70/70 + 10/70

MAT 121 – Exam2 – Spring 2015

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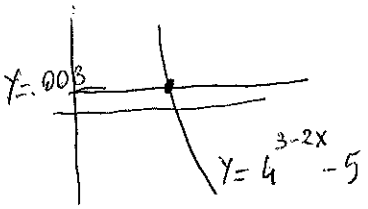
Name KEY

Instructions. Complete 7 out of the following 10 exercises, as indicated. Exercise 11 is for extra points. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**. You can use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used. **SHOW YOUR WORK NEATLY, PLEASE (no work, no credit)**.

Complete 1 of the exercises 1-2

1. Solve for x: $4^{3-2x} - 5 = 0.003$.



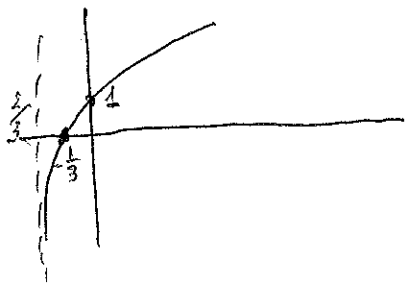
$4^{3-2x} = 5.003 \Rightarrow \log_4 4^{3-2x} = \log_4 5.003$

$\Rightarrow 3-2x = \log_4 5.003 \Rightarrow -2x = \log_4 5.003 - 3$

$\Rightarrow x = \frac{3}{2} - \frac{1}{2}\log_4 5.003 \simeq \boxed{.919}$

$\log_4(5.003) \simeq 1.161$

2. Identify the x-intercept of the function $f(x) = \log_3(3x+2)$.



X-INTERCEPT: $Y = 0$

$\log_3(3x+2) = 0 \Rightarrow 3x+2 = 1$

NOTE: $3^{\log_3(3x+2)} = 3^0$

THEN: $3x = -1 \Rightarrow \boxed{x = -\frac{1}{3}}$

IN GGB: $f(x) = \log(3, 3x+2)$

IN OLD TI: $f(x) = \dfrac{\log(3x+2)}{\log 3}$

Complete 2 of the exercises 3-5

3. Evaluate the function $f(x) = 2.3 \log_{1.2}(2x + 1)$ at $x = 5$.

$f(5) = 2.3 \log_{1.2}(2(5)+1) \approx 30.25$

≈ 30.249622722

4. What is the value of the function $f(x) = 3.7(2^{-4.7x})$ at $x = 1.5$?

$f(1.5) = 3.7(2^{-4.7(1.5)}) \approx .027921597$

$\approx .028$

5. Identify the value of the function $f(x) = \ln(5 - 7x)$ at $x = -2$.

$f(-2) = \ln(5 - 7(-2)) = \ln 19 \approx 2.944438979$

≈ 2.944

9

Complete both exercises 6-7

6. A water reservoir is exponentially leaking its content, so that half of its content is lost in 3 hours. How long does it take the reservoir to dry up?

t = TIME IN HOURS, C = WATER IN RESERVOIR, C_0 = INITIAL CONTENT

$C = C_0 e^{-kt} \Rightarrow \frac{C_0}{2} = C_0 e^{-k\cdot 3} \Rightarrow e^{-3k} = \frac{1}{2} \Rightarrow -3k = \ln\left(\frac{1}{2}\right)$

$\Rightarrow k = -\frac{1}{3}\ln\left(\frac{1}{2}\right) = \ln\left(\left(\frac{1}{2}\right)^{-\frac{1}{3}}\right) = \ln\sqrt[3]{2} \Rightarrow C = C_0 e^{-\ln\sqrt[3]{2}\, t} \Rightarrow$

$\Rightarrow C = C_0\left(\sqrt[3]{2}\right)^{-t}$ OR $C = C_0 2^{-\frac{t}{3}}$

DRY-UP $\Rightarrow C \approx 0 \Rightarrow C < 1$. NOT ENOUGH INFORMATION AS THIS IS.

ONE COULD ASSUME THAT $C = \frac{C_0}{1000} < 1$ AND SOLVE: $\frac{C_0}{1000} = C_0\left(\sqrt[3]{2}\right)^{-t}$

$\Rightarrow \left(\sqrt[3]{2}\right)^{-t} = \frac{1}{1000} \Rightarrow -t = \log_{\sqrt[3]{2}}(.001) \Rightarrow t \approx 30$ HOURS

7. A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1000 animal and that the growth of the pack will be modeled by the logistic curve

$$p(t) = \frac{1000}{1 + 9e^{-0.1656t}}$$

where t is measured in months.

(a) What was the population after 5 months?

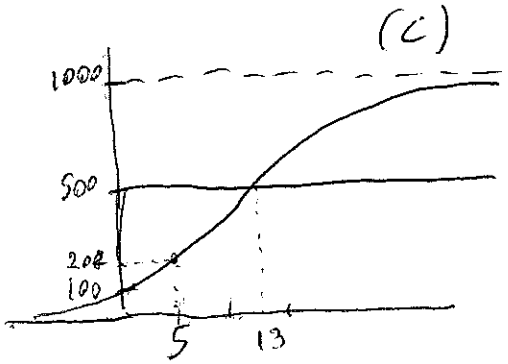
(b) After how many months will the population be 500?

(c) Graph this model and estimate the horizontal asymptotes.

(a) $p(5) = 202$; (b) $p(t) = 500 \Rightarrow$

$\Rightarrow t \approx 13.3$ MONTHS

(c)



(C) $Y = 1000$ (CARRYING CAPACITY) IS A HORIZONTAL ASYMPTOTE, BUT ALSO $Y = 0$.

Complete 1 of the exercises 8-9

8. The pH of an acidic solution is a measure of the concentration of the acid particles in the solution, with smaller values of the pH indicating higher acid concentration. In a laboratory experiment, the pH of a certain acid solution is changed by dissolving over-the-counter antacid tablets into the solution. In this experiment, the pH changes according to the equation

$$\text{pH} = 3.5 - \log\left(\frac{x-3}{2x}\right),$$

where x is the number of grams of antacid added to the solution. What is the pH of the solution after the addition of 3.5 grams of antacid tablet?

ASKED FOR $pH(3.5) \approx 4.646$

9. The chemical acidity of a solution is measured in units of pH: $\text{pH} = -\log\left[\text{H}^+\right]$, where $\left[\text{H}^+\right]$ is the hydrogen ion concentration in the solution. What is $\left[\text{H}^+\right]$ if pH = 4.6?

$pH = -\log[H^+] \Rightarrow [H^+] = 10^{-pH}$

$[H^+](4.6) = 10^{-4.6} \approx 2.511886 \times 10^{-5} \approx .000025$

Complete this exercise: you can download the data-sheet from the coursework section in EagleWeb.

10. In the following table it is reported the number (in thousand) of full-time employed people in USA that held a Bachelor degree or higher, for odd years between 2005 and 2013 (use the xls file on EagleWeb).

X	Year	BA or Higher (thousand people)	BA only (thousand people)
5	2005	30607	19843
7	2007	33157	21673
9	2009	33080	21271
11	2011	34353	21834
13	2013	36982	23606

Consider x to be the number of years after 2000, and y to be the full-time employed US population (in thousand) holding only a BA. Use technology to answer to the following questions.

(a) Find the cubic and the quadratic models that are the best fit for these data. (Round your answer to five decimal places).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate full-time employed US population holding only a BA in 2015. Round to the nearest employed graduate using the greatest integer function.

(a) WE MUST ONLY USE THE 3RD COLUMN FOR Y AND THE X HAS VALUE ZERO IN 2000.

QUADRATIC: $Y = 15.16071\,X^2 + 111.45714\,X + 19292.98214$

$R^2 = .81997$

CUBIC: $Y = 35.84375\,X^3 - 952.62054\,X^2 + 8334.0139\,X - 2449.83666$

$R^2 = .98286$

(b) CUBIC IS BETTER, BECAUSE IT HAS A LARGER R^2.

(c) $X = 15$ AND $Y = 29193.39994 \Rightarrow$ 29,193,399 FULL-TIME EMPLOYED PEOPLE WITH ONLY BA.

Extra points

11. The amount of time (in hours per week) a student utilizes a math-tutoring center roughly follows the normal distribution $y = 0.93e^{\frac{(x-6.2)^2}{0.75}}$, $2 \le x \le 9$, where x is the number of hours.

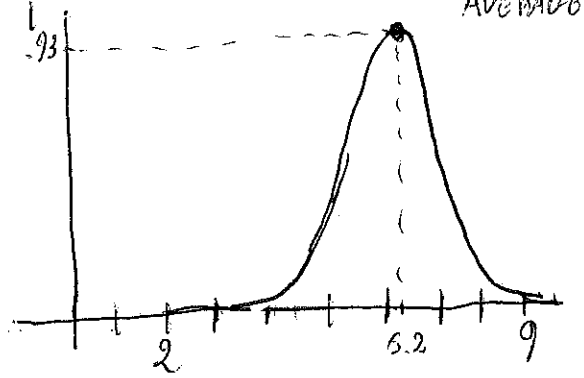
(a) Graph this distribution.

(b) Using part (a) estimate the average number of hours per week a student uses the math-tutoring center.

NOTE: NORMAL DISTRIBUTION CURVES ARE $Y = a e^{-\frac{(x-\mu)^2}{\sigma}}$

HERE: $a = .93$, $\mu = 6.2$ AVERAGE, $\sigma = .75$ STANDARD DEVIATION

(a) - (b)



AVERAGE INPUT AT MAXIMUM: $x = 6.2$