Math 099 - Summer 2011 - Test 2

Instructor: Dr. Francesco Strazzullo Name Key

Instructions. Only calculators are allowed on this examination. Point values of each problem are indicated. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Solve the following linear equations

   (a) (15*pts*) $4x + 6 = 2(1 + 3x) + 1$.

$4x+6 = 2(1)+2(3x)+1$

$4x+6 = 2+6x+1$

$4x+6 = 3+6x$

$-4x-3 \quad -3-4x$

$3 = 2x$

$\frac{3}{2} = \frac{2x}{2}$

$x = \frac{3}{2}$

CHECK:

$4\left(\frac{3}{2}\right)+6 \stackrel{?}{=} 2\left(1+3\left(\frac{3}{2}\right)\right)+1$

$6+6 \stackrel{?}{=} 2+2\left(\frac{9}{2}\right)+1$

$12 \stackrel{?}{=} 2+9+1$ ✓

   (b) (15*pts*) $\frac{2}{3}(x-1) = \frac{4}{15}x + 3$.

LCD$\left(\frac{2}{3}, \frac{4}{15}\right)$ is $3 \cdot 5 = 15$

$15\left(\frac{2}{3}\right)(x-1) = 15\left(\frac{4}{15}x+3\right)$

$10(x-1) = 15\left(\frac{4}{15}x\right)+15(3)$

$10x-10 = 4x+45$

$-4x+10 \quad -4x+10$

$\frac{6x}{6} = \frac{55}{6} \rightarrow x = 9\frac{1}{2}$

$= 9.1\overline{6}$

CHECK:

$\frac{2}{3}\left(\frac{55}{6}-1\right) \stackrel{?}{=} \frac{4}{15}\left(\frac{55}{6}\right)+3$

$\frac{2}{3}\left(\frac{55-6}{6}\right) \stackrel{?}{=} \frac{2 \cdot 11}{3 \cdot 3}+3$

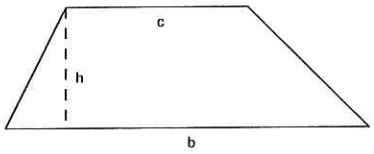
$\frac{2}{3}\left(\frac{49}{6}\right) \stackrel{?}{=} \frac{22}{9}+3$

$\frac{1 \cdot 49}{3 \cdot 3} \stackrel{?}{=} \frac{22+3(9)}{9}$

$\frac{49}{9} \stackrel{?}{=} \frac{22+27}{9}$ ✓

2. The formula for the area of a trapezoid is

$$A = \frac{1}{2}(b+c)\,h$$



(a) (15*pts*) Solve the above formula for c.

$A = \left(\frac{1}{2}b + \frac{1}{2}c\right)h$

$A = \frac{1}{2}bh + \frac{1}{2}ch$

$-\frac{1}{2}bh \quad -\frac{1}{2}bh$

$\frac{h}{2}c = A - \frac{h}{2}b$

$\left(\frac{2}{h}\right)\frac{h}{2}c = \frac{2}{h}\left(A - \frac{h}{2}b\right)$

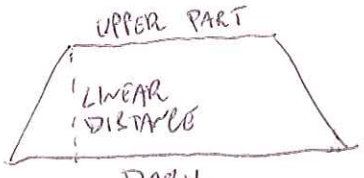
$c = \frac{2A}{h} - b$

(b) (10*pts*) You can approximate the shape of your vehicle's windshield with a trapezoid whose area is 13.5 ft^2. Knowing that the dash of your vehicle is 7 ft and the linear distance between the dash and the roof is 5 ft, find the length of the upper part of your windshield.

WE CAN USE THE GIVEN FORMULA $A = \frac{1}{2}(b+c)h$

OR THE ONE OBTAINED AT (a).

DATA:



$c = ?$; $h = 5$

$b = 7$; $A = 13.5$

PLUG: $13.5 = \frac{1}{2}(7+c)(5)$

$13.5 = \left(\frac{7}{2} + \frac{1}{2}c\right)(5)$

$13.5 = \frac{35}{2} + \frac{5}{2}c$

$-17.5 \quad -17.5$

$-4 = \frac{5}{2}c \rightarrow c = -\frac{8}{5} = -1.6$ FT

ALGEBRAIC CHECK

$13.5 \stackrel{?}{=} \frac{1}{2}(7-1.6)(5)$

$13.5 \stackrel{?}{=} \frac{1}{2}(5.4)(5)$

$13.5 = (2.7)(5)$ ✓

THE UPPER PART OF THE WINDSHIELD SHOULD HAVE A NEGATIVE LENGTH, WHICH IS ABSURD. THE MODEL IS WRONG

3. (15*pts*) The total cost of an air-compressor rental is \$45 for the first day plus \$30 for each additional day. Write two equations with inputs for each described with an inequality. Define your variables so y depends on x.

INPUT: X DAYS OF RENTAL ; OUTPUT: Y COST IN DOLLARS FOR RENTAL

$$Y = \begin{cases} 45 & , \quad 0 < X \leq 1 \\ 45 + 30\,(X-1) & , \quad X > 1 \end{cases}$$

X TOTAL DAYS, $(X-1)$ DAYS AFTER THE FIRST DAY.

4. (15*pts*) Solve the inequality

$$-11 \geq -5x + 2(x+1) > -20$$

and graph its solution set, then write it in interval notation.

$-11 \geq -5X + 2X + 2 > -20$

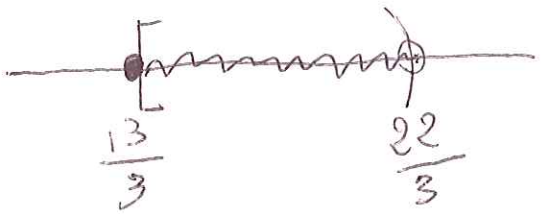
$-11 \geq -3X + 2 > -20$

$-2 \qquad -2 \quad -2$

$-13 \geq -3X > -22$

$\frac{-13}{-3} \leq \frac{-3X}{-3} < \frac{-22}{-3}$

$\frac{13}{3} \leq X < \frac{22}{3}$



CHECK: $X = 5$

$-11 \overset{?}{\geq} -5(5) + 2(5+1) \overset{?}{>} -20$

$-11 \overset{?}{\geq} -25 + 2(6) \overset{?}{>} -20$

$-11 \geq -13 > -20$ ✓

SOLUTION SET:

$\left[\frac{13}{3}, \frac{22}{3}\right)$

OR

$[4.\overline{3}, 7.\overline{3})$

5. (15*pts*) The conversion formula from Fahrenheit to Celsius degrees is

$$C = \frac{5}{9}(F - 32).$$

To melt iron, the temperature of a furnace must be at least 1540 °C but at most 1650 °C. What range of Fahrenheit temperatures must be maintained?

RANGE IN CELSIUS

$$1540 \leq C \leq 1650$$

AT LEAST ↑ ↑ AT MOST

PLUG FORMULA:

$$1540 \leq \frac{5}{9}(F-32) \leq 1650$$

$$\left(\frac{9}{5}\right)1540 \leq \left(\frac{9}{5}\right)\frac{5}{9}(F-32) \leq \left(\frac{9}{5}\right)1650$$

$$2772 \leq F - 32 \leq 2970$$

$+32 \qquad +32 \qquad +32$

$$2804 \leq F \leq 3002$$

TO MELT IRON, THE TEMPERATURE OF A FURNACE MUST BE BETWEEN 2804 °F AND 3002 °F.