MAT 102 – Final Test - Fall 2013

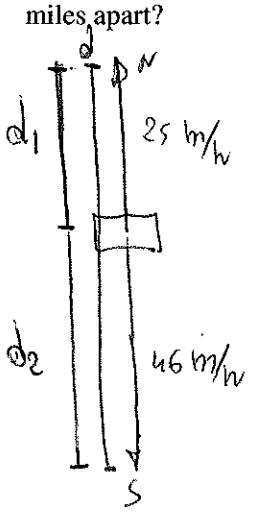
Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Each problem is worth 10 points. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded to two decimal places. You might need some of the following formulas:

- $h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$, with $g = 32\frac{ft}{sec^2} \approx 9.8\frac{m}{sec^2}$
- $d = vt$ and $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$ and $A^2 - B^2 = (A - B)(A + B)$
- $\log_a(MN) = \log_a(M) + \log_a(N)$ and $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$
- $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$
- $R = \log\left(\frac{I}{I_0}\right)$ and $D = 10\log\left(\frac{I}{I_0}\right)$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Two cars leave a house at the same time. One travels due nord at an average speed of 25 miles per hour, and the other travels due south at an average speed of 46 miles per hour. After how many minutes will the two cars be about 28 miles apart?



$d = d_1 + d_2 = 28$

$d = vt$

$d_1 = 25 \cdot t$

$d_2 = 46 \cdot t$

$\Rightarrow 28 = 25t + 46t \Rightarrow 71t = 28$

$\Rightarrow t = \frac{28}{71} \approx .39$ HOURS

OR (MULT. BY 60) 23 MINUTES

2. Solve the following quadratic equation.

$$3x^2 + 14x = 5$$
$$-5 \quad -5$$

$3x^2+14x-5=0$ $\left(\begin{array}{l}\text{PRODUCT } 3(-5)=-15\\ \text{SUM } 14\end{array}\right) \Rightarrow 15, -1$

$3x^2+15x-x-5=0$

$3x(x+5)-1(x+5)=0$

$(3x-1)(x+5)=0$

$3x-1=0 \Rightarrow x=+\frac{1}{3}$

$x+5=0 \Rightarrow x=-5$

3. A ball is kicked upward from a 1-foot height at a speed of 23 feet per second. How long does it take the ball to be 6 feet above the ground?

$h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0 \Rightarrow h = -\frac{1}{2}(32)t^2 + 23t + 1$

$h = 6 \Rightarrow -16t^2 + 23t + 1 = 6 \Rightarrow -16t^2 + 23t - 5 = 0 \Rightarrow$

$-6 \quad -6$

$\Rightarrow t = \frac{-23 \pm \sqrt{23^2 - 4(-16)(-5)}}{2(-16)} = \frac{23 \pm \sqrt{209}}{32} \approx$ 0.27 seconds, 1.17 seconds

4. Consider the following equation of a line.

$$3x + 5y = 7y + 6$$

a. Rewrite this equation in slope-intercept form. Reduce all fractions to lowest terms.
b. Find the equation, in slope-intercept form, for the line which is **perpendicular** to this line and passes through the point $(1, 2)$. Reduce all fractions to lowest terms.

a) $3x - 6 = 7y - 5y \Rightarrow y = \frac{3}{2}x - 3$

b) PERPENDICULAR LINE $\Rightarrow$ OPPOSITE RECIPROCAL SLOPES $\Rightarrow m = -\left(\frac{3}{2}\right)^{-1} = -\frac{2}{3}$

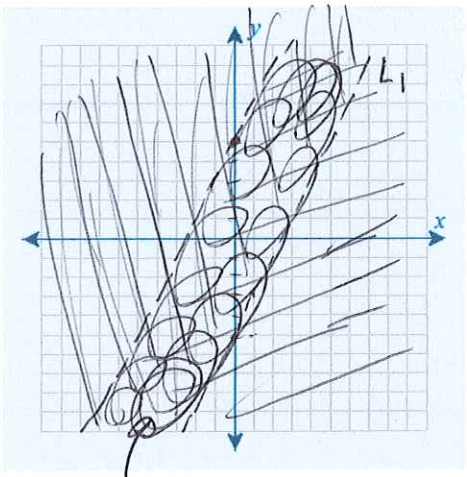
$y = mx + b \Rightarrow y = -\frac{2}{3}x + b \Rightarrow 2 = -\frac{2}{3} + b \Rightarrow b = 2 + \frac{2}{3} = \frac{8}{3}$

PLUG POINT $(1, 2)$

$y = -\frac{2}{3}x + \frac{8}{3}$

5. Graph the solution set of the following inequality:

$$|2x - y| < 5$$



SOLUTION SET

$|A| < B \Rightarrow \begin{cases} A < B \\ \text{AND} \\ A > -B \end{cases}$

$|A| > B \Rightarrow \begin{bmatrix} A < -B \\ \text{OR} \\ A > B \end{bmatrix}$

$\Rightarrow \begin{cases} 2X - Y < 5 \\ \text{AND} \\ 2X - Y > -5 \end{cases}$ BOUNDARY

LINES ARE PARALLEL AND DASHED

SOLVE: 1) $2X - Y < 5 \Rightarrow Y > 2X + 5$

2) $2X - Y > -5 \Rightarrow Y < 2X + 5$

"AND" $\Rightarrow$ INTERSECTION $\Rightarrow$ OVERLAP

6. Solve the polynomial equation

$$x^3 - x^2 - 7x = -3$$

$P(X) = X^3 - X^2 - 7X + 3 = 0$

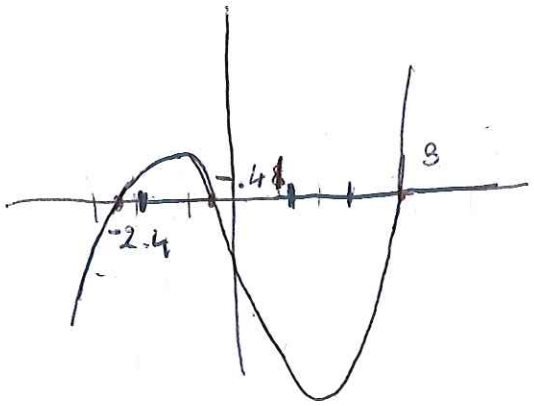
$P(3) = 3^3 - 3^2 - 7(3) + 3 = 30 - 30 = 0$ ✓

$\boxed{X = 3}$ IS A ROOT

SYNT. DIV.

	1	−1	−7	3
3		3	6	−3
	1	2	−1	0

✓



$P(X) = (X - 3)(X^2 + 2X - 1)$

$X = \dfrac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \dfrac{-2 \pm \sqrt{8}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$= \boxed{-1 \pm \sqrt{2}}$ OR $\boxed{\begin{matrix} .4142 \\ -2.4142 \end{matrix}}$

7. A certain species of deer is to be introduced into a forest, and wildlife experts estimate the population will grow according to the model $P(t) = (621)5^{1.2t}$, where t represents the number of years from the time of introduction. How long does it take this population to be 1500 deer?

$(621)\,5^{1.2t} = 1500 \Rightarrow 5^{1.2t} = \frac{1500}{621} = \frac{500}{207}$

TAKE ln ON BOTH SIDES: $\ln(5^{1.2t}) = \ln\left(\frac{500}{207}\right)$

$1.2t \cdot \ln 5 = \ln\left(\frac{500}{207}\right) \Rightarrow t = \frac{\ln\left(\frac{500}{207}\right)}{1.2\ln 5} \approx .4566$

ABOUT HALF YEAR.

8. One day in a city they felt an earthquake of intensity I , which measured 4.3 on the Richter scale. Two days later in that same city they felt another earthquake with intensity 20 times stronger than the first one. What was the magnitude of the second earthquake on the Richter scale?

$R = \log\left(\frac{I}{I_0}\right) \Rightarrow 4.3 = \log\left(\frac{I}{I_0}\right)$

"SECOND EARTHQUAKE INTENSITY" $= 20I \Rightarrow R = \log\left(\frac{20I}{I_0}\right) = \log 20 + \log\frac{I}{I_0}$

$\Rightarrow R = \log 20 + 4.3 \approx 5.6$ RICHTER DEGREES.

9. Josh is saving up money for a down payment on a car. He currently has $2092, but knows he can get a loan at a lower interest rate if he can put down $2600. If he invests the $2092 in an account that earns 4.6% annually, continuously compounded, how long will it take Josh to accumulate the $2600?

CONTINUOUSLY COMPOUNDED $\Rightarrow A = Pe^{rt}$

$2600 = 2092e^{.046t} \Rightarrow \ln\left(\frac{650}{523}\right) = \ln\left(e^{.046t}\right)$

$\Rightarrow .046t = \ln\left(\frac{650}{523}\right) \Rightarrow t = \frac{1}{.046}\ln\left(\frac{650}{523}\right) \approx 4.73$ YEARS

10. A bakery decides to produce a new kind of pastries. The table shows the profit, in dollars, for various levels of production.

Pastries (units)	20	30	40	50	65	80	85	95	100
Profit (dollars)	50	120	155	130	60	160	180	178	130

Consider x to be the number of pastries produced, and y to be the profit in hundred dollars. Use technology to answer to the following questions.

(a) Find the quadratic and the quartic models that are the best fit for these data. (Round your answer to five decimal places).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate the production level that maximizes the profit.

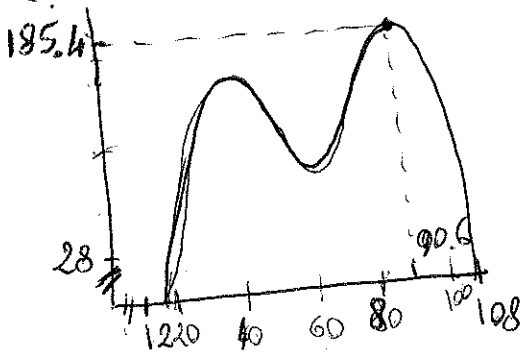
(a) QUAD: $y = -.01106x^2 + 2.25663x + 39.49784$, $R^2 = .33851$

QUART: $y = -0.00013x^4 + .03085x^3 - 2.64385x^2 + 92.3502x - 975.34559$, $R^2 = .84714$

(b) QUARTIC IS BETTER.

(c) USE GRAPH

2ND + TRACE + 4



MAX AT $X = 90.6$

MUST CHECK TABLE FOR BEST APPROXIMATION

X	Y
90	185.19
91	185.29

THEN MAXIMUM PROFIT OF $\$185.19$

WHEN PRODUCING 91 PASTRIES.