Math 102 - Spring 2010 - Test 1

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Instructions. Only calculators are allowed on this examination. Each problem is 10 points worth. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. A company charting its profits notices that the relationship between the number of units sold, x, and the profit, P, is linear. If 200 units sold results in \$3750 and 425 units sold results in \$6000 profit, write the equation that models its profit.   X = UNITS SOLD , Y = PROFIT IN \$

$$\text{SLOPE} = \frac{6000-3750}{425-200} = 10$$

SLOPE - POINT FORM : $Y - 6000 = 10(X - 425)$

(NOT NEEDED $Y = 10X + 1750$)

2. The total U.S. population during 1940 to 1970, for selected years, is shown in the table below, with the population given in thousands.

Year	1940	1942	1945	1950	1960	1970
Population	125,931	120,320	120,052	125,973	167,726	209,948

(a) Find the average annual rate of change in population during 1945–1960, with the appropriate units.

"AVERAGE RATE OF CHANGE" $= \text{SLOPE} = \dfrac{167726 - 120052}{1960 - 1945} = 3178.267$ THOUSAND PEOPLE PER YEAR

(b) Use the slope from part (a) and the population from 1945 to write the equation of the line associated with 1945 and 1960.

LET X BE YEARS AFTER 1945, Y BE POPULATION IN THOUSAND PEOPLE

$$Y = 3178.267\,X + 120052$$

3. The total U.S. soda drinks advertising and promotional expenditures can be modeled by the equation $y = 241.145x + 431.88$, where y is measured in millions of dollars and x is the number of years from 1980. If this model remains accurate, what are the expected advertising and promotional expenditures in 2015?

YEAR 2015 IS $X = 2015 - 1980 = 35$

$Y(35) = 241.145 \cdot 35 + 431.88 = 8871.955$

IN 2015 THIS MODEL PROJECTS THE EXPENDITURES TO BE OF ABOUT 8,871.955 MILLION DOLLARS.

4. The total prescription drugs sales in a local pharmacy for the years 1975–2008 can be modeled by the function $s = 3.5t + 27.2$, where s is in thousand dollars and t is the number of years after 1975. During what year does the model estimate the sales to be \$142,700?

SALES OF \$142,700 IS $S = 142.7$

$$142.7 = 3.5t + 27.2 \rightarrow \frac{3.5t}{3.5} = \frac{115.5}{3.5} \rightarrow$$
$$-27.2 \qquad\qquad -27.2$$

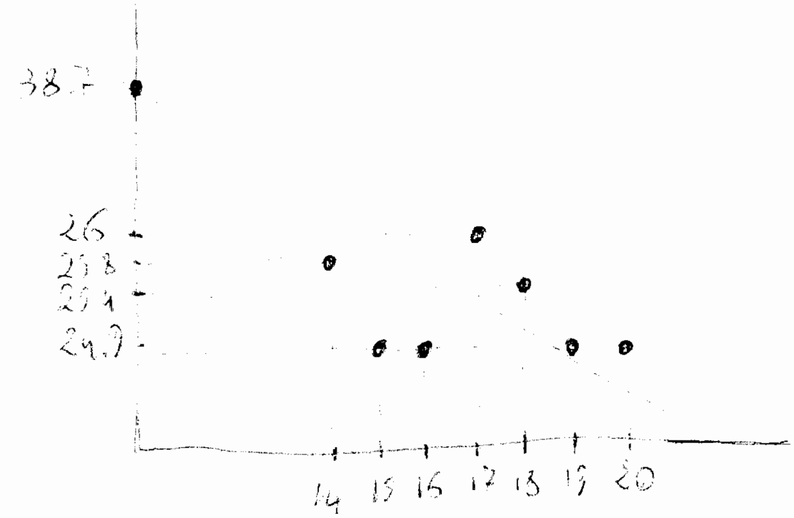
$\rightarrow t = 33$, WHICH IS YEAR $1975 + 33 = 2008$

THIS MODEL ESTIMATES THE EXPENDITURES TO BE \$142,700 IN 2008.

5. The table gives the percent of U.S. residents who reported smoking for selected years.

Year	1985	1999	2000	2001	2002	2003	2004	2005
Smoking	38.7	25.8	24.9	24.9	26.0	25.4	24.9	24.9

(a) Make a scatter plot of the data, with x equal to the number of years past 1985 and y the percent of smokers. (Clearly report the coordinates of the points)



(b) Using your calculator, find the linear model which is the best fit for the data.

$Y = -.7197X + 32.6439$

(c) Use the unrounded model to estimate the percentage of U.S. smokers in 2008.

YEAR 2008 IS $X = 2008 - 1985 = 23$

$Y(23) = 21.089$

THE UNROUNDED MODEL ESTIMATES A 21.09 % OF US SMOKERS IN 2008.

6. Solve the system of linear equations $\begin{cases} 4x & -5y & = -17 \\ 3x & +2y & = -7 \end{cases}$

ELIMINATION: 3 EQ1 $12x - 15y = -51$

-4 EQ2 $-12x - 8y = 28$

$0 \quad \frac{-23y}{-23} = \frac{-23}{-23} \rightarrow y = 1$

PLUG $y=1$ IN EQ2: $3x + 2 \cdot 1 = -7 \rightarrow \frac{3x}{3} = \frac{-9}{3} \rightarrow$
$\quad\quad -2 \quad -2$

$\rightarrow x = -3$

SOLUTION $(-3, 1)$

7. A pharmacist wants to mix two solutions to obtain 150 cc of a solution that has 7% concentration of a certain medicine. If one solution has a 12% concentration of the medicine and the second has a 3% concentration, how much of each solution should he mix?

x = QUANTITY IN CC OF SOLUTION AT 12% (.12)

y = QUANTITY IN CC OF SOLUTION AT 3% (.03)

QUANTITY: $\begin{cases} x + y = 150 \longrightarrow y = 150 - x \\ .12x + .03y = .07 \cdot 150 \end{cases}$
RATES:

PLUG IN $\rightarrow .12x + .03(150 - x) = 10.5 \rightarrow$

$\rightarrow .12x + 4.5 - .03x = 10.5 \rightarrow \frac{.09x}{.09} = \frac{6}{.09} \rightarrow x = 66.7$
$\quad\quad -4.5 \quad\quad -4.5$

PLUG IN $y = 150 - x \rightarrow y = 150 - 66.7 = 83.3$

THE PHARMACIST NEEDS 66.7 CC OF SOLUTION AT 12% AND 83.3 CC OF SOLUTION AT 3%.

8. Wholesalers' willingness to sell laser printers is given by the supply function $p = 50.5 + .8q$, and retailers' willingness to buy the printers is given by $p = 400 - .7q$, where p is the price per printer in dollars and q is the number of printers. What price will give market equilibrium for printers?

$$\begin{cases} p = 50.5 + .8q \\ p = 400 - .7q \end{cases} \to 50.5 + .8q = 400 - .7q \to \frac{1.5q}{1.5} = \frac{349.5}{1.5} \to$$

$$-50.5 + .7q \qquad -50.5 + .7q$$

$$\to q = 233 \xrightarrow{\text{PLUG BACK}} p = 50.5 + .8 \cdot 233 = 236.9$$

THE MARKET EQUILIBRIUM IS OBTAINED WHEN THE PRICE IS SET AT \$236.90 PER PRINTER.

9. Give the solution in interval notation for the inequality $2x + 1 \leq 5x - 2$.

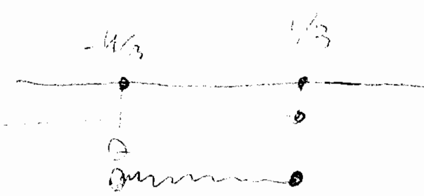
$$2x + 1 \leq 5x - 2 \qquad \frac{-3x}{-3} \leq \frac{-3}{-3} \to x \geq 1$$

$$-5x - 1 \qquad -5x - 1$$

SWITCH

SOLUTION: $[1, +\infty)$

10. Solve the double inequality $1 \leq 2 - 3x < 6$.



$$1 \leq 2 - 3x < 6$$

$$-2 \qquad -2 \qquad -2$$

$$\frac{-1}{-3} \leq \frac{-3x}{-3} < \frac{4}{-3} \to \frac{1}{3} \geq x > -\frac{4}{3}$$

SWITCH

IN INTERVAL NOTATION: $\left(-\frac{4}{3}, \frac{1}{3}\right]$