# MAT 121 – Exam3 – Spring 2018

**Instructor**: Dr. Francesco Strazzullo                                        **Name** KEY

I certify that I did not receive third party help in *completing* this test (sign) ______________

**Instructions.** Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test. You can also use a graphing tool and/or a computer algebra system like GeoGebra, but you **cannot use any "solve" tool**. When solving a problem graphically sketch the graph you used.
**SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

**1.** Tori just got a ride on ATL ferris wheel. What are her linear and angular speeds if the diameter of the wheel is 200 feet and one "flight" is equal to four revolutions, lasting about 15 minutes? Round your solutions to two decimal places.

"4-REVOLUTIONS" ⇒ $S = 4 \cdot 2\pi R = 8\pi R$ ; IN $T = 15$ min $= 900$ sec.

$2R = 200$ FT ⇒ $R = 100$ FT.

$V$ = LINEAR SPEED $= \frac{S}{T} = \frac{800\pi}{900}$ FT/SEC $= \frac{8}{9}\pi$ FT/SEC $\approx 2.79$ FT/SEC

$\omega$ = ANGULAR SPEED $= \frac{V}{R} = \frac{8}{900}\pi$ RAD/SEC $\approx 0.03$ RAD/SEC

$= \frac{\theta}{T}$

**2.** Determine the amplitude, period, and phase shift of the following trigonometric model.

$$y = 3 - 5\cos\left(\frac{\pi}{8}x - \frac{3\pi}{5}\right)$$

If there is no phase shift, state "no phase shift". If there is a phase shift, state the direction of the phase shift and the number of units (as a positive number) the graph is shifted.

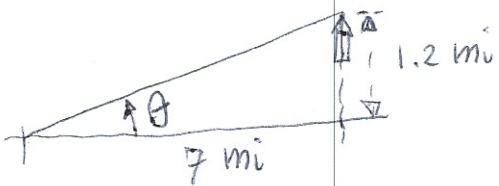
IF $Y = K + A\cos(Bx + C)$ THEN AMPLITUDE $= |A|$ ; PERIOD $= \frac{2\pi}{|B|}$

AND "PHASE SHIFT" $= \left|\frac{C}{B}\right|$ ( $\frac{C}{B} > 0$ → TO THE LEFT )

AMPLITUDE $= 5$ ; PERIOD $= \frac{2\pi}{\pi/8} = 16$ ;

PHASE SHIFT $= \frac{3\pi/5}{\pi/8} = \frac{24}{5} = 4.8$ UNITS TO RIGHT

**3.** Taylor is watching a Space X launch from an observation spot 7 miles away. Find the angle of elevation from Taylor to the spacecraft, when it is at a height of 1.2 miles. Write your answer in degrees rounded to two decimal places.




$$\theta = \tan^{-1}\left(\frac{1.2}{7}\right) \approx 9.73^\circ$$

**4.** Use trigonometric identities to simplify the expression.

$$\csc^2(\beta) + \csc^2(\beta)\cos^2(\beta)$$

$$\csc^2(\beta)\cdot\left(1+\cos^2(\beta)\right) = \frac{1}{\sin^2\beta}\left(1+\left(1-\sin^2(\beta)\right)\right)$$

$$= \frac{2-\sin^2(\beta)}{\sin^2(\beta)} = \frac{2}{\sin^2(\beta)} - 1 = 2\csc^2(\beta) - 1$$

**5.** Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equation.

$$\cos^2(x) - 4\sin(x) = 3\sin(x) - 2$$

Please, **indicate all possible solutions**. Use the parameter $k$ to represent any integer multiple of the function's period in the interval $[0, 2\pi)$. Write your answer in radians, as an exact answer when possible, else round to four decimal places. If there is no solution, please state "No Solution".

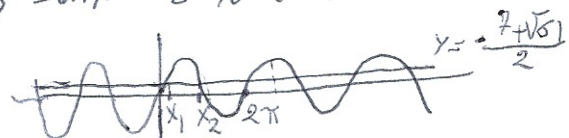
$$(1 - \sin^2(x)) - 7\sin x + 2 = 0$$

$$\sin^2 x + 7\sin x - 3 = 0$$

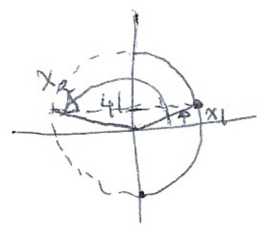
$$z = \sin x \Rightarrow z^2 + 7z - 3 = 0 \Rightarrow z = \frac{-7 \pm \sqrt{49+12}}{2} \Rightarrow$$

$$z = \frac{-7 \pm \sqrt{61}}{2} \approx \begin{cases} - \; -7.41 \Rightarrow \sin x = z \approx -7.4 \text{ NOT POSSIBLE} \\ + \; 0.41 \Rightarrow \sin x = z \approx 0.41 \text{ POSSIBLE.} \end{cases}$$

$$\sin x = \frac{-7+\sqrt{61}}{2}$$




$$x_1 = \sin^{-1}\left(\frac{-7+\sqrt{61}}{2}\right) \approx 23.899^\circ \text{ OR } 0.4171$$




$$x_2 = \pi - x_1 \approx 2.7245$$

$x_1$ AND $x_2$ IN $[0, 2\pi)$ THEN

$$x = x_1 + 2k\pi \approx .4171 + 2k\pi$$

OR

$$x = x_2 + 2k\pi \approx 2.7245 + 2k\pi$$

OR $\rightarrow x = -.4171 + (2k+1)\pi$

**6.** Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equation.

$$2\sin\left(2x - \frac{\pi}{4}\right) - 1 = \cos\left(\frac{3\pi}{4} - 2x\right)$$

Please indicate only the solutions in the interval $[0, 2\pi)$. Write your answer in radians, as an exact answer when possible, else round to four decimal places. If there is no solution, please state "No Solution".

COFUNCTION RELATION: $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$ THEN

$$\cos\left(\frac{3}{4}\pi - 2x\right) = \sin\left(\frac{\pi}{2} - \left(\frac{3}{4}\pi - 2x\right)\right) = \sin\left(\frac{\pi}{2} - \frac{3}{4}\pi + 2x\right)$$

$$= \sin\left(2x - \frac{\pi}{4}\right) \quad \text{THEN.}$$

$$2\sin\left(2x - \frac{\pi}{4}\right) - 1 = \sin\left(2x - \frac{\pi}{4}\right) \Rightarrow$$

$$\Rightarrow \sin\left(2x - \frac{\pi}{4}\right) = 1$$

THEREFORE: $(k \in \mathbb{Z})$ $\quad 2x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \Rightarrow$

$$\Rightarrow 2x = \frac{3}{4}\pi + 2k\pi \Rightarrow x = \frac{3}{8}\pi + k\pi$$

TO FIND $x$ IN $[0, 2\pi)$, TAKE $k$ POSITIVE:

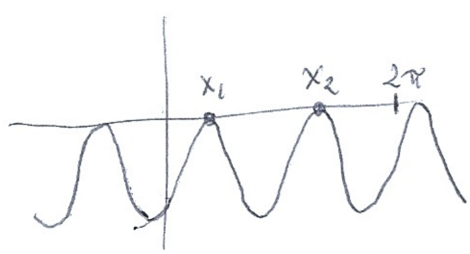
$x_1 = \frac{3}{8}\pi + 0\pi = \frac{3}{8}\pi \approx 1.1781$

$x_2 = \frac{3}{8}\pi + \pi = \frac{11}{8}\pi \approx 4.3197$

ALL OTHER CHOICES WILL BE OUT OF $[0, 2\pi)$

GRAPHICALLY: $y = 2\sin\left(2x - \frac{\pi}{4}\right) - 1 - \cos\left(\frac{3}{4}\pi - 2x\right)$

LOOK FOR ZEROS:



**7.** A rock leaves a slingshot at an angle of θ in reference to the horizontal. The initial velocity is $v_0 = 15$ feet per second. The rock hits a target 120 feet away. Find one value of θ if the range of the rock is given by the equation

$$r = \frac{1}{32} v_0^2 \sin(2\theta).$$

Please write your answer in degrees or radians rounded to the nearest hundredth

$r = 120$ FT ; $V_0 = 15$ FT/SEC THEN IT SHOULD BE

$$120 = \frac{1}{32}(15)^2 \cdot \sin(2\theta)$$

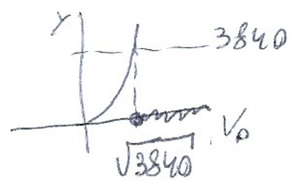
$$\sin(2\theta) = \frac{120 \cdot 32}{15^2} = \frac{256}{15} \approx 17.07 \quad \text{NOT POSSIBLE.}$$

THERE IS NONE ANGLE THAT WOULD ALLOW REACHING A 120 FT AWAY TARGET IF THE INITIAL SPEED IS ONLY 15 FT/SEC.

---

NOTE: MINIMUM SPEED REQUIRED IS $V_0$ SUCH THAT

$$\frac{120 \cdot 32}{V_0^2} \le 1 \quad \text{OR} \quad V_0^2 \ge 3840$$




THAT IS $(V_0 \ge 0)$ $\quad V_0 \ge \sqrt{3840} \approx 61.9677$

IF $V_0 = 62$ FT/SEC, THEN $\sin(2\theta) = \frac{3840}{62^2} = \frac{960}{961} \approx .999$

AND $2\theta = \sin^{-1}\left(\frac{960}{961}\right) \Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{960}{961}\right) \approx 43.693^\circ$