MAT 121 – Exam2 – Spring 2018

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test: only a textbook can be used, or a cheat-sheet approved by your instructor. Personal notebooks cannot be used. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the polynomial inequality $(2x+5)(x-1)(3-4x) > 0$. Write your answer in interval notation.

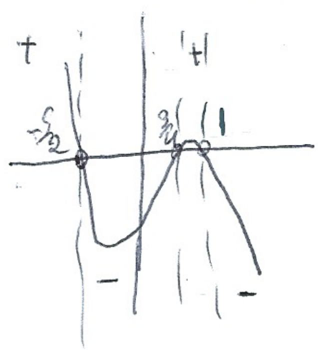
NUMERICALLY: ZEROS: $x = -\frac{5}{2}, 1, \frac{3}{4}$ GRAPHICALLY

$-\infty$ $-5/2$ $3/4$ 1 ∞

X	-3	0	7/8	2
Y	(-)(-)(+) +	(+)(-)(+) −	(+)(-)(-) +	(+)(+)(-) −

SOL

$$\boxed{\left(-\infty, -\frac{5}{2}\right) \cup \left(\frac{3}{4}, 1\right)}$$



2. Use polynomial long division to rewrite the following rational function in the form $f(x) = q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder. Then write the equations of any asymptote.

$$f(x) = \frac{5x^5 - 3x^2 + 1}{x^3 + 8} = 5x^2 + \frac{1 - 43x^2}{x^3 + 8}$$

$\boxed{y = 5x^2 \text{ NON-VERT. ASYMPTOTE}}$

$$\begin{array}{r|l} & 5x^2 \\ \hline x^3+8 & 5x^5 - 3x^2 + 1 \\ & 5x^5 + 40x^2 \\ \hline & 0 - 43x^2 + 1 \end{array}$$

V.A. POSSIBLE: $x^3 + 8 = 0 \Rightarrow x = \sqrt[3]{-8} = -2$
ONLY REAL SOLUTION

$\frac{r(-2)}{q(-2)} = \frac{\#}{0} \Rightarrow$ $\boxed{x = -2 \text{ IS A VERT. ASYMP.}}$

3. Consider the factored polynomial

$$f(x) = (x+2)^4 (x-1)^3 (x+3).$$

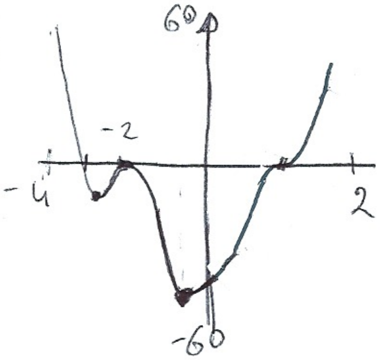
Step 1. Determine the degree and y-intercept (write the y-intercept as an ordered pair).
Step 2. Determine the x-intercept(s) at which f crosses the axis. If there are none, state "none".
Step 3. Determine the zero(s) of f at which it "flattens out". If there are none, state "none".
Step 4. Determine the turning points of f (specifying if maximum or minimum). If there are none, state "none".

$\boxed{1}$ DEGREE $= 4+3+1 = 8$; KNOWN-TERM $= 2^4 \cdot (-1)^3 \cdot 3 = -48 \Rightarrow$ Y-INT $= (0, -48)$

$\boxed{2}$ ODD MULTIPLICITY $\Rightarrow X = 1, -3 \Rightarrow \boxed{(1,0), (-3,0)}$

$\boxed{3}$ MULTIPLICITY $> 2 \Rightarrow X = -2, 1 \Rightarrow \boxed{(-2,0), (1,0)}$

$\boxed{4}$



MINIMA: $(-2.823, -4.537)$, $(-.177, -50.838)$

MAXIMUM: $(-2, 0)$

4. Solve the rational inequality. Write your answer in interval notation (use two decimal places if needed).

$$2x+1 \le \frac{3}{x+6}$$

STD FORM $\Rightarrow \dfrac{(2x+1)(x+6)-3}{x+6} \le 0 \Rightarrow \dfrac{2x^2+12x+x+6-3}{x+6} \le 0$

$Y = \dfrac{2x^2+13x+3}{x+6} \le 0 \Rightarrow$ ZERO'S: $2x^2+13x+3=0 \Rightarrow X = \dfrac{-13 \pm \sqrt{13^2-4(3)(2)}}{4} \Rightarrow$

$x = \dfrac{-13 \pm \sqrt{145}}{4} \approx -.24, -6.26$

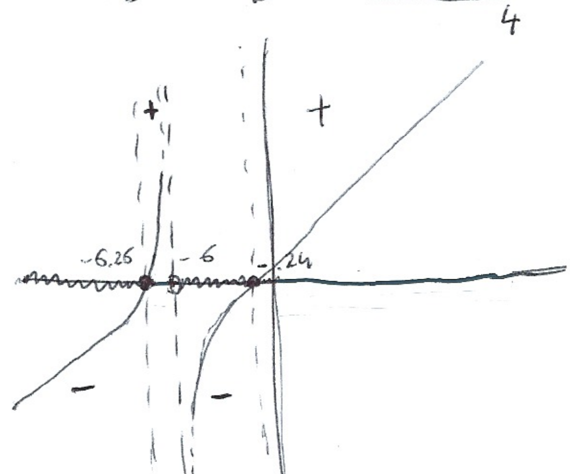
V. ASYMPT. $X+6=0 \Rightarrow X=-6$

CHECK: $\dfrac{2(-6)^2+13(-6)+3}{-6+6} = \dfrac{\neq 0}{0}$ ✓

NUMERICALLY or GRAPH $\Rightarrow$

	$-\infty$	$\frac{-13-\sqrt{145}}{4}$	-6	$\frac{-13+\sqrt{145}}{4}$	∞
X	-10	-6.1	0	10	
Y	$-$	$+$	$-$	$+$	

$\boxed{Y \le 0}$



$$\boxed{\left(-\infty, \frac{-13-\sqrt{145}}{4}\right] \cup \left(-6, \frac{-13+\sqrt{145}}{4}\right]}$$

5. The half-life of kryptonite is approximately 15.4 years.

Step 1. Determine a so that $A(t) = A_0 a^t$ describes the amount of kryptonite left after t years, where A_0 is the amount at time $= 0$. Round to 6 decimal places if needed.

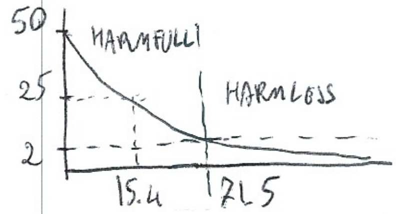
Step 2. How much of a 50 gram sample of kryptonite would remain after 10 years? Round to 3 decimal places.

Step 3. Superman would be harmed by 2 or more grams of kryptonite. How long would it take for the remaining amount of a 50 gram sample of kryptonite to be harmless to Superman? Round to 1 decimal place.

$\boxed{1}$ $\frac{1}{2}A_0 = A(15.4) = A_0 a^{15.4} \Rightarrow a^{15.4} = \frac{1}{2} \Rightarrow a = \left(\frac{1}{2}\right)^{1/15.4} \approx \boxed{0.955988}$

$\boxed{2}$ $A_0 = 50g \Rightarrow A(t) = 50(.955988)^t \Rightarrow A(10) = 50(.955988)^{10} \approx \boxed{31.878}$

$\boxed{3}$ $A(t) < 2$



SOLVE $A(t) = 2$

$(.955988^t) \cdot 50 = 2 \Rightarrow \frac{2}{50} = \frac{1}{25} \Rightarrow$

$t = \log_{0.955988}\left(\frac{1}{25}\right) \approx 71.5$

$\boxed{\text{IT IS HARMLESS AFTER 71.5 YEARS.}}$

6. In an effort to control vegetation overgrowth, 7 goats are released in an isolated area free of predators. After 2 years, it is estimated that the goats population has increased to 16. Assuming exponential population growth, what will the population be after another 6 months? (Use the greatest integer function round rule).

EXP. POPULATION GROWTH: $P = P_0 e^{kt} = P_0 a^t$ GOATS, IN t YEARS

$P_0 = 7 \Rightarrow 16 = P(2) = P_0 a^2 = 7 \cdot a^2 \Rightarrow a^2 = \frac{16}{7} \Rightarrow a = \frac{4}{\sqrt{7}} \Rightarrow$

$\Rightarrow P = 7\left(\frac{4}{\sqrt{7}}\right)^t \approx 7(1.511858)^t$

ANOTHER 6 MONTHS $= \frac{1}{2}$ MORE YEARS $\Rightarrow t = 2.5 \Rightarrow$

$\Rightarrow P(2.5) \approx 19.67 \Rightarrow P(2.5) = \boxed{19 \text{ GOATS}}$

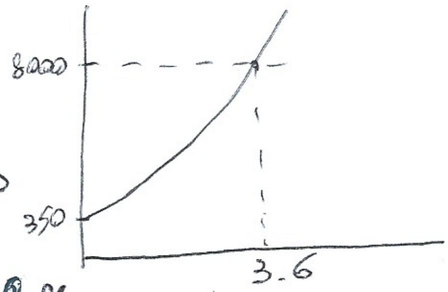
7. Jacob hopes to earn \$350 in interest in 3.6 years time from \$8,000 that he has available to invest. To decide if it's feasible to do this by investing in an account that compounds weekly, he needs to determine the annual interest rate such an account would have to offer for him to meet his goal. What would the minimum annual rate of interest have to be? Round percentages to two decimal places.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ WITH $P = 8000$, $n = 52$ (WEEKLY), $t = 3.6$, $A = 8350$

$$8000\left(1+\frac{r}{52}\right)^{52\cdot 3.6} = 8350 \Rightarrow$$

$$\Rightarrow \left(1+\frac{r}{52}\right)^{187.2} = \frac{167}{160} \Rightarrow 1+\frac{r}{52} = \left(\frac{167}{160}\right)^{\frac{1}{187.2}} \Rightarrow$$

$$\Rightarrow r = 52\left(\left(\frac{167}{160}\right)^{\frac{1}{187.2}} - 1\right) \approx .0119 = 1.19\,\%$$



WITH EARLY APPROXIMATION $\frac{1}{187.2} \approx .005342$ ONE GETS $r = 1.19\%$.

8. Solve the following logarithmic equation. If needed, round your answer to 4 decimal places.

$\log_4(x-3) + \log_4(3x-1) = 1$

DOMAIN $x-3>0 \Rightarrow x>3 \Rightarrow (3,\infty)$ AND $\Rightarrow (3,\infty)\cap\left(\frac{1}{3},\infty\right) = (3,\infty)$

OPTIONAL $3x-1>0 \Rightarrow x>\frac{1}{3} \Rightarrow \left(\frac{1}{3},\infty\right)$

$$\log_4\left((x-3)(3x-1)\right) = 1 \Rightarrow (x-3)(3x-1) = 4 \Rightarrow 3x^2 - x - 9x + 3 = 4 \Rightarrow$$

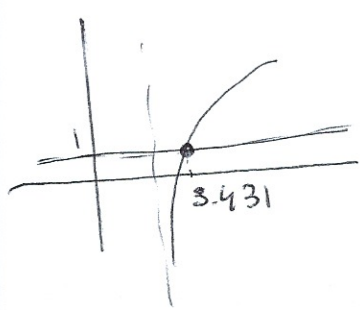
$$\Rightarrow 3x^2 - 10x - 1 = 0 \Rightarrow x = \frac{10 \pm \sqrt{112}}{6} = \frac{10 \pm 4\sqrt{7}}{6} = \frac{5 \pm 2\sqrt{7}}{3} \Rightarrow$$

$x \approx -.097,\ 3.431$

CHECK:

- $\log_4(-.097-3)$ UNDEFINED! $\Rightarrow \frac{5-2\sqrt{7}}{3}$ IS NOT A ROOT.
- $\log_4(3.431-3)$ AND $\log_4(3(3.431)-1)$ DEFINED $\Rightarrow x = \frac{5+2\sqrt{7}}{3}$ IS A ROOT.

GRAPHICALLY



MAT 121 – Exam2 – Extra – Spring 2018

Instructor: Dr. Francesco Strazzullo **Name** KEY

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Solve the rational inequality. Write your answer in interval notation (use two decimal places if needed).

$$\frac{5x}{x+2} < \frac{x-3}{x^2-4}$$

Y_1 Y_2

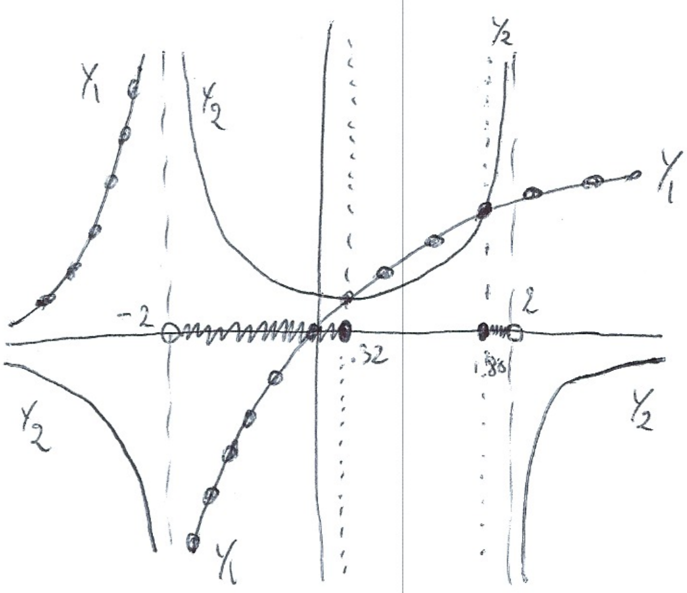
GRAPHICALLY

SPECIAL POINTS/X-VALUES:

INTERCEPTS: $X \approx .32, 1.88$

VERT. ASYMPTOTES:

$X = \pm 2$



$Y_1 < Y_2$ $Y_1 \neq Y_2$ EXCLUDES BOUNDARY POINTS (INTERSECTS)

$(-2, .32) \cup (1.88, 2)$

ALGEBRAIC/NUMERICAL

1) STD FORM: $LCD = x^2-4 = (x-2)(x+2)$

$$\frac{5x(x-2)-(x-3)}{(x-2)(x+2)} < 0$$

$$Y = \frac{5x^2-11x+3}{(x-2)(x+2)} < 0$$

2) X-INTERCEPTS: $5x^2-11x+3=0 \Rightarrow$

$$\Rightarrow x = \frac{11 \pm \sqrt{121-60}}{10} = \frac{11 \pm \sqrt{61}}{10} \approx 0.32,\ 1.88$$

V.A.: $x = 2, -2$

3) TEST:

	$-\infty$ to -2	-2 to $\frac{11-\sqrt{61}}{10}$	$\frac{11-\sqrt{61}}{10}$ to $\frac{11+\sqrt{61}}{10}$	$\frac{11+\sqrt{61}}{10}$ to 2	2 to ∞
X	-3	0	1	1.9	3
Y	$+$	$-$	$+$	$-$	$+$

4) $\left(-2, \frac{11-\sqrt{61}}{10}\right) \cup \left(\frac{11+\sqrt{61}}{10}, 2\right)$