MAT 320 – Spring 2019 – Exam2

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I certify that I did not receive third party help in *completing* this test (sign) ______________________________

Instructions. Technology is allowed on this exam. Each problem is worth 10 points. If you use formulas or properties from our book, make a reference. When using technology describe which commands (or keys typed) you used or print out and attach your worksheet.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1) Let $A = \begin{bmatrix} 4 & 5 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 3 \\ 0 & 1 \end{bmatrix}$. Find $3A - 2B$. =

$$= \begin{bmatrix} 12 & 15 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 12 & 6 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 12-12 & 15-6 \\ 3-0 & 0-2 \end{bmatrix} = \begin{bmatrix} 0 & 9 \\ 3 & -2 \end{bmatrix}$$

2) Find the transpose of $M = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 4 & 6 \\ 2 & 0 & -2 \\ 1 & 4 & 1 \end{bmatrix}$.

$$M^T = \begin{bmatrix} 3 & -1 & 2 & 1 \\ 1 & 4 & 0 & 4 \\ 0 & 6 & -2 & 1 \end{bmatrix}$$

3) You can use technology **only to check your results and compute arithmetic operations.** Find the determinant of

$$A = \begin{bmatrix} 8 & 0 & 5 & -5 \\ 0 & 3 & 0 & 0 \\ -10 & 2 & -7 & 8 \\ 0 & 2 & 0 & 1 \end{bmatrix}.$$

BY 2ND ROW $|A| = (-1)^{2+2}(3)\begin{vmatrix} 8 & 5 & -5 \\ -10 & -7 & 8 \\ 0 & 0 & 1 \end{vmatrix} =$ BY 3RD ROW

$= 3(1)\begin{vmatrix} 8 & 5 \\ -10 & -7 \end{vmatrix} = 3(-56+50) = -18$

4) Evaluate the determinant of $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ in two different ways: a) by expanding along the second row, then

b) by expanding along the third column.

(a) $|A| = (-1)^{2+1}(1)\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} + (-1)^{2+2}(-3)\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} + (-1)^{2+3}(1)\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}$

$= -(2-0) - 3(1-0) - (0-4) = -2-3+4 = -1$

(b) $|A| = 0 + (-1)^{2+3}(1)\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} + (-1)^{3+3}(1)\begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix}$

$= -(0-4) + (-3-2) = 4-5 = -1$ ✓

5) Let $A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & -1 & 2 & 3 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 3 & 4 \\ 0 & -1 & 0 & 0 \end{bmatrix}$. Determine a **basis** for Col(A), the **column space** of A.

$$\mathrm{col}(A) = \mathrm{SPAN}(\{A_1, \dots, A_n\}) = \mathrm{SPAN}(A_1, A_2, A_3, A_4)$$

$$\mathrm{RREF}\ A = \begin{bmatrix} I_4 \\ \vec{0}^T \end{bmatrix} \Rightarrow \text{rank} = 4 \text{ ALL INDEP.}$$

6) Let $B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Determine a basis for Row(B), the **row space** of B.

$$\mathrm{ROW}(B) = \mathrm{Col}(B^T) = \mathrm{SPAN}(\{\mathrm{RREF}(B)_1^T, \dots, \mathrm{RREF}(B)_{\mathrm{rank}(B)}^T\})$$

$$\mathrm{RREF}(B) = \begin{bmatrix} I_3 \\ \vec{0}^T \end{bmatrix} \Rightarrow$$

$$\Rightarrow \mathrm{Row}(B) = \mathrm{SPAN}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \mathbb{R}^3$$

7) Let $A = \begin{bmatrix} 3 & 2 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 0 & 2 & 1 & -1 \end{bmatrix}$.

(a) Determine a basis for Col(A).

(b) Determine a basis for Null(A).

$$\text{RREF}(A) = \begin{bmatrix} I_3 & \begin{matrix} 1 & 3/2 \\ -1 & -5/4 \\ 0 & -5/4 \end{matrix} \end{bmatrix} \Rightarrow \text{(a) } \text{Col}(A) = \text{SPAN}(\mathcal{E}_3) = \mathbb{R}^3$$

$$\text{(b) } \text{NULL}(A) = \{\vec{x} \in \mathbb{R}^5 \mid A\vec{x} = \vec{0}\} = \text{SPAN}\left(\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3/2 \\ 5/4 \\ 5/4 \\ 0 \\ 1 \end{bmatrix}\right)$$

$$\vec{x} = x_4 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3/2 \\ 5/4 \\ 5/4 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{(a) BASIS} = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}; \quad \text{(b) BASIS} = \left\{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 5 \\ 0 \\ 4 \end{bmatrix}\right\}$$

8) You can use technology **only to check your results and compute arithmetic operations.** Use Row Reduction to verify that $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is invertible and use A^{-1} to solve the system $Ax = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

$$[A \mid I_3] \xrightarrow{R_3 - \frac{1}{2}R_1} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right] \xrightarrow[R_3 + \frac{1}{2}R_2]{\frac{1}{2}R_1,\ -R_2} \left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 \end{array}\right] \xrightarrow[2R_3]{R_1 - \frac{1}{2}R_2 - R_3}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right] \Rightarrow A \text{ IS INVERTIBLE AND } A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\text{SOLUTION} \quad \vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\text{CHECK:} \quad A\vec{x} = \begin{bmatrix} -2 + 3 + 0 \\ 0 - 3 + 0 \\ -1 + 3 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \quad \checkmark$$

9) After checking the conditions for doing it, solve the following system using Cramer's rule.

$$\begin{cases} 3x - 2y + z = 1 \\ 2x - y - z = -1 \\ x - y + 3z = 3 \end{cases}$$

$\Rightarrow |A| \neq 0$ THEN $\vec{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ IS GIVEN BY $x_i = \dfrac{|B^i|}{|A|}$, WHERE $\begin{cases} B^i_j = A_j, & j \neq i \\ B^i_i = \vec{b} \end{cases}$

HONOR: $X + Y - W = 2$

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow \underset{\text{TECH}}{|A| = 1}$$

HONOR: $$A_H = \begin{bmatrix} 3 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \\ 1 & -1 & 3 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} = \left|\begin{array}{c|c} A & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 1 & 1 & 0 \end{matrix} & -1 \end{array}\right| = -1 \cdot |A| = -1$$

$$B^1 = \begin{bmatrix} 1 & -2 & 1 \\ -1 & -1 & -1 \\ 3 & -1 & 3 \end{bmatrix} \Rightarrow |B^1| = 0 \Rightarrow x_1 = 0$$

$$B^2 = \begin{bmatrix} 3 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & 3 & 3 \end{bmatrix} \Rightarrow |B^2| = 0 \Rightarrow x_2 = 0$$

$$B^3 = \begin{bmatrix} 3 & -2 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |B^3| = 1 \Rightarrow x_3 = 1$$

SOLUTION: $\vec{X} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

CHECK: $A\vec{X} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ ✓

HONOR

$$B^1_H = \left[\begin{array}{c|c} B^1 & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 2 & 1 & 0 \end{matrix} & -1 \end{array}\right] \Rightarrow |B^1_H| = -1|B^1| = 0 \Rightarrow x_1 = 0$$

$$B^2_H = \left[\begin{array}{c|c} B^2 & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 1 & 2 & 0 \end{matrix} & -1 \end{array}\right] \Rightarrow |B^2_H| = -1 \cdot |B^2| = 0 \Rightarrow x_2 = 0$$

$$B^3_H = \left[\begin{array}{c|c} B^3 & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 2 & 1 & 2 \end{matrix} & -1 \end{array}\right] \Rightarrow |B^3_H| = -1 \cdot |B^3| = -1 \Rightarrow x_3 = \frac{-1}{-1} = 1$$

$$B^4_H = \left[\begin{array}{c|c} A & \begin{matrix} 1 \\ -1 \\ 3 \end{matrix} \\ \hline \begin{matrix} 1 & 1 & 0 \end{matrix} & 2 \end{array}\right] \Rightarrow |B^4_H| = 2|A| = 2 \Rightarrow x_4 = \frac{2}{-1} = -2$$

HONOR SOLUTION: $\vec{X} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \end{bmatrix}$;

CHECK: $A_H \vec{X} = \begin{bmatrix} 1 \\ -1 \\ 3 \\ -1(-2) \end{bmatrix}$ ✓