MAT 102 - Exam3 - Fall 2013

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Instructions. Each problem is worth 10 points. If you solve a problem graphically then draw the graph you used. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded to two decimal places. You might need some of the following formulas:

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$, and $A^2 - B^2 = (A-B)(A+B)$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the following polynomial equation.

$$x^3 - 7x^2 + 6x = 0$$

GRAPH OR ALGEBRA:

$x(x^2 - 7x + 6) = 0$

$x(x-6)(x-1) = 0$

$x = 0$

$x - 6 = 0 \Rightarrow x = 6$

$x - 1 = 0 \Rightarrow x = 1$

$\boxed{x = 0, 6, 1}$

2. Solve the following polynomial inequality, writing your answer in interval notation.

$$\underbrace{(2x-3)(x+4)(5-x)}_{f(x)} \le 0$$

BY GRAPH OR ALGEBRA: "BOUNDARY POINTS" = "X-INTERCEPTS" ⇒

$2x - 3 = 0 \Rightarrow x = 3/2$

$x + 4 = 0 \Rightarrow x = -4$

$5 - x = 0 \Rightarrow x = 5$

NOTE THAT "≤" IMPLIES THAT B.P. ARE IN THE SOLUTION SET.

B.P.	-4	3/2	5	
TEST X	-5	0	2	6
SIGN OF f(x)	+	−	+	−

SOLUTION

$\boxed{\text{SOLUTION SET: } [-4, 3/2] \cup [5, \infty)}$

20)

3. Use polynomial long division to rewrite the following fraction in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

$$f(x) = \frac{4x^4 - 6x^3 + 8x^2 + 4x + 3}{2x^2 + 1}$$

$$\begin{array}{r|l}
 & 2x^2 - 3x + 3 \\
\hline
2x^2+1 & 4x^4 - 6x^3 + 8x^2 + 4x + 3 \\
 & -4x^4 \qquad\quad -2x^2 \\
\hline
 & -6x^3 + 6x^2 + 4x + 3 \\
 & 6x^3 \qquad\quad +3x \\
\hline
 & 6x^2 + 7x + 3 \\
 & -6x^2 \qquad -3 \\
\hline
 & 7x
\end{array}$$

$$f(x) = \boxed{2x^2 - 3x + 3 + \frac{7x}{2x^2+1}}$$

4. Find (if any) the x-intercept(s) and the turning point(s) of the following polynomial function (report them as ordered pairs).

$$s(x) = x^3 - 5x^2 + 9x - 45$$

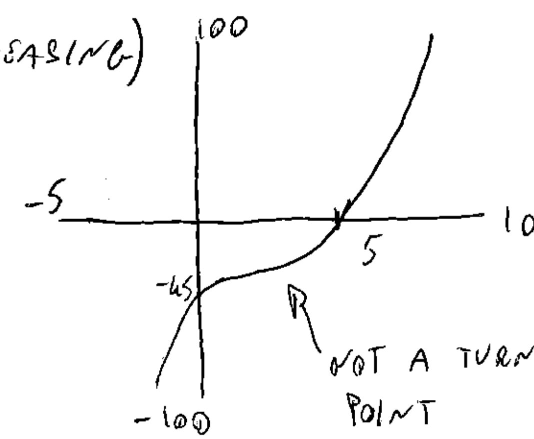
TURNING POINTS: NONE (ALWAYS INCREASING)

X-INTERCEPT: $(5, 0)$

NOTE: $s(x) = x^2(x-5) + 9(x-5)$

$= (x^2+9)(x-5)$

$= (x-3i)(x+3i)(x-5)$



NOT A TURNING POINT

BECAUSE THERE IS ONLY ONE X-INTERCEPT

20

5. Knowing that $-5i$ is one of its roots, factor the following polynomial function completely

$$f(x) = x^4 - 6x^3 - 2x^2 - 150x - 675$$

BY GRAPH OR TABLE ONE CAN FIND INTEGRAL ROOTS: $X = -3, 9$.

BY ALGEBRA;

$X = -5i$ ROOT $\Rightarrow X = +5i$ ROOT $\Rightarrow f(x) = g(x)\cdot(x-5i)(x+5i) = g(x)\,(x^2+25)$

$$\begin{array}{r|l} & x^2 - 6x - 27 = g(x) \\ \hline x^2+25 & x^4 - 6x^3 - 2x^2 - 150x - 675 \\ & -x^4 \quad\quad -25x^2 \\ \hline & -6x^3 - 27x^2 - 150x - 675 \\ & 6x^3 \quad\quad +150x \\ \hline & -27x^2 \quad - 675 \\ \hline & 0 \end{array}$$

$g(x) = x^2 - 6x - 27 = (x-9)(x+3)$

COMPLETE FACTORIZATION AS IN THE FUNDAMENTAL THEOREM OF ALGEBRA

$f(x) = (x-9)(x+3)(x-5i)(x+i5)$

6. Given the following rational function:

$$f(x) = \frac{x^4 - 5x^3 - 8x + 40}{40\,(x^2 - 25)}$$

(a) Find the domain of $f(x)$ and the equations for the vertical asymptotes, if any.
(b) Find equations for the horizontal or oblique asymptotes of $f(x)$, if any.
(c) Sketch the graph of $f(x)$.

(a) DOMAIN: $40(x^2-25) \neq 0 \Rightarrow x \neq \pm 5 \Rightarrow (-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$40(x^2-25) = 0 \Rightarrow 0 = x^2 - 25 = (x-5)(x+5)$

POSSIBLE ASYMPTOTES ARE $x = 5$, $x = -5$

SIMPLIFIED FORM

LONG DIVISION: $f(x) = \frac{1}{40}\left(\frac{x^3(x-5) - 8(x-5)}{(x-5)(x+5)}\right) = \frac{1}{40}\ \frac{x^3-8}{x+5}$

$$\begin{array}{r|l} & x^2 - 5x + 25 \\ \hline x+5 & x^3 - 8 \\ & -x^3 - 5x^2 \\ \hline & -5x^2 - 8 \\ & -5x^2 + 25x \\ \hline & 25x - 8 \\ & -25x - 125 \\ \hline & -133 \end{array}$$

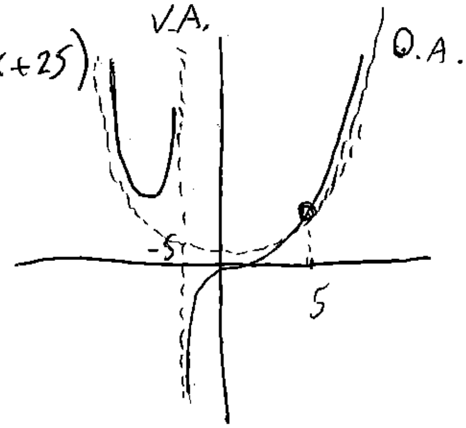
AT $x = 5$ IS DEFINED $\Rightarrow x = 5$ NOT A V.A

AT $x = -5$ IS $\frac{\#}{0} \Rightarrow \boxed{x = 5 \ \text{V.A}}$

SIMPLIFIED FORM

(b) OBLIQUE ASYMP. $Y = \frac{1}{40}(x^2 - 5x + 25)$

(c)



$$\boxed{\begin{array}{l} f(x) = \dfrac{x^2-5x+25}{40} + \dfrac{-133}{40(x+5)} \\ \text{ON DOMAIN} \end{array}}$$

7. The daily cost for producing and selling donuts at a local store is modeled by the function

$$C(x) = 0.00167x^3 - 0.067x^2 + 0.03x + 20,$$

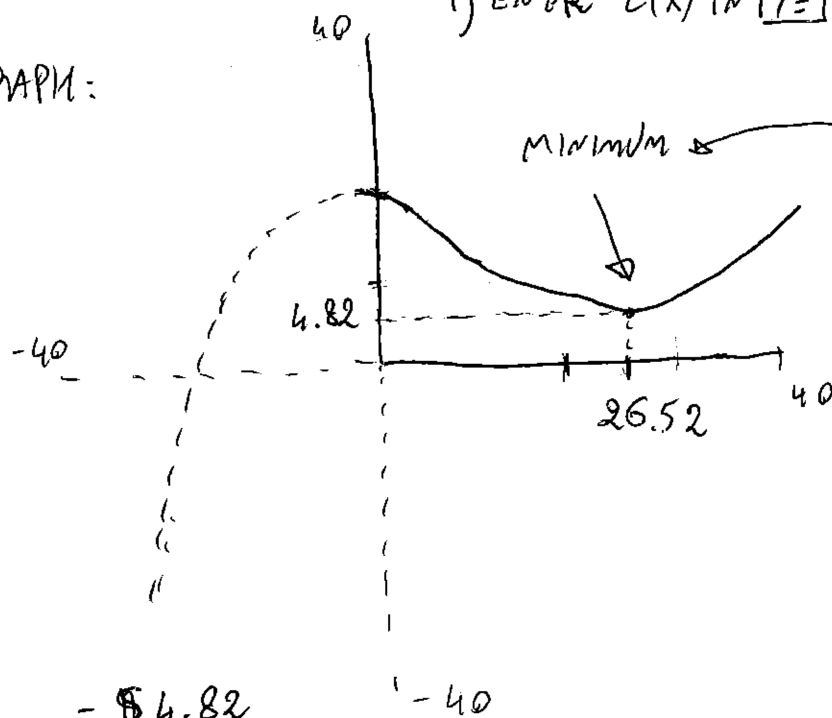
where C is in dollars and x represents the number of dozens of donuts sold in one day. How many donuts should be produced in one day in order to minimize the cost?

X = "DOZENS OF DONUTS" MUST BE POSITIVE

1) ENTER C(X) IN $\boxed{Y=}$; 2) $\boxed{2^{ND}}$ + $\boxed{TRACE}$ + $\boxed{3}$

3) FOLLOW INSTRUCTIONS

GRAPH:



$C_{MINIMUM} = \$4.82$

$X_{MINIMUM} = 26.52$

X = 1 ⇒ 12 DONUTS

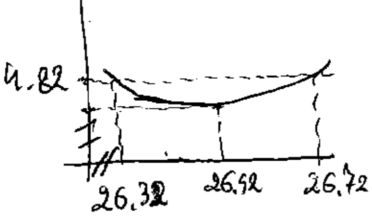
→ MINIMUM COST FOR $(26.52)(12) \approx$ $\boxed{318 \text{ DONUTS}}$

NOTE THAT, BY THE DOZEN, ONE SHOULD APPROXIMATE X_{MIN} WITH AN INTEGER, THEREFORE WE SHOULD TEST

X	26	27
C(X)	4.84	4.838

. BECAUSE IN BOTH CASES WE HAVE A LARGER COST OF \$4.84, WE MUST LOOK CLOSELY AT THE INTERSECTION BETWEEN C(X) AND Y = 4.82.

IN PRACTICE (ROUNDING TO THE CENT) C(X) = 4.82 FOR



$26.32 \le X \le 26.72$ THAT IS WHEN PRODUCING/SELLING BETWEEN 316 AND 320 DONUTS (AND 318 IS THE MIDPOINT)