Math 102 - Fall 2011 - Test 2

KEY

Instructor: Dr. Francesco Strazzullo   Name

Instructions. Only calculators are allowed on this examination. Each problem is worth 10 points. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. You want to order some pizza for your friends. You have a budget of $80. There are medium pizzas at $5 each and large ones at $9 each. The delivery can carry at most 30 boxes.
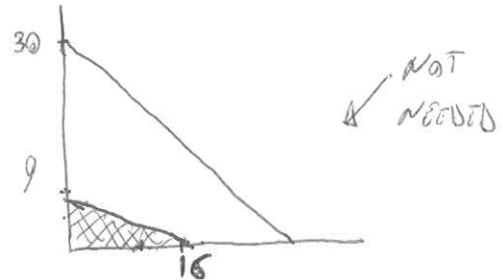
   (a) Set up a system of linear inequality describing the constraints for your order.

   (b) List three possible orders.

a) COSTS: X MIDIUM   Y LARGE   BUDGET

$$5X + 9Y \le 80$$

DELIVERY: $X + Y \le 30$

30

9

16

NOT NEEDED

b) $(0,8)$, $(1,8)$, $(16,0)$

2. Graph the solution set of the following system of linear inequalities, labeling the corner point and highlighting the contour.

$$\begin{cases} 3x & -2y & \le 4 \\ 2x & +y & > 3 \end{cases} \longrightarrow Y > -2x + 3$$

SOLVE FOR Y $\dfrac{-2Y}{-2} \le \dfrac{-3X}{-2} + \dfrac{4}{-2} \longrightarrow Y \ge \dfrac{3}{2}X - 2$

$\begin{cases} Y \ge \dfrac{3}{2}X - 2 & (\text{ABOVE B.L.}) \\ Y > -2X + 3 & (\text{ABOVE B.L.}) \end{cases}$

BOUNDARY LINES:

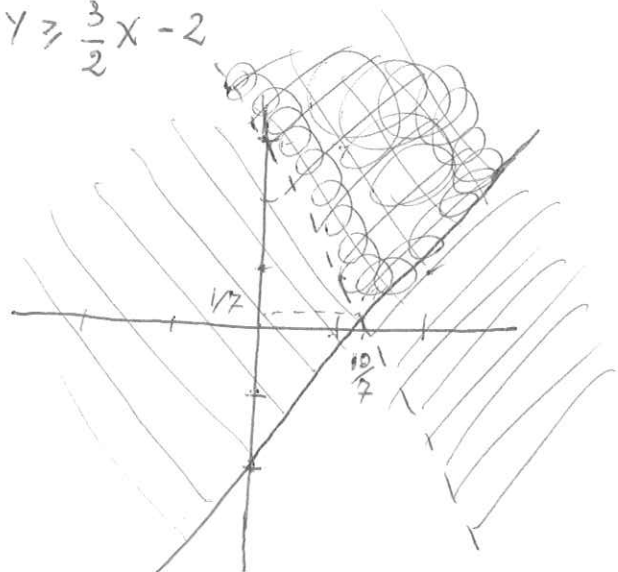SOLID $\begin{cases} Y = \dfrac{3}{2}X - 2 \\ \end{cases}$
DASHED $\begin{cases} Y = -2X + 3 \end{cases}$

CORNER POINT (GRAPH. OR ALG.)

$-2X + 3 = \dfrac{3}{2}X - 2 \longrightarrow -\dfrac{7}{2}X = -5$

$\longrightarrow X = \left(-\dfrac{2}{7}\right)(-5) = \dfrac{10}{7} \sim 1.43$

$\longrightarrow Y = -2\left(\dfrac{10}{7}\right) + 3 = \dfrac{1}{7} \sim .43$

1

20)

3. You found a bag of dollar bills and you return it to the police. Later on, an officer tells you that the bag contained three kinds of bills: an O amount of \$1 bills, an F amount of \$5 bills, and an L amount of \$50 bills. The bag contained 110 bills worth \$1620. Moreover, there were three times \$50 bills than \$5 bills. Write a system of three linear equations in three variables modeling the bag content, then find out each amount of bills.

ITEMS BILLS	QUANTITY AMOUNT	RATE \$/EA	Q·R VALUE \$
ONE	O	1	1O
FIVE	F	5	5F
FIFTY	L	50	50L
TOTAL	110		1620

$$\begin{cases} O + F + L = 110 \\ O + 5F + 50L = 1620 \\ 3F - L = O \end{cases}$$

" THREE TIMES MORE L THAN F "

IF $F = 1$, THEN $L = 3 \cdot 1 = 3$

IF $F = 2$, THEN $L = 3 \cdot 2 = 6$

THUS $L = 3F$

↓ STANDARD FORM

$$3F - L = O$$

↓ MATRIX

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 110 \\ 1 & 5 & 50 & \vdots & 1620 \\ 0 & 3 & -1 & \vdots & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & \vdots & 70 \\ 0 & 1 & 0 & \vdots & 10 \\ 0 & 0 & 1 & \vdots & 30 \end{bmatrix} \xrightarrow{SOLUTION}$$

$$\rightarrow \begin{cases} O = 70 \\ F = 10 \\ L = 30 \end{cases}$$

THERE WERE 70 \$1 BILLS, 10 \$5 BILLS, AND 70 \$50 BILLS.

10

4. Solve the following systems of linear equations.

(a)
$$\begin{cases} x - 2y - 3z + w = 1 \\ 2x \quad\quad - z \quad\quad = 4 \\ x + 2y - 4z + w = -6 \\ -y + z + 2w = -1 \end{cases}$$

4×5 ↓ MATRIX

$$\begin{bmatrix} 1 & -2 & -3 & 1 & | & 1 \\ 2 & 0 & -1 & 0 & | & 4 \\ 1 & 2 & -4 & 1 & | & -6 \\ 0 & -1 & 1 & 2 & | & -1 \end{bmatrix} \xrightarrow{RREF} \begin{matrix} x & y & z & w \\ \begin{bmatrix} 1 & 0 & 0 & 0 & | & 133/54 \\ 0 & 1 & 0 & 0 & | & -41/27 \\ 0 & 0 & 1 & 0 & | & 25/27 \\ 0 & 0 & 0 & 1 & | & -31/18 \end{bmatrix} \end{matrix}$$

SOLUTION →
$$\begin{cases} x = 133/54 \approx 2.46 \\ y = -41/27 \approx -1.52 \\ z = 25/27 \approx .93 \\ w = -31/18 \approx -1.7 \end{cases}$$

(b)
$$\begin{cases} x \quad\quad - 2z = 6 \\ x + y - 6z = 4 \end{cases}$$

2×4 ↓ MATRIX

$$\begin{bmatrix} 1 & 0 & -2 & | & 6 \\ 1 & 1 & -6 & | & 4 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -2 & | & 6 \\ 0 & 1 & -4 & | & -2 \end{bmatrix}$$ SOLUTION →

$$\rightarrow \begin{cases} x - 2z = 6 \rightarrow x = 2z + 6 \\ y - 4z = -2 \rightarrow y = 4z - 2 \\ z = z \end{cases}$$

OR $(2z+6, \ 4z-2, \ z)$

3

5. Solve the equation $4x^2 + 4x - 15 = 0$. (Show your work)

BY GRAPH OR QUADRATIC FORMULA $\left(x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$

HERE BY GROUPING:

PRODUCT: $-15(4) = -60 = -2^2 \cdot 3 \cdot 5 \rightarrow 10, -6$ ⎤ → SPLIT THE "x" TERM:

SUM: $4 = 10 - 6$ ✓

$4x^2 + 10x - 6x - 15 = 0 \rightarrow 2x(2x+5) - 3(2x+5) = 0$

$\rightarrow (2x+5)(2x-3) = 0$ ⎰ $2x+5 = 0 \rightarrow 2x = -5 \rightarrow x = -5/2$

⎱ $2x - 3 = 0 \rightarrow 2x = 3 \rightarrow x = 3/2$

6. A ball is kicked from the 7th row of a stadium, 60 feet above the ground, at a speed of 90 feet per second. The height h of the ball depends on the time t after the kick, according to the quadratic model $h = -16t^2 + 90t + 60$.

   (a) How long does it take the ball to hit the ground?

   (b) After how many seconds will the ball be 83 feet above the ground?

⎤ WE COULD USE THE GRAPH.

a) GROUND → $h = 0$ →

$\rightarrow -16t^2 + 90t + 60 = 0$ DIVIDE BY -2

$\rightarrow 8t^2 - 45t - 30 = 0$

$t = \dfrac{45 \pm \sqrt{(-45)^2 - 4(8)(-30)}}{2(8)} \rightarrow$

$\rightarrow t = \dfrac{45 \pm \sqrt{2985}}{16} = \dfrac{45 \pm 54.64}{16}$ ⎰ $t = -.6$ NOT POSSIBLE

                                                        ⎱ $t = 6.23$ SECONDS

b) $h = 83 \rightarrow -16t^2 + 90t + 60 = 83 \rightarrow -16t^2 + 90t - 23 = 0$

$t = \dfrac{-90 \pm \sqrt{90^2 - 4(16)(-23)}}{2(-16)}$ ⎰ $t = 5.36$ SECONDS

                                                        ⎱ $t = .27$ SECONDS

4

20

7. The Callmeback phone company charges $15.50 plus 2.5 cents per minute calls for the first 500 minutes and $28 plus 15.5 cents per minutes above 500, every month.   y   x

(a) Write the function that gives the monthly charge in dollars as a function of the minutes used.

WE NEED TO CHANGE THE RATES FROM ¢/min TO $/min : $2.5 \frac{¢}{m} = .025 \text{ }\$/m$

$15.5 \text{ }¢/m = .155 \text{ }\$/m$.

$$Y = \begin{cases} Y_1 & 15.5 + .025X \quad , \quad 0 \le X \le 500 \\ Y_2 & 28 + .155(X-500) \quad , \quad X > 500 \end{cases}$$

(b) What is the monthly charge if 20 minutes are used?

$X = 20 \rightarrow Y = Y_1 = 15.5 + .025(20) = 16$ DOLLARS

$0 \le 20 \le 500$

(c) What is the monthly charge if 620 minutes are used?

$X = 620 > 500 \rightarrow Y = Y_2 = 28 + .155(620-500) = 46.60$ DOLLARS

10)