MAT 221- Fall 2016 - Exam 1 - Addendum

Instructor: Dr. Francesco Strazzullo　　　　　　Name KЗX

Instructions. You are expected to use a graphing calculator or a software, but **not a CAS**. You can use one letter-size cheat sheet. Sketch any graph that you use, **approximating up to the third decimal place**. Each problem is worth 10 points, unless otherwise specified.

Always use the appropriate wording and units of measure in your answers (when applicable).

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. You discovered a new virus. You have an initial population of 150 units that follows the model

$$P(t) = 75\left(10e^{0.25t} + 2\right),$$

where t is the time in hours. Use the model above to estimate the following (truncating to the integer).

(a) (*5 points*) The *average rate of growth* of the population during the second half of the first day.

$12 \le t \le 24$

$$\underset{\text{ON } [a,b]}{A.R.G.} = \frac{P(b)-P(a)}{b-a} = \frac{P(24)-P(12)}{24-12}$$

$$= \frac{\cancel{750}^{125}}{\cancel{12}_{2}}(e^6 - e^3) = \frac{125}{2}(e^6 - e^3) \approx 23{,}958 \text{ UNIT/HOUR}$$

(b) (*10 points*) The *instantaneous rate of growth* of the population at the end of the first day.

$t = 24$

$$I.R.G. = P'(t) = 750 \cdot (.25)e^{.25t} = 187.5\,e^{.25t}$$

$$P'(24) = 187.5\,e^6 \approx 75{,}642 \text{ UNITS/HOUR}$$

2. In Spring and Summer 2016, the daily profit P, in dollars, of producing and selling x liters of lemonade is also depending on the daily level of production $x = g(t)$, where t represents the day with $t = 1$ on May the 1^{st}, and

$$P = \frac{t^3 - 2\cos\left(\frac{\pi}{90}t\right)}{g(t)}.$$

(a) (5 *points*) Express verbally in writing the information $g(5) = 3.75$, $P(5) = 64$, and $g'(5) = .75$. (*Use the units of measure*)

$t=5 \Rightarrow$ MAY 5^{TH}. ON THIS DAY 3.75 LITERS ($\approx$ 1 GALLON) OF LEMONADE WERE PRODUCED/SOLD AT AN INCREASING RATE OF .75 LITERS PER DAY. ON MAY THE 5^{TH} THE DAILY PROFIT WAS 64 DOLLARS.

(b) (10 *points*) Assuming the values in part (2a), at which daily rate is the profit changing on May the 5^{th}? Is the daily profit increasing or decreasing?

$$P(t) = \frac{f(t)}{g(t)} \Rightarrow P'(t) = \frac{f'(t)g(t) - f(t)g'(t)}{[g(t)]^2}$$

$$f(t) = t^3 - 2\cos\left(\frac{\pi}{90}t\right) \Rightarrow f'(t) = 3t^2 + \frac{\pi}{45}\sin\left(\frac{\pi}{90}t\right) \Rightarrow$$

$$\Rightarrow P'(5) = \frac{\left(3(5)^2 + \frac{\pi}{45}\sin\left(\frac{\pi}{90}\cdot 5\right)\right)\cdot(3.75) - \left(5^3 - 2\cos\left(\frac{\pi}{90}\cdot 5\right)\right)\cdot(.75)}{(3.75)^2}$$

≈ 13.44 DOLLAR/DAY ($>0 \Rightarrow$ INCREASING PROFIT)

(c) (10 *points*) Assuming the values in part (2a), approximate the daily profit on May the 6^{th}.

$P(t) \approx L(t) = P(5) + P'(5)\cdot(t-5)$. THEN

$P(6) \approx P(5) + P'(5) = 64 + 13.44 = 77.44$ DOLLARS

MAT 221 - Fall 2016 - Exam 1 - Lab (at most 20 minutes)

Instructor: Dr. Francesco Strazzullo　　　　Name KEY

Instructions. You are expected to use a graphing calculator or a software to complete this part. You can use one letter-size cheat sheet. Sketch any graph that you use, **approximating up to the third decimal place**. Each problem is worth 10 points.

SHOW YOUR WORK NEATLY, PLEASE.

1. Consider the function $f(x) = 2x^3 - \cos(3\pi x)$.

   (a) Approximate the first derivative of $f(x)$ at $x = 1$.

   USE GRAPH TO ESTIMATE OR TECHNOLOGY TO COMPUTE

   $$f'(1) = \left.\frac{dy}{dx}\right|_{x=1} = 6$$

   (b) Compute the slope-intercept form of the equation of the tangent and the normal lines to $f(x)$ at $x = 1$.

   $a = 1, \quad f(a) = f(1) = 3$:

   TANGENT: $L(x) = f(a) + f'(a)(x-a) = 3 + 6(x-1) = 6x - 3$

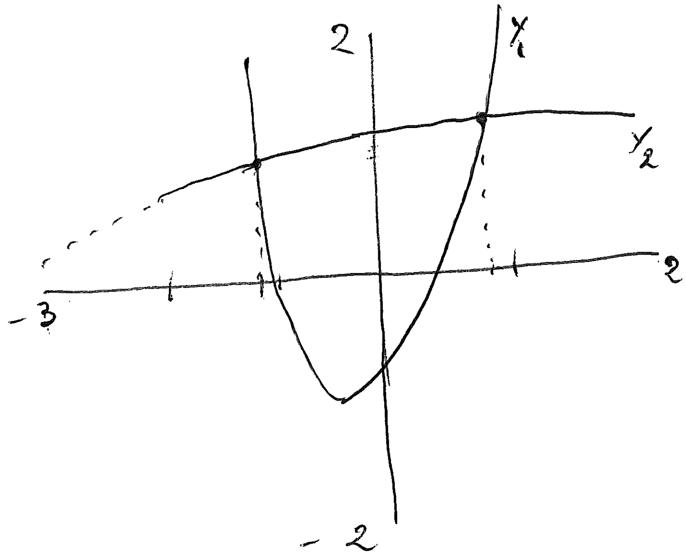
   NORMAL: $N(x) = f(a) - \frac{1}{f'(a)}(x-a) = 3 - \frac{1}{6}(x-1) = -\frac{1}{6}x + \frac{19}{6}$

   (c) Use the previous step to complete the following table, approximating up to the third decimal place, and to check the local linearity of $f(x)$ at $x = 1$.

a	.9	.999	1	1.001	1.1
$f(x)$	2.046	2.994	3	3.006	3.25
$L(x)$	2.4	2.994	3	3.006	3.6

   VERY CLOSE

2. Find in the interval $[-2, 1]$ the solutions of the equation $\overbrace{x^4 - \cos(2x+1)}^{Y_1} = \overbrace{\sqrt[4]{x+4}}^{Y_2}$.



$X \cong -1.127, \; 0.868$

MAT 221- Fall 2016 - Exam 1 - Conceptual

Instructor: Dr. Francesco Strazzullo Name____________________________

Instructions. You are expected to use a graphing calculator or a software, but **not a CAS**. You can use one letter-size cheat sheet. Sketch any graph that you use, **approximating up to the third decimal place**. Each problem is worth 10 points, unless otherwise specified. **Exercise** (5c) **is only for the honor section.**

Always use the appropriate wording and units of measure in your answers (when applicable).

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Over the interval $(-2, 5)$, sketch the graph of a function $f(x)$ such that:

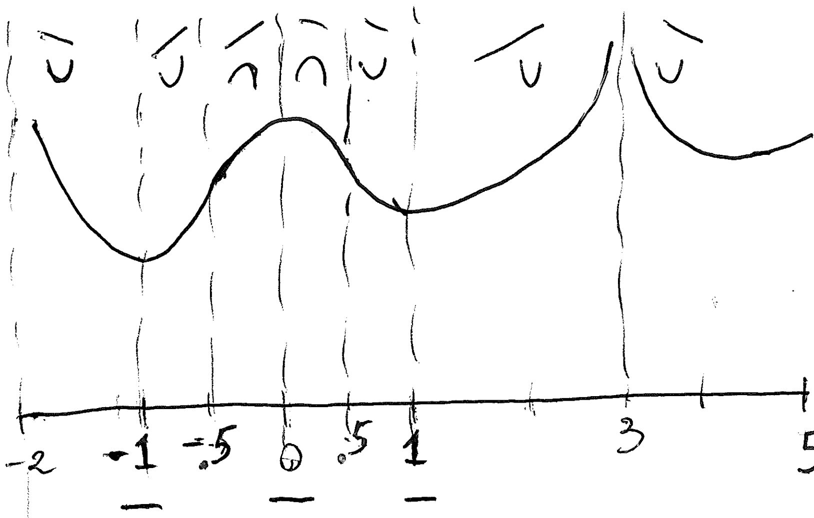
   (a) $f'(-1) = f'(0) = f'(1) = 0$ and $f'(x)$ is undefined at $x = 3$; HORIZ TANG. AND CUSP OR HOLE AT X=3

   (b) $f'(x) > 0$ for $-1 < x < 0$ and $1 < x < 3$; INCREASING /

   (c) $f''(x) > 0$ for $-2 < x < -0.5$, $0.5 < x < 3$ and $3 < x < 5$; CONCAVE UP U

   (d) $f''(x) < 0$ for $-0.5 < x < 0.5$. CONCAVE DOWN ∩

   (e) $f'(x) < 0$ for $-2 < x < -1$, $0 < x < 1$ and $3 < x < 5$; DECREASING \



2. For $f(x) = 3x^5 + 2x^3 - 3x + 4$, compute $f'(x)$ and $f''(x)$.

$$f'(x) = \frac{d}{dx}\left[3x^5 + 2x^3 - 3x + 4\right] = 3(5x^4) + 2(3x^2) - 3 \cdot 1 + 0$$

$$= 15x^4 + 6x^2 - 3$$

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[15x^4 + 6x^2 - 3\right] = 15(4x^3) + 6(2x) + 0$$

$$= 60x^3 + 12x$$

3. Differentiate the following functions (each problem is worth 10 points):

(a) $h(x) = \dfrac{x^2 - x}{x^3 + 1}$

QUOTIENT RULE $\dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{u'v - uv'}{v^2}$

$$h'(x) = \frac{(2x-1)(x^3+1) - (x^2-x)(3x^2)}{(x^3+1)^2}$$

$$= \frac{2x^4 + 2x - x^3 - 1 - 3x^4 + 3x^3}{(x^3+1)^2}$$

$$= \frac{-x^4 + 2x^3 + 2x - 1}{(x^3+1)^2}$$

NOTE THAT $x = -1$ IS NOT A ROOT OF THE NUMERATOR, THUS WE CAN'T SIMPLIFY THIS RATIONAL EXPRESSION

(b) $g(t) = (t^4 + t)e^t$

PRODUCT RULE: $\dfrac{d}{dx}[uv] = u'v + uv'$

$$g'(t) = (4t^3+1)e^t + (t^4+t)e^t$$

$$= (t^4 + 4t^3 + t + 1)e^t$$

4. The daily cost C, in thousand dollars, of producing x units of a certain commodity is also depending on the daily level of production $x = g(t)$, where t represents the day of a particular quarter of the year. During the second quarter (April–June) assume $t = 1$ on April the 1^{st} and

$$C = (t^2 - 2t + 1)\, g(t).$$

(a) (5 *points*) Express verbally in writing the information $g(3) = 2{,}500$, $C(3) = 10{,}000$, and $g'(3) = 50$. (*Use the units of measure*) $t=3$ CORRESPONDS TO APRIL THE 3^{rd}.

ON APRIL THE 3^{rd} $(g(3)=2500)$ ABOUT 2500 UNITS WERE PRODUCED $(g'(3)=50)$ AT AN INCREASING RATE OF 50 UNITS PER DAY. ON THIS DAY, THE DAILY COST WAS OF $(C(3)=10{,}000)$ K\$ 10,000 (OR \$ 10,000,000)

(b) (10 *points*) Assuming the values in part (4a), at which daily rate is the cost changing on April the 3^{rd}? Is the daily cost increasing or decreasing?

"DAILY RATE OF COST" $= \dfrac{dC}{dt} \overset{\text{PRODUCT RULE}}{=} (2t-2)g(t) + (t^2-2t+1)g'(t)$

ON APRIL THE 3^{RD} $t=3$: $C'(3) = 4\,g(3) + 4\,g'(3) =$

$= 10{,}200$ K\$/DAY.

THE DAILY COST ON APRIL THE 3^{RD} IS INCREASING OF 10,200 K\$ PER DAY.

(c) (10 *points*) Assuming the values in part (4a), approximate the daily cost of production on April the 4^{th}.

APRIL 4^{TH} IS CLOSE TO APRIL 3^{RD}; $a=3$, $t=4$

$C(t) \approx L(t) = C(3) + C'(3)\cdot(t-3) \Rightarrow C(4) \approx L(4) =$

$= 10{,}000 + 10{,}200 = 20{,}200$ K\$.

USING THE APPROXIMATION OF $g(t)$ WE GET IT WRONG;

$g(t) \approx \ell(t) = g(3) + g'(3)(t-3) \Rightarrow C(t) = (t^2-2t+1)\cdot g(t) \approx$

$\approx (t^2-2t+1)\cdot \ell(t) \Rightarrow C(4) \approx 9\cdot(2500+50) = 22{,}950$ K\$

5. You discovered a new virus. You have an initial population of 150 units that follows the model

$$P(t) = 75\left(10t^{0.25} + 2\right),$$

where t is the time in hours. Use the model above to estimate the following (truncating to the integer).

(a) (5 *points*) The *average rate of growth* of the population during the first day. → FROM $t=0$ TO $t=24$

$$AR = \frac{P(24)-P(0)}{24} \approx 69 \text{ UNITS PER HOUR}$$

(THEREFORE ON AVERAGE ITS GROWING FAST)

(b) (10 *points*) The *instantaneous rate of growth* of the population at the end of the first day. $t=24$

$$P'(t) = 75\left(10(0.25)t^{-.75}\right) \quad \text{(NOT NEEDED)}$$

$$P'(24) \approx 17 \text{ UNITS PER HOUR}$$

(THEREFORE THE GROWTH SLOWS DOWN AT THE END OF THE FIRST DAY)

(c) (10 *points* **only for the Honor section**) Use a linear approximation to estimate the population on the first hour of the second day. → $t=25$

$$P(25) \approx P(24) + P'(24) \approx 1810 + 17 = 1827 \text{ UNITS.}$$