Math 102 - Spring 2010 - Test 4

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. Only calculators are allowed on this examination. *Always use the appropriate wording and units of measure in your answers (when applicable).*

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. (15 points) During years 1990 and 2009 the number of homicides per 100,000 people is given by the function $y = -0.000710x^4 + 0.0265x^3 - 0.300x^2 + 0.684x + 9.337$, with x equal to the number of years from 1990.

   (a) Use technology to find during which year(s) the number of homicides per 100,000 people was 5.59.

   USE TABLE TO SEE WHEN $Y = 5.59$, OR THE INTERSECTION POINT $Y_1 = Y$ AND $Y_2 = 5.59$. FOR $8 < X < 9$, $X = 15$. SINCE BETWEEN 1990 AND 2009 X RANGES FROM 0 TO 19, THEN THE NUMBER OF HOMICIDES PER 100,000 PEOPLE IS 5.59 DURING 1998 AND 2005.

   (b) Use technology to find during which year(s) the number of homicides per 100,000 was maximum.

   USED WINDOW $[0, 20]$, $[0, 10]$. THEN $\boxed{2^{ND}}$, $\boxed{TRACE}$ $\boxed{4: MAXIMUM}$ AND FOUND $X = 1.38$, $Y = 9.78$.

   IT MEANS THE THE MAXIMUM NUMBER OF HOMICIDES PER 100,000 PEOPLE (WHICH IS 9.78) OCCURS DURING 1991 (AND BEFORE 1992).

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2. For the following rational functions, use algebra to find (if any) the vertical asymptotes and use technology to find (if any) the horizontal asymptotes.

(a) (11 points) $f(x) = \dfrac{4-x^2}{4x^2+16}$

V.A.: $4x^2+16=0 \to x^2 = -\dfrac{16}{4} = -4 \to x = \pm 2\sqrt{i}$ NOT REAL

THEREFORE NONE V.A.

H.A.: SAME DEGREE, THUS $y = $ "QUOTIENT OF LEADING COEFF" $= \dfrac{-1}{4} = -\dfrac{1}{4}$

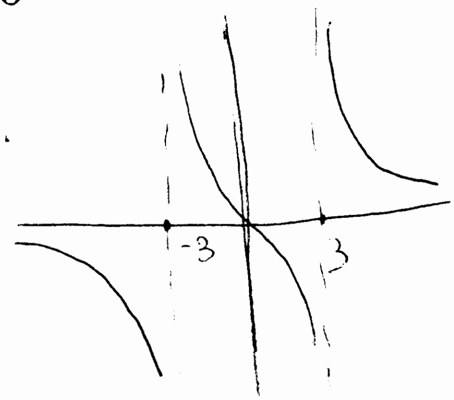
IS THE HORIZ. ASYMPTOTE.

(b) (11 points) $f(x) = \dfrac{7x}{3x^2-27}$

V.A.: $3x^2-27=0 \to x^2 = \dfrac{27}{3} = 9 \to x = \pm\sqrt{9} = \pm\sqrt{3}$ COULD

BE V.A. CHECK GRAPHICALLY:

SO $x=3$ AND $x=-3$ ARE V.A.



H.A.: "DEGREE DENOMINATOR" > "DEG. NUM"

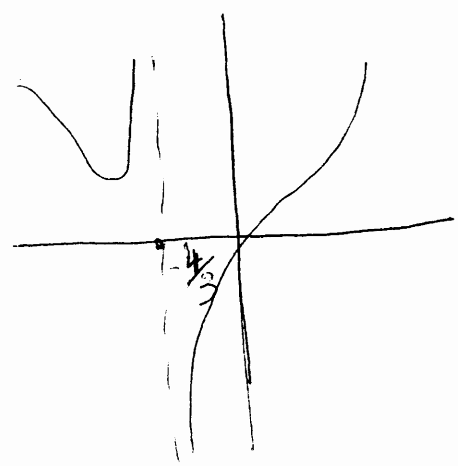
SO $y=0$ IS THE HORIZ. ASYMPTOTE,

AS SHOWN BY THE GRAPH.

(c) (11 points) $f(x) = \dfrac{x^3-1}{4+3x}$

V.A.: $4+3x=0 \to x = -\dfrac{4}{3}$,

WHICH IS A V.A. BY THE GRAPH



H.A.: NONE

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3. (15 points) Suppose the concentration of a drug (as percent) in a patient bloodstream t hours after injection is given by

$$C(t) = \frac{200t}{2t^2 + 32}.$$

Graph the function $C(t)$ using the window $[0, 20]$ by $[0, 20]$.

(a) What is the drug concentration 1 hour after injection? 5 hours?

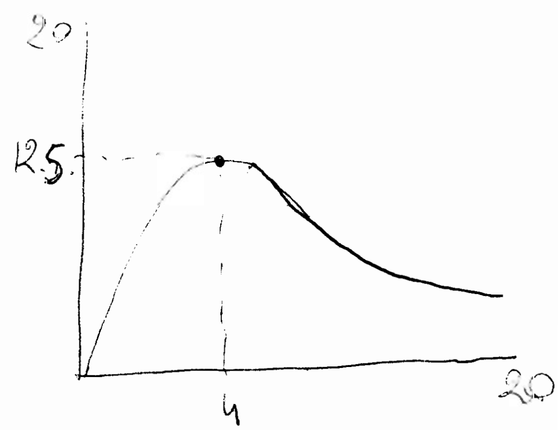
$C(1) = \frac{200}{34} \approx 5.88\ \%$

$C(5) = \frac{1000}{82} \approx 12.19\ \%$

(b) What is the highest percent concentration? In how many hours will it occur?

AS FOR EXERCISE 1b. MAXIMUM CONCENTRATION OF 12.5 %

AFTER 4 HOURS.



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4. (15 points) A car rental agency rents compact, mid-size, and luxury cars. Its goal is to purchase 90 cars with a total of \$2,270,000 and to earn a daily rental of \$3150 from all cars. The compact cars cost \$18,000 each and earn \$25 per day in rental, the mid-size cars cost \$25,000 each and earn \$25 per day, and the luxury cars cost \$40,000 each and earn \$55 per day.

X = # COMPACT
Y = # MID
Z = # LUXURY

(a) Write the system of linear equations describing the goals of this agency.

TOTAL NUMBER OF CARS: $X + Y + Z = 90$

TOTAL INVESTMENT: $18{,}000X + 25{,}000Y + 40{,}000Z = 2{,}270{,}000$

TOTAL REVENUE: $25X + 25Y + 55Z = 3150$

NOTICE A TYPO FOR THE MIDSIZE CAR RENTAL RATE, SUPPOSED TO BE 35 \$/DAY.

(b) Find the number of each type of car the agency should purchase to meet its goal.

$$A = \begin{bmatrix} 1 & 1 & 1 & 90 \\ 18000 & 25000 & 40000 & 2270000 \\ 25 & 25 & 55 & 3150 \end{bmatrix}$$

$$\mathrm{RREF}(A) \approx \begin{bmatrix} 1 & 0 & 0 & 61 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 30 \end{bmatrix}$$

THEREFORE $Y = -1$, WHICH IS NOT POSSIBLE, AND THE AGENCY CAN NOT REACH ITS GOALS.

NOTICE THAT WITH RATE 35 FOR MIDSIZE (Y), ONE WOULD HAVE $X = 40$, $Y = 30$, $Z = 20$.

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5. Solve the following systems of linear equations.

(a) (11 points) $\begin{cases} x + y - z - 4w = 6 \\ 3x - 4y + z + 2w = 5 \\ 5x + 2y + 6z - 3w = 1 \\ x \quad - 3z \quad = 4 \end{cases}$

$$A = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -4 & 6 \\ 3 & -4 & 1 & 2 & 5 \\ 5 & 2 & 6 & -3 & 1 \\ 1 & 0 & -3 & 0 & 4 \end{array}\right], \quad RREF(A) = \begin{bmatrix} 1 & 0 & 0 & 0 & 361/328 \\ 0 & 1 & 0 & 0 & -217/164 \\ 0 & 0 & 1 & 0 & -317/328 \\ 0 & 0 & 0 & 1 & -431/328 \end{bmatrix}$$

THEREFORE:

$$\begin{cases} X = 361/328 \approx 1.1 \\ Y = -217/164 \approx -1.32 \\ Z = -317/328 \approx -.97 \\ W = -431/328 \approx -1.31 \end{cases}$$

(b) (11 points) $\begin{cases} x - 3y + 2z = 12 \\ 2x - 6y + z = 7 \end{cases}$

$$A = \left[\begin{array}{ccc|c} 1 & -3 & 2 & 12 \\ 2 & -6 & 1 & 7 \end{array}\right], \quad RREF(A) = \begin{bmatrix} 1 & -3 & 0 & 2/3 \\ 0 & 0 & 1 & 17/3 \end{bmatrix}$$

THEREFORE

$$\begin{cases} X - 3Y = 2/3 \\ Z = 17/3 \end{cases} \rightarrow \begin{cases} X = 3Y + 2/3 \\ Y = Y \\ Z = 17/3 \end{cases}$$

$2/3 \approx .7$

$17/3 \approx 5.7$

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