MAT 121 – Final – Spring 2017

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use your own cheat sheet after I approve it, or the one on Eagleweb. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Consider the following equation of a line.

$$2 + \frac{y - 5x}{3} = 1$$

Find the equation, in slope-intercept form, for the line which is **parallel** to this line and passes through the point $(3, -4)$.

$Y - 5X = 3(1-2) \Rightarrow Y = 5X - 3 \Rightarrow m = 5$ (PARALLEL)

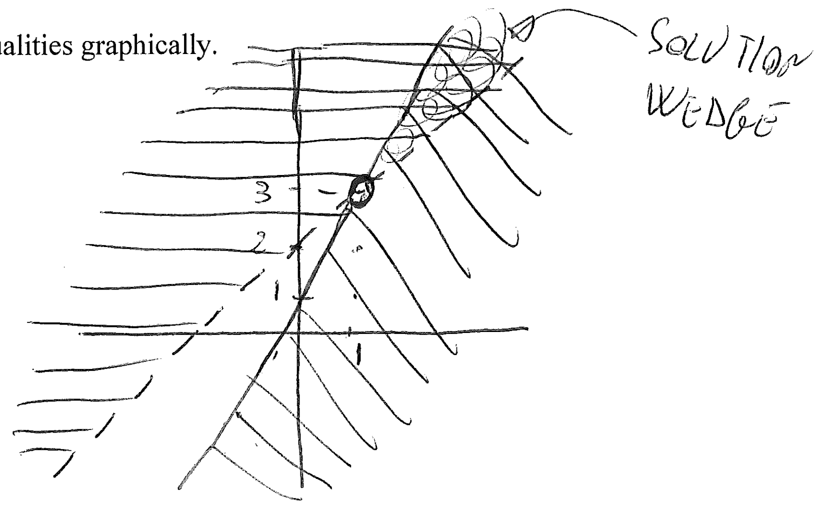
$Y = 5X + b$ PLUG POINT THEN $\Rightarrow -4 = 5(3) + b \Rightarrow b = -19$

$-15 \quad -15$

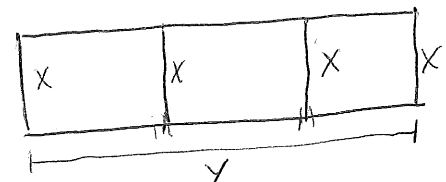
$\boxed{Y = 5X - 19}$

2. Solve the system of two linear inequalities graphically.

$\underbrace{y \leq}_{\text{SOLID BELOW}} 2x + 1 \underbrace{\text{and}}_{\text{INTERSECTION OVERLAP}} \underbrace{y >}_{\text{DASHED ABOVE}} x + 2$



3. A rancher has 800 feet of fencing to put around a rectangular field and then subdivide the field into 3 identical smaller rectangular plots placing two fences parallel to one of the field's shorter sides. Find the dimensions that maximize the enclosed area. Write your answers as fractions reduced to lowest terms.



$4X + 2Y = 800 \Rightarrow Y = 400 - 2X$

MAXIMIZE: $A = XY$

$\Rightarrow A = x(400 - 2x) = -2x^2 + 400x$ MAX AT VERTEX:

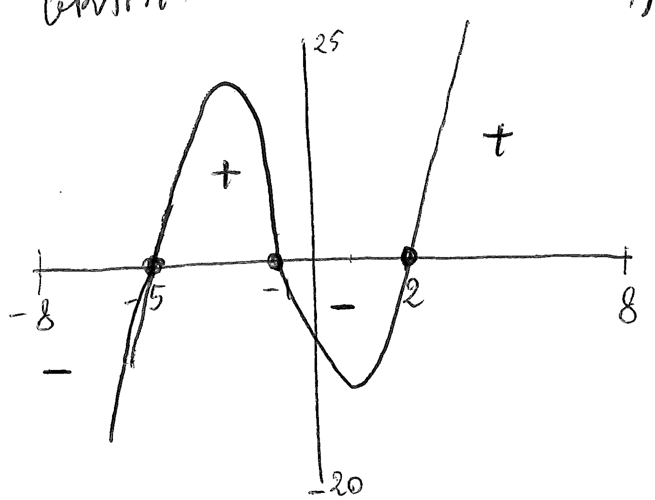
$x = h = -\frac{b}{2a} = -\frac{400}{2(-2)} = 100$ FEET.

THEN $Y = 400 - 2(100) = 200$.

EACH PLOT IS 100 BY $\frac{200}{3}$ FEET, THE TOTAL FIELD IS 100 BY 200 FT.

4. Solve the polynomial inequality $(x - 2)(x + 5)(x + 1) \leq 0$. Write your answer in interval notation.

GRAPH: BOUNDARY POINTS (ROOTS), $X = -5, -1, 2$, ARE INCLUDED ("=")



SOLUTION: $(-\infty, -5] \cup [-1, 2]$

5. Use polynomial long division to rewrite the following rational function in the form $f(x) = q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

$$f(x) = \frac{x^4 - 6x^3 - 10x^2 + 2x + 1}{x^2 + 4}$$

(a) describe the domain of $f(x)$ in interval notation,
(b) determine the equations of all (if any) asymptotes, and
(c) sketch the graph of $f(x)$ (you can use a graphing utility and copy the graph here).

(a) $d(x) \neq 0$: $x^2 + 4 \neq 0$ FOR ALL REALS   DOMAIN $= (-\infty, \infty)$

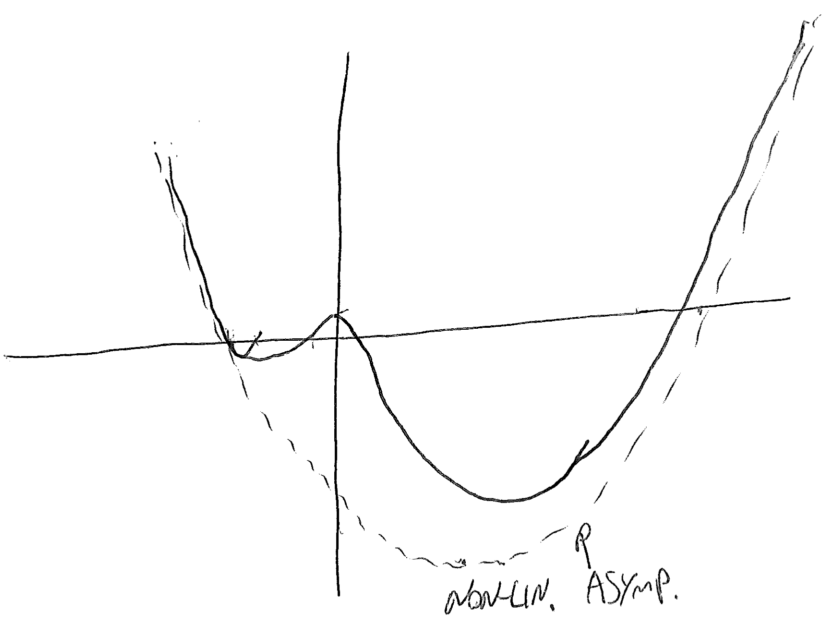
(b) YOU CAN USE A CAS TO DIVIDE:

$$\begin{array}{r|l}
 & x^2 - 6x - 14 \\
\hline
x^2+4 & x^4 - 6x^3 - 10x^2 + 2x + 1 \\
 & -x^4 \qquad\quad -4x^2 \\
\hline
 & -6x^3 - 14x^2 + 2x + 1 \\
 & 6x^3 \qquad\quad +24x \\
\hline
 & -14x^2 + 26x + 1 \\
 & 14x^2 \qquad +56 \\
\hline
 & 26x + 57
\end{array}$$

$$f(x) = \underbrace{x^2 - 6x - 14} + \frac{26x+57}{x^2+4}$$

(c) $q(x) = x^2 - 6x - 14$ QUADRATIC ASYMPTOTE

NO VERTICAL ASYMPTOTE BECAUSE THE DOMAIN IS $\mathbb{R}$



6. Chester hopes to earn \$1200 in interest in 3.5 years from \$20,000 that he has available to invest. To decide if it's feasible to do this by investing in an account that compounds quarterly, he needs to determine the annual interest rate such an account would have to offer for him to meet his goal. What would the annual rate of interest have to be? Round to two decimal places.

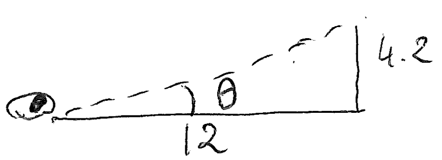
$A = P + I = 20000 + 1200 = 21200$; QUARTERLY $\Rightarrow n = 4$; $t = 3.5$

$A = P\left(1 + \frac{r}{n}\right)^{nt} \Rightarrow 20000\left(1 + \frac{r}{4}\right)^{4(3.5)} = 21200 \Rightarrow$

$\Rightarrow \left(1 + \frac{r}{4}\right)^{14} = \frac{53}{50} \Rightarrow 1 + \frac{r}{4} = \sqrt[14]{\frac{53}{50}} \Rightarrow r = 4\left(\sqrt[14]{\frac{53}{50}} - 1\right) \approx$

$\approx 0.01668 \Rightarrow r = 1.67\%$

7. Ethan is watching a satellite launch from an observation spot 12 miles away. Find the angle of elevation from Ethan to the satellite, which is at a height of 4.2 miles. Write your answer in degrees rounded to two decimal places.



$\tan\theta = \frac{4.2}{12} = .35 \Rightarrow$

$\Rightarrow \theta = \tan^{-1}(.35) \approx 19.29^\circ$

8. Determine the amplitude, period, and phase shift of the following trigonometric equation.

$$y = 5 - 3\sin\left(3\pi x + \frac{\pi}{3}\right) = A\sin(Bx + C) + D$$

AMPLITUDE $= |A| = |-3| = 3$

PERIOD $= \frac{2\pi}{B} = \frac{2\pi}{3\pi} = \frac{2}{3}$

PHASE SHIFT (UNITS) $= \frac{C}{B} = \frac{\pi/3}{3\pi} = \frac{1}{9}$

$C > 0 \Rightarrow$ P. S. TO LEFT.

9. Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{cases} -2x - 4y + z = -1 \\ 3x + 5y - 4z = 3 \\ 4x - 3y + 2z = 1 \end{cases}$$

AUGMENTED $= \left[\begin{array}{ccc|c} -2 & -4 & 1 & -1 \\ 3 & 5 & -4 & 3 \\ 4 & -3 & 2 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/9 \\ 0 & 1 & 0 & -1/9 \\ 0 & 0 & 1 & -5/9 \end{array}\right] \Rightarrow$

SYSTEM $\begin{cases} x = 4/9 \\ y = -1/9 \\ z = -5/9 \end{cases}$

UNIQUE SOLUTION $\left(\frac{4}{9}, \frac{-1}{9}, -\frac{5}{9}\right) \approx (.\bar{4}, -.\bar{1}, -.\bar{5})$

10. The table below gives the 4-year graduation rate for traditional students enrolled at a State College, per selected graduation years.

Year	1964	1972	1978	1985	1990	1995	2000	2005	2010	2016
Graduation Rate (%)	28.6	28.9	26.2	29.2	29.8	30.3	32	33.4	35	37.6

Consider x to be the number of years after 1960, and y to be 4-year graduation rate (%) for the graduating class of that year.

(a) Use technology to compute the cubic and the power regression models for these data.
(b) Use the best model among those of the previous step to estimate the 4-year graduation rates for the 2014 and the 2017 graduating classes (rounded to the first decimal place).

(a) CUBIC: $Y = -.00005X^3 + .0099X^2 - .26138X + 29.65061$

$R^2 = .95572$

POWER: $Y = 22.96052 \cdot X^{.09211}$, $R^2 = .69097$

(b) BEST MODEL IS THE CUBIC, WITH LARGER R^2.

2014 → $X = 54$, $Y = 36.7\%$

2017 → $X = 57$, $Y = 37.9\%$