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MAT 102 - Exam2 - Fall 2014

Instructor: Dr. Francesco Strazzullo

Name

Key

Instructions. Technology is allowed on this exam. Each problem is worth 10 points. When using technology describe which commands (or keys typed) you used AND sketch any graph used. You might need some of the following formulas:

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $(x - h)^2 + (y - k)^2 = R^2$
- $(A \pm B)^2 = A^2 \pm 2AB + B^2$
- $A^2 - B^2 = (A - B)(A + B)$
- $h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$, with $g = 32 \frac{ft}{sec^2} \approx 9.8 \frac{m}{sec^2}$
- $y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$, with $h = -\frac{b}{2a}$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Consider the following equation of a line.

$$-2x + 3y = 4x + 3$$

- Rewrite this equation in slope-intercept form. Reduce all fractions to lowest terms.
- Find the equation, in slope-intercept form, for the line which is **perpendicular** to this line and passes through the point (1, 2). Reduce all fractions to lowest terms.

a) $3y = 4x + 3 + 2x \rightarrow \frac{3y}{3} = \frac{6x}{3} + \frac{3}{3} \rightarrow \boxed{y = 2x + 1}$

b) SLOPE FROM (a): 2 \rightarrow PERPENDICULAR LINE SLOPE: $m = -\frac{1}{2}$
 LOOKING FOR $y = -\frac{1}{2}x + b$ THROUGH (1, 2).

PLUG POINT: $2 = -\frac{1}{2}(1) + b \rightarrow b = 2 + \frac{1}{2} = \frac{5}{2}$

$$\boxed{y = -\frac{1}{2}x + \frac{5}{2}}$$

2. Write the **slope-intercept form** of the equation for the line that passes through the point (-2, 3) and has slope -4.

$m = -4 \rightarrow y = -4x + b$
 PLUG POINT: (-2, 3) $\rightarrow 3 = -4(-2) + b \rightarrow$
 $-8 \quad -8$

$\rightarrow b = -5 \rightarrow \boxed{y = -4x - 5}$

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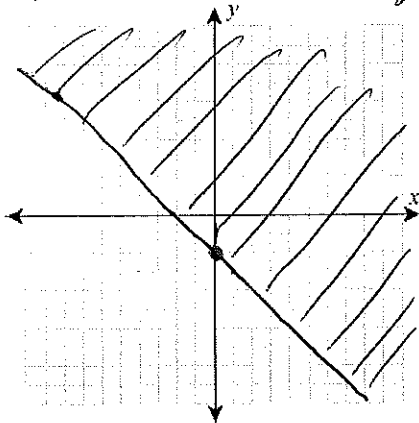
3. Solve the system of two linear inequalities graphically.

$$2x - y \leq 3x + 2 \text{ or } y > -3x + 1$$

Step 1. First linear inequality.

$$\begin{aligned} 2x - y &\leq 3x + 2 \\ -2x &\quad -2x \quad \quad -1 \quad \quad -1 \quad \quad -1 \\ -y &\leq x + 2 \\ y &\geq -x - 2 \end{aligned}$$

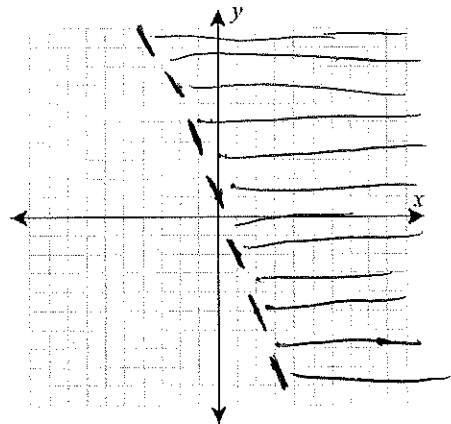
SOLID LINE, REGION ABOVE LINE



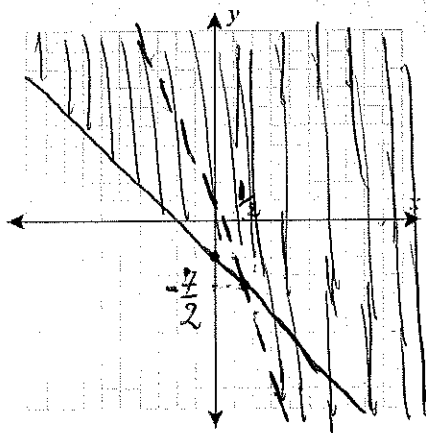
Step 2. Second linear inequality.

$$y > -3x + 1$$

DASHED LINE, REGION ABOVE LINE



Step 3. Graph both inequalities, identify the coordinates of the corner point, and highlight the solution set of this system of linear inequalities. Also, mark your selection A or B.



~~A~~ the union of the individual solution sets
BECAUSE THERE IS "OR"

B) the intersection of the individual solution sets

CORNER POINT:

I) PLUG THE BOUNDARY LINES IN A TI-84

$$Y_1 = -x - 2$$

$$Y_2 = -3x + 1$$

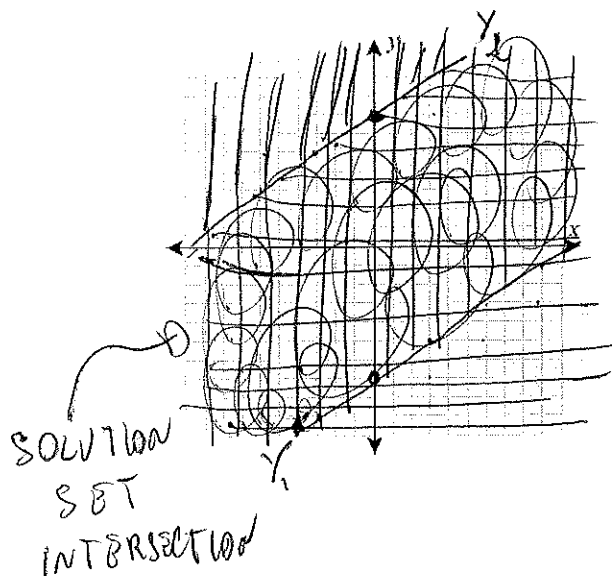
II) TYPE: 2ND + TRACE + INTERSECT

III) TYPE 3-TIMES ENTER

$$IV) (1.5, -3.5) = \left(\frac{1}{2}, -\frac{7}{2} \right)$$

4. Graph the solution set of the following linear inequality:

$$|3x - 2y| \leq 14 \rightarrow \begin{cases} 3x - 2y \leq 14 & \text{INEQ. 1} \\ \text{AND} \\ 3x - 2y \geq -14 & \text{INEQ. 2} \end{cases}$$



$$\text{INEQ. 1: } -2y \leq -3x + 14 \rightarrow \frac{-2y}{-2} \leq \frac{-3x + 14}{-2} \rightarrow$$

$$\rightarrow y \geq \frac{3}{2}x - 7 : \text{SOLID, ABOVE B.L.}$$

$$\text{INEQ. 2: } -2y \geq -3x - 14 \rightarrow \frac{-2y}{-2} \geq \frac{-3x - 14}{-2} \rightarrow$$

$$\rightarrow y \leq \frac{3}{2}x + 7 : \text{SOLID, BELOW B.L.}$$

5. Find the standard form of the equation for the circle with center $(-2, 3)$ and radius 5.

STANDARD FORM OF THE EQUATION OF THE CIRCLE WITH CENTER (h, k) AND RADIUS R . $\rightarrow (x - h)^2 + (y - k)^2 = R^2$

$$(x - (-2))^2 + (y - 3)^2 = 5^2$$

$$(x + 2)^2 + (y - 3)^2 = 25$$

6. The cost in dollars for producing x pairs of shoes is modeled by the quadratic function

$$C(x) = 0.05x^2 - 15x + 2200.$$

Find the production level that minimizes the cost.

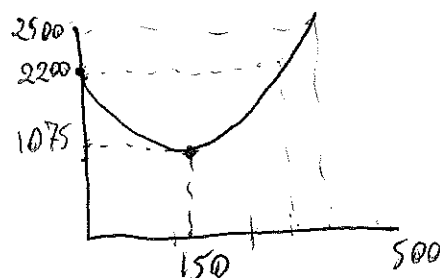
$C(x)$ IS QUADRATIC MODEL, WITH GRAPH A PARABOLA CONCAVE UPWARD,
MINIMUM AT THE VERTEX (h, k) , WHERE $h = -\frac{b}{2a} = -\frac{-15}{2(0.05)} = 150$.
USING A TI-84:

I) $Y_1 = C(x)$

II) ADJUST WINDOW TO SEE THE MINIMUM:

$$[Y_{MIN}, Y_{MAX}] = [0, 2500], [X_{MIN}, X_{MAX}] = [0, 500]$$

III) 2^{ND} + TRACE + MINIMUM



150 PAIRS OF SHOES (THEN 300 SHOES...) WILL MINIMIZE THE COST.

7. Graph the following function.

$$t(x) = \begin{cases} 2x + 3 & \text{if } x > 0 \\ 2x^2 + 1 & \text{if } x \leq 0 \end{cases}$$

LINEAR ON THE RIGHT OF $x=0$
QUADRATIC AT $x=0$ AND ON
THE LEFT OF $x=0$

AT $x=0$: $2(0) + 3 = 3$

AT $x=0$: $t(0) = 2(0)^2 + 1 = 1$

