS ANY, IT WILL BE AMONG THE FACTORS OF $6 \cdot (-12) = -72$

USE TABLE

$f(1) = -60 \neq 0$; $f(-1) = 10 \neq 0$; ... ; $f(-3) = -168 \neq 0$;

$f(4) = 0 \rightarrow \boxed{X = 4 \text{ IS AN INTEGRAL ROOT}}$



TO CHECK THAT THERE ARE NO MORE INTEGRAL ROOTS, DIVIDE $f(x)/(x-4)$

$$\begin{array}{r|l} & 6x^2 + 11x + 3 \\ x-4 & 6x^3 - 13x^2 - 41x - 12 \\ & 6x^3 - 24x^2 \\ \hline & 11x^2 - 41x - 12 \\ & -11x^2 - 44x \\ \hline & 3x - 12 \\ & 3x - 12 \\ \hline & 0 \end{array}$$

$\rightarrow f(x) = (x-4)(6x^2 + 11x + 3) \rightarrow$ FACTOR PRODUCT $18 = 3 \cdot 6$, SUM $11 \rightarrow 9, 2$

$6x^2 + 9x + 2x + 3 = 3x(2x+3) + (2x+3) =$

$= (2x+3)(3x+1)$

$2x+3=0 \rightarrow x = -3/2$ (NOT INTEGER)

$3x+1=0 \rightarrow x = -1/3$ (NOT INTEGER)

2. Factor the quartic polynomial $P(x) = 2x^4 + 7x^3 - 13x^2 + 7x - 15$. You can use the fact that $x^2 + 1$ divides $P(x)$.

LONG DIVISION:

$2x^4/x^2 \quad 7x^3/x^2 \quad -15x^2/x^2$

$$\begin{array}{r|l} & 2x^2 + 7x - 15 \\ x^2+1 & 2x^4 + 7x^3 - 13x^2 + 7x - 15 \\ & 2x^4 + 2x^2 \\ \hline & 7x^3 - 15x^2 + 7x - 15 \\ & 7x^3 - 7x \\ \hline & -15x^2 + 15 \\ & -15x^2 - 15 \\ \hline & 0 \end{array}$$

$\rightarrow P(x) = (x^2+1)(2x^2+7x-15)$

FACTOR $2x^2 + 7x - 15$

PRODUCT $2 \cdot (-15) = -30$

SUM 7

$\rightarrow 10, -3 \rightarrow$ *

*

$2x^2 + 7x - 15 =$

$= 2x^2 + 10x - 3x - 15$

$= 2x(x+5) - 3(x+5)$

$= (2x-3)(x+5)$

THEREFORE:

$$\boxed{P(x) = (x^2+1)(2x-3)(x+5)}$$

3. Perform the long division $\dfrac{3x^3 + 12x^2 + 3x + 7}{3x^2 + 2}$.

$3x^3/3x^2 \quad 12x^2/3x^2$

$$
\begin{array}{r|l}
 & x + 4 \\
\hline
3x^2+2 & 3x^3 + 12x^2 + 3x + 7 \\
 & 3x^3 \qquad\quad + 2x \\
\hline
 & \qquad 12x^2 + x + 7 \\
 & \qquad 12x^2 \qquad + 8 \\
\hline
 & \qquad\qquad x - 1
\end{array}
$$

$$\boxed{\frac{3x^3+12x^2+3x+7}{3x^2+2} = x+4+\frac{x-1}{3x^2+2}}$$

4. The profit in dollars of a company selling x coffee mugs can be modeled by the function $y = -0.00007x^4 + 0.007x^3 - .14x^2 - .65x - 10$.

(a) Find out how many mugs must be sold to have a profit of \$30.

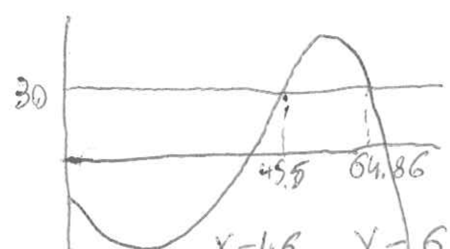
$Y_2 = 30$

$Y_1 = (-7E{-}5)X^4 + .007X^3 - .14X^2 - .65X - 10$

USE TABLE TO FIGURE OUT THE WINDOW:

$[0, 80]$ BY $[-40, 80]$, THEN TWICE

$\boxed{2ND} + \boxed{TRACE} + \boxed{5}$



$X = 46$, $X = 65$

(b) Find out the level of production that maximizes the profit and what the maximum profit is.

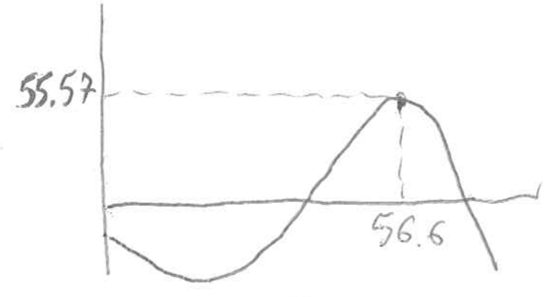
WITH THE SETTING AS IN (a), WE LOOK FOR THE MAXIMUM

$\boxed{2ND} + \boxed{TRACE} + \boxed{4}$

CHECK WHICH PRODUCTION GIVES HIGHEST PROFIT, WITH TABLE:

$X = 56 \rightarrow Y = 55.46$

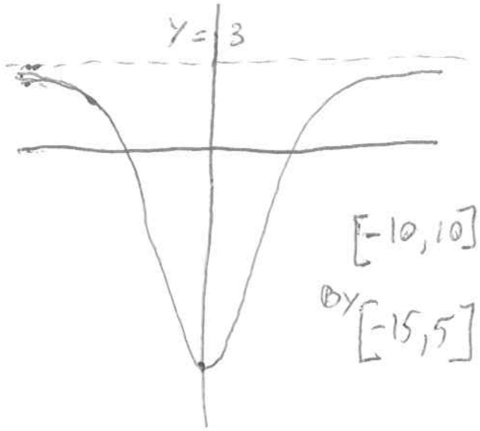
$X = 57 \rightarrow Y = 55.52$



MAXIMUM PROFIT IS \$55.52, WHEN PRODUCING 57 COFFEE MUGS.

5. For the following rational functions, use algebra to find the domain, then use algebra or technology to find (if any) the asymptotes (vertical, horizontal, or slant). (Each part is worth 10 points)

(a) $f(x) = \dfrac{3x^2 - 12}{x^2 + 1}$



I) DOMAIN: $X^2 + 1 \neq 0 \rightarrow$ SOLVE

$X^2 + 1 = 0 \rightarrow X = \pm i$ NOT REAL $\rightarrow$

II) $\rightarrow$ $\boxed{\text{DOMAIN: } (-\infty, +\infty)}$ $\rightarrow$ $\boxed{\text{NO VERT. ASYMP.}}$

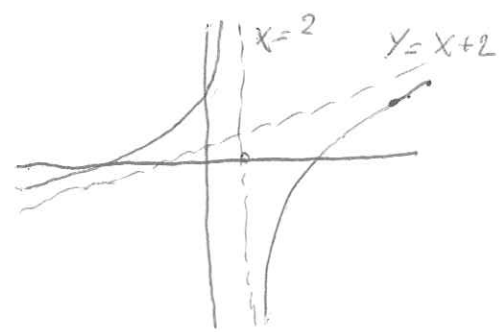
III) HORIZ OR SLANT ASYMPTOTE:

$$\begin{array}{r|l} & 3 \\ \hline X^2+1 & 3X^2 - 12 \\ & 3X^2 + 3 \\ \hline & -15 \end{array} \rightarrow f(x) = 3 + \frac{-15}{X^2+1}$$

$Y = 3$ HORIZONTAL ASYMPTOTE.

NONE SLANT.

(b) $f(x) = \dfrac{x^2 - 16}{x - 2}$



I) DOMAIN: $X - 2 \neq 0 \rightarrow X \neq 2 \rightarrow (-\infty, 2) \cup (2, +\infty)$

II) VERTICAL ASYMPTOTE:

CANDIDATE: $X = 2 \xrightarrow{\text{PLUG}} f(2) = \frac{-12}{0} \rightarrow$ V.A.

$\boxed{\text{VERTICAL ASYMPTOTE: } X = 2}$

III)

$$\begin{array}{r|l} & X + 2 \\ \hline X-2 & X^2 - 16 \\ & X^2 - 2X \\ \hline & 2X - 16 \\ & 2X - 4 \\ \hline & -12 \end{array} \rightarrow f(x) = X + 2 + \frac{-12}{X-2}$$

$Y = X + 2$ SLANT ASYMPTOTE

6. The number of new notions (per week) retained by a junior student at college t weeks after the beginning of the semester is given by

$$N(t) = \frac{210t}{20 + 3t^2}.$$

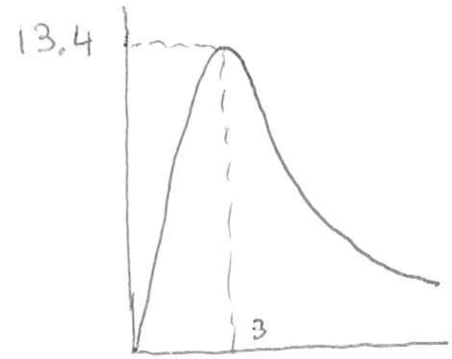
(a) What is the number of (new weekly) notions retained after one week? After nine weeks?

ONE WEEK $\to$ $t = 1 \to N(1) \doteq 9.13 \approx 9$ NEW NOTIONS

NINE WEEKS $\to$ $t = 9 \to N(9) = 7.18 \approx 7$ NEW NOTIONS

(b) What is the highest number of (new weekly) notions retained? How long after the beginning of the semester will it occur?

$\boxed{2^{ND}}$ + $\boxed{\text{TRACE}}$ + $\boxed{4}$



$[0,14]$ BY $[0,14]$

THE HIGHEST NUMBER OF NEW WEEKLY NOTIONS RETAINED IS 13 AND IT OCCUR BETWEEN THE 2ND AND 3RD WEEK

7. Let $f(x) = x^2 + 3$ and $g(x) = 5 - 2x$. Compute $(g + f)(x)$ and $(f \cdot g)(x)$.

I) $(g+f)(x) = g(x) + f(x) = (5-2x) + (x^2+3) = 5 - 2x + x^2 + 3 = x^2 - 2x + 8$

II) $(f \cdot g)(x) = f(x)\, g(x) = (x^2+3)(5-2x) = 5x^2 - 2x^3 + 15 - 6x$

$= -2x^3 + 5x^2 - 6x + 15$

8. Let $f(x) = 4x + 1$ and $g(x) = 4 + x^2$. Compute $(g \circ f)(x)$ and $f(g(-2))$.

I) $(g \circ f)(x) = g(f(x)) = 4 + (4x+1)^2 = 4 + (4x+1)(4x+1)$

$= 4 + 16x^2 + 8x + 1$

$= 16x^2 + 8x + 5$

II) $g(-2) = 4 + (-2)^2 = 8 \rightarrow f(g(-2)) = f(8) = 4(8) + 1 = 33$

9. Find the inverse of the function $f(x) = 6 + 3x$ and check your result.

I) SOLVE $Y = 6 + 3X$ FOR X:

$3X = Y - 6 \rightarrow X = \dfrac{Y-6}{3}$

II) SWAP X AND Y: $Y = \dfrac{X-6}{3}$ IS $f^{-1}(x) = \dfrac{X-6}{3}$

CHECK: 1) $f(f^{-1}(x)) = 6 + 3\left(\dfrac{X-6}{\cancel{3}}\right) = 6 + X - 6 = X$ ✓

2) $f^{-1}(f(x)) = \dfrac{(6+3X)-6}{3} = \dfrac{\cancel{6}+3X-\cancel{6}}{3} = \dfrac{\cancel{3}X}{\cancel{3}} = X$ ✓