80 + 14 EXTRA

MAT 121 – Exam3 – Spring 2014

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Complete 8 out of the following 12 exercises, as indicated. Exercise 13 and the last two questions of exercise 12 are for extra points. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

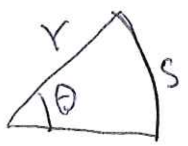
Complete 2 of the exercises 1-3

1. Rewrite 405° in radian measure as a multiple of π.

$1^\circ = \frac{\pi}{180}$ RAD THEN $405^\circ = 405\left(\frac{\pi}{180}\right)$ RAD $= \frac{9}{4}\pi$ RAD

OR 2.25π RAD

2. Find the length of the arc, S, on a circle of radius 3 centimeters intercepted by a central angle of 120°. Round to two decimal places.



$S = r\theta$, BUT θ MUST BE IN RADIAN !

$\theta = 120 \cdot \frac{\pi}{180} = \frac{2}{3}\pi$

THEN $S = 3 \cdot \frac{2}{3}\pi = 2\pi \approx 6.28$ cm

3. Find the area of the sector of the circle with radius 4 inches and central angle $\frac{5\pi}{6}$.

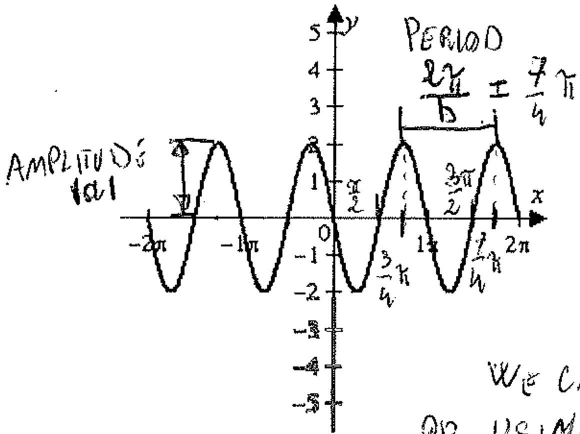
$A = \frac{1}{2}r^2\theta$ (θ IN RADIAN)

$A = \frac{1}{2}(4)^2 \cdot \frac{5}{6}\pi = \frac{20}{3}\pi \approx 20.944$ in^2

20

Complete 2 of the exercises 4-6

4. Find a, b, and c for the function $f(x) = a\ \cos(bx - c)$ such that the graph of $f(x)$ matches the graph below.



PERIOD $\frac{2\pi}{b} = \frac{7}{4}\pi - \frac{3}{4}\pi = \pi \Rightarrow \frac{2\pi}{b} = \pi \Rightarrow b = 2$

AMPLITUDE: $|a| = 2$.
BECAUSE AT $X = 0$ $f(X)$ IS DECREASING THEN $a > 0$. $\Rightarrow$

$\Rightarrow a = 2$

WE CAN FIND c BY PLUGGING a, b, AND A POINT, OR USING TWO CONSECUTIVE ZEROS: $\cos(\frac{\pi}{2}) = \cos(-\frac{\pi}{2}) = 0$.

BECAUSE TWO CONSECUTIVE ZERO'S OF $f(X)$ ARE AT $X = 0$ AND $X = \frac{\pi}{2}$. THEN

CHECK $\Rightarrow \begin{cases} 2(0) - c = \frac{\pi}{2} \Rightarrow c = -\frac{\pi}{2} \\ 2(\frac{\pi}{2}) - c = -\frac{\pi}{2} \Rightarrow c = -\frac{\pi}{2} \end{cases}$ ✓

THIS CAN BE TO FIND b.

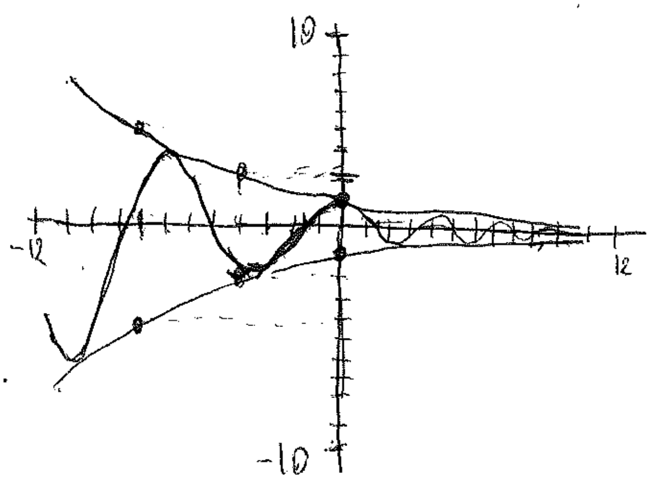
THEN $f(X) = 2\cos(2X + \frac{\pi}{2})$.

ONE COULD USE GGB WITH SLIDERS.

5. Use a graphing utility to graph in the same viewing window the damping factor and the function

$$f(x) = 2^{-\frac{x}{4}}\cos x.$$

Sketch the graphs and describe the behavior of the function as x increases without bound.



DAMPING FACTOR: $g(X) = 2^{-\frac{X}{4}}$

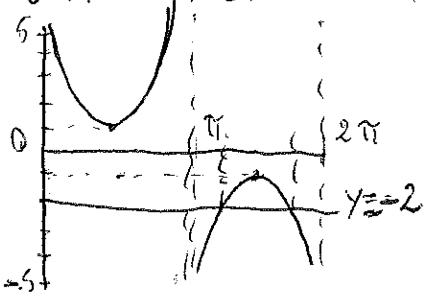
$-g(X) \le f(X) \le g(X)$

AS $X \to \infty$, $f(X) \to 0$

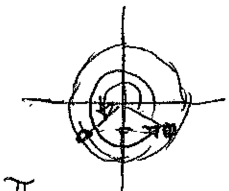
10 OR 20

6. Determine two solutions of the equation $\csc x = -2$ such that $0 \le x < 2\pi$.

GRAPHICALLY OR BY ALGEBRA: $CSC(X) = \frac{1}{\sin X}$



$\frac{1}{\sin X} = -2 \Rightarrow \sin X = -\frac{1}{2}$



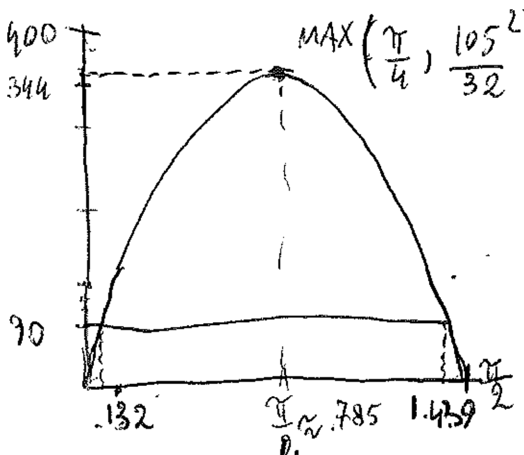
$\Rightarrow X = \arcsin\left(-\frac{1}{2}\right) \begin{cases} X = \frac{7}{6}\pi \approx 3.665 \\ X = \frac{11}{6}\pi \approx 5.76 \end{cases}$

Complete 2 of the exercises 7-9

7. Ignoring the effects of air resistance, the range of a projectile fired at an angle θ with the horizontal and with an initial velocity of v_0 feet per second is $r = \frac{1}{32} v_0^2 \sin(2\theta)$ where r is measured in feet. A golfer strikes a golf ball at 105 feet per second. At what angle must the golfer hit the ball so that it travels (a) 90 feet? (b) it has the maximum range?

$V_0 = 105 \text{ FT/SEC} \Rightarrow Y = \frac{105^2}{32}\sin(2\theta) \Rightarrow Y \approx 344.531 \sin(2\theta)$

GRAPHICALLY OR BY ALGEBRA



a) SOLVE THE EQUATION:

$\frac{105^2}{32}\sin(2\theta) = 90 \Rightarrow$

$\Rightarrow \sin(2\theta) = \frac{64}{245} \Rightarrow$

$\Rightarrow 2\theta = \arcsin\left(\frac{64}{245}\right)$ AND

$2\theta = \pi - \arcsin\left(\frac{64}{245}\right) \Rightarrow$

$\Rightarrow \theta = \frac{1}{2}\arcsin\left(\frac{64}{245}\right) \approx .132$ AND $\theta = \frac{1}{2}\left(\pi - \arcsin\left(\frac{64}{245}\right)\right) \approx 1.439$.

b) THE MAX IS ACHIEVED WHEN $\sin(2\theta)$ IS AT MAX, THAT IS:

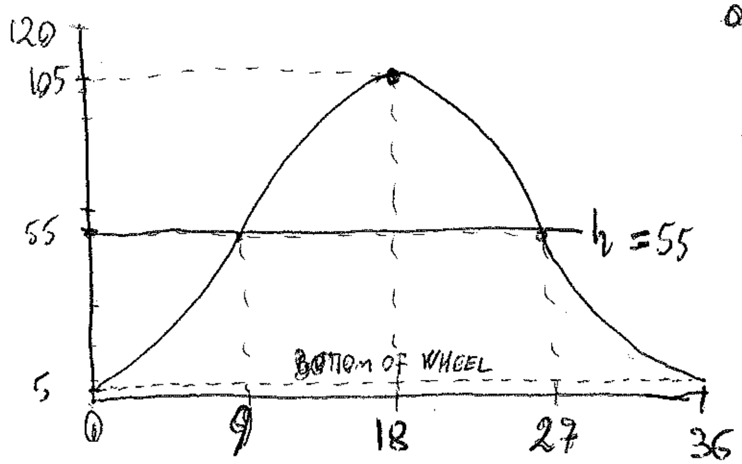
$\sin(2\theta) = 1 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} \approx .785$

10 OR 20

8. A Ferris wheel is built such that the height h (in feet) above the ground of a seat on the wheel at time t (in seconds) can be modeled by $h(t) = 55 + 50\sin\left(\frac{\pi}{18}t - \frac{\pi}{2}\right)$. The wheel makes one revolution every 36 seconds and the ride begins when $t = 0$. During the first 36 seconds of the ride, when will a person, who starts at the bottom of the Ferris wheel, be (a) 55 feet above the ground? (b) at the maximum height?

NOTE, THE PERIOD IS $\frac{2\pi}{b}$, THAT IS $\frac{2\pi}{\pi/18} = 36$ (WHICH IS ONE REVOLUTION)

GRAPHICALLY OR BY ALGEBRA



FIRST 36 SECONDS: $0 \le t \le 36$

a) SOLVE THE EQUATION: $h(t) = 55$

$$\cancel{55} = \cancel{55} + 50\sin\left(\frac{\pi}{18}t - \frac{\pi}{2}\right) \Rightarrow$$

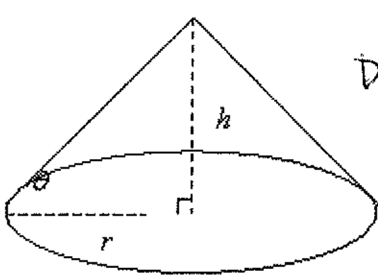
$$\Rightarrow \sin\left(\frac{\pi}{18}t - \frac{\pi}{2}\right) = 0 \Rightarrow \begin{cases} \frac{\pi}{18}t - \frac{\pi}{2} = 0 \\ \frac{\pi}{18}t - \frac{\pi}{2} = \pi \end{cases}$$

$$\Rightarrow \begin{cases} \frac{t}{18} = \frac{1}{2} \Rightarrow t = 9 \\ \frac{t}{18} = \frac{3}{2} \Rightarrow t = 27 \end{cases} \text{SECONDS}$$

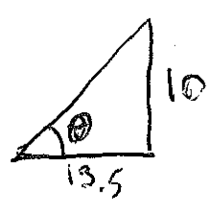
b) MAXIMUM FOR MAXIMUM SINE:

$$\frac{\pi}{18}t - \frac{\pi}{2} = \frac{\pi}{2} \Rightarrow \frac{t}{18} = 1 \Rightarrow t = 18 \text{ SEC.}$$

9. A granular substance such as sand naturally settles into a cone-shaped pile when poured from a small aperture. Its height depends on the humidity and adhesion between granules. The angle of elevation of a pile, θ, is called the angle of repose. If the height of a pile of sand is 10 feet and its diameter is approximately 27 feet, determine the angle of repose. Round your answer to the nearest degree.



DATA: $h = 10$, $2r = 27 \Rightarrow r = 13.5$



$$\frac{h}{r} = \tan\theta \Rightarrow \theta = \arctan\left(\frac{h}{r}\right)$$

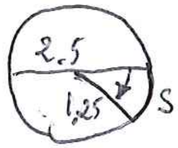
$$\Rightarrow \theta = \arctan\left(\frac{10}{13.5}\right) \approx .638 \text{ RAD}$$

$$\approx 37^\circ$$

10 OR 20

Complete 1 of the exercises 10-11

10. A car is traveling along Route 66 at a rate of 75 miles per hour, and the diameter of its wheels are 2.5 feet. Find the number of revolutions per minute the wheels are turning. Round your answer to one decimal place.



LINEAR SPEED = $75 \, mi/h = 75 \left(\frac{5280 \text{ FT}}{60 \text{ min}} \right) = 6600 \text{ FT}/\text{min}$

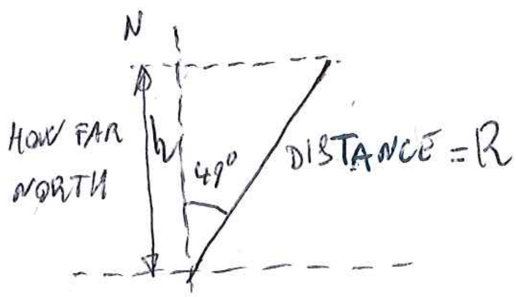
LINEAR SPEED = $\frac{\text{DISTANCE}}{\text{TIME}} = V$

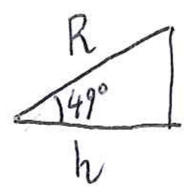
DISTANCE = "MULTIPLE OF CIRCUMFERENCE" AND

REVOLUTIONS = $\frac{\text{DISTANCE}}{\text{CIRCUMFERENCE}} = \frac{\text{TIME} \cdot V}{\text{CIRCUMFERENCE}}$. TAKING TIME = 1 min

REVOLUTIONS = $\frac{6600}{2.5\pi} \approx 840.3$ ROUNDS PER MINUTE (OR RPM).

11. A jet is traveling at 600 miles per hour at a bearing of 49°. After flying for 1.2 hours in the same direction, how far north will the plane have traveled? Round your answer to the nearest mile.





$h = R \cos 49°$

R = DISTANCE = SPEED · TIME
= 600 (1.2) = 720 mi

THEN $h = 720 \cdot \cos 49° \approx 472$ MILES NORTH

10

Complete this exercise: you can download the data-sheet from the coursework section in EagleWeb.

12. The table shows the mean monthly temperature T (in degrees Fahrenheit) and the mean monthly precipitation P (in inches) for Honolulu (Hawaii) where t is the month, with $t = 1$ corresponding to January. (Data Source: National Climatic Data Center)

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
T	73	73	74	76	77	80	81	82	82	80	78	75
P	2.7	2.4	1.9	1.1	0.8	0.4	0.5	0.5	0.7	2.2	2.3	2.9

(a) Use the sine regression feature of a graphing utility to find sine models to fit each set of data. Report these models and the corresponding correlation coefficients.
(b) Which data is best fit by its sine regression?
(c) What is the period of each model?
(d) **(Extra 2 points)** What is the amplitude of each model? Interpret the meaning of the amplitude for each model in the context of the problem.
(e) **(Extra 2 points)** At what values of t does each sine model reach its maximum and minimum? What do these values represent in the context of the problem?

(a) $T \approx 77.304 + 4.424 \sin(0.501t - 2.448)$, $R^2 = .9874$

$P \approx 1.618 + 1.3 \sin(0.488t + 1.505)$, $R^2 = .9554$

(b) T IS BEST FIT BECAUSE IT HAS A LARGER CORRELATION COEFF.

(c) FOR $y = a \sin(bx - c)$ THE PERIOD IS $\frac{2\pi}{b}$, HERE:

$T \to$ PERIOD $= \frac{2\pi}{.501} \approx 12.541$ MONTHS (FEW DAYS MORE THAN 1 YEAR)

$P \to$ PERIOD $= \frac{2\pi}{.488} \approx 12.875$ MONTHS (ALMOST 1 MONTH MORE THAN 1 YEAR)

(d) AMPLITUDE $= |a|$ REPRESENTS HOW FAR FROM THE EQUILIBRIUM ONE CAN BE.

$T \to$ AMPLITUDE $= 4$ °F, THAT IS THE TEMPERATURE CAN BE 4 °F ABOVE OR BELOW THE AVERAGE TEMPERATURE OF 77 °F.

$P \to$ AMPL. $= 1.3$ in, THAT IS THE AVERAGE MONTHLY PRECIPITATION CAN BE ABOVE OR BELOW THE MEAN OF 1.6 in OF RAIN PER MONTH.

(e) USING GGB (OR ANY GRAPH): NOTE THE ANALOGY WITH (d).

$T \to$ MAX $= (8, 81)$, MIN $(2, 73)$ $\to$ MAX IN AUGUST, MIN IN FEB

$P \to$ MAX $= (0, 2.9)$ IN JAN., MIN $(6.6, .3)$ MIDDLE OF JUNE.

THE HOTTEST MONTH IS AUGUST, THE COOLEST FEB, THE WETTEST JAN AND THE DRIEST JUNE.

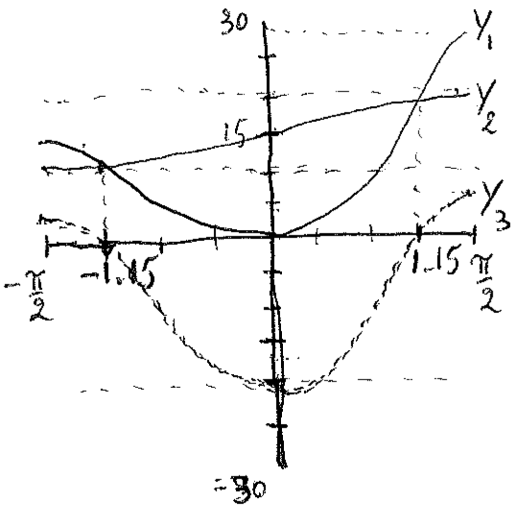
10 + 4 EXTRA

Extra points

13. Use a graphing utility to approximate the solutions (to three decimal places) of the given equation in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$\overbrace{6\sin^3 x + 18\sin^2 x}^{Y_1} = \overbrace{5\sin x + 15}^{Y_2}$$

GRAPH OR ALGEBRA:



THE GRAPH CAN EITHER BE FOR THE INTERSECTION OF Y_1 AND Y_2 OR THE X-INTERCEPTS (ZEROS) OF $Y_3 = Y_1 - Y_2$.

ALGEBRA:

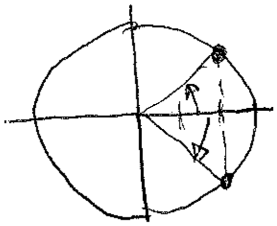
$$Y_3 = 6\sin^3 X + 18\sin^2 X - 5\sin X - 15$$
$$= 6\sin^2 X(\sin X + 3) - 5(\sin X + 3)$$
$$= (\sin X + 3)(6\sin^2 X - 5).$$

BECAUSE $\sin X \geq -1$, $\sin X + 3 \neq 0$. THEN $Y_3 = 0$ ONLY IF

$6\sin^2 X - 5 = 0 \Rightarrow 6\sin^2 X - 3 - 2 = 0 \Rightarrow 3(2\sin^2 X - 1) = 2 \Rightarrow$

$\Rightarrow 3\cos(2X) = 2 \Rightarrow \cos(2X) = \frac{2}{3} \Rightarrow 2X = \arccos\left(\frac{2}{3}\right)$ AND

$2X = -\arccos\left(\frac{2}{3}\right) \Rightarrow X = \pm\frac{1}{2}\arccos\left(\frac{2}{3}\right) \approx \pm 1.15$



10 EXTRA