Math 221- Fall 2012 - Test 1 - Lab (20 minutes)

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Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. You are expected to use a graphing calculator or a software to complete this part. You can use our class notes or alternatively a maximum of 4 size-letter sheets of notes. Sketch any graph that you use, **approximating up to the third decimal place**. Each problem is worth 10 points.

SHOW YOUR WORK NEATLY, PLEASE.

1. Consider the function $f(x) = 3x^2 + \sin(2\pi x)$.

   (a) Complete the following table. NEED $Y = f(X)$, THEN $\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{6} + a$

a	.5	.999	1	1.001	1.5
$f'(a)$	-3.283	12.277	12.283	12.289	2.717

   (b) Express the equation of the tangent and the normal lines to $f(x)$ at $x = 1$.

TANGENT LINE: AT $(a, f(a))$ $L(X) = f(a) + f'(a)(X - a)$

NORMAL LINE AT $(a, f(a))$: $N(X) = f(a) - \frac{1}{f'(a)}(X - a)$

$\frac{1}{f'(1)} = \frac{1}{12.283} = .081$

$a = 1,\ f(a) = f(1) = 3 \cdot (1)^2 + \sin 2\pi = 3,\ f'(a) = f'(1) = 12.283$

$L(X) = 3 + 12.283(X - 1) = 12.283X - 9.283$

$N(X) = 3 - .081(X - 1) = -.081X + 3.081$

   (c) Use the previous step to complete the following table and check the local linearity of $f(x)$ at $x = 1$, approximating up to the third decimal place.

x	.5	.999	1	1.001	1.5
$f(x)$	.75	2.988	3	3.012	6.75
$L(x)$	-3.142	2.988	3	3.012	9.142

$f(X) \approx L(X)$

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2. Solve the inequality $x + e^{1-2x} \ge x^2$

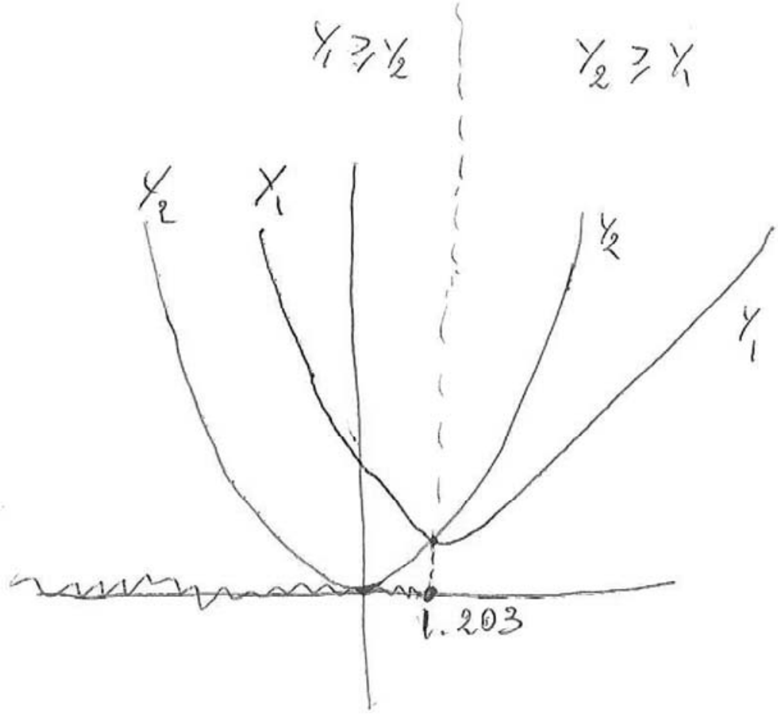
$Y_1 = x + e^{1-2x}$

$Y_2 = x^2$

$\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{5}$

INTERSECTION POINT

$Y_1 \ge Y_2$



$X \le 1.203$

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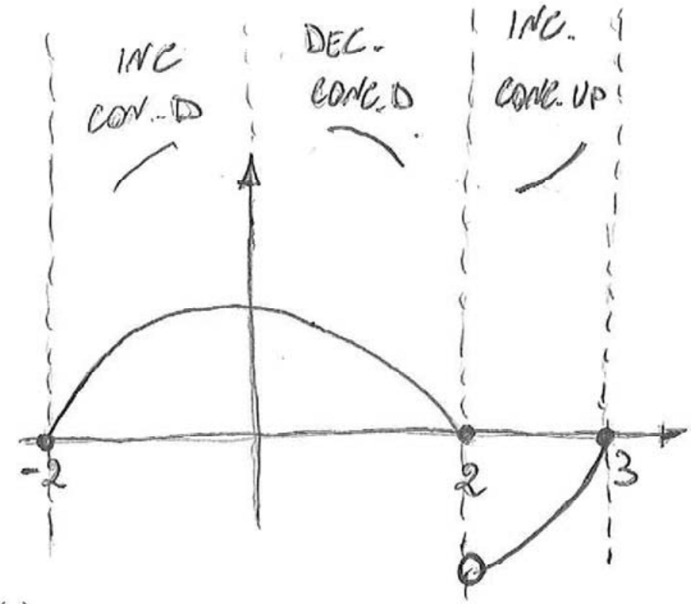
Math 221- Fall 2012 - Test 1 (50 minutes)

Instructor: Dr. Francesco Strazzullo Name: KEY

Instructions. You are expected to use a graphing calculator or a software, but not a CAS. You can use our class notes or alternatively a maximum of 4 size-letter sheets of notes. Sketch any graph that you use, **approximating up to the third decimal place**. Each problem is worth 10 points.

Always use the appropriate wording and units of measure in your answers (when applicable).
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Over the interval $[-2, 3]$, sketch the graph of a function $f(x)$ such that:
   - (a) $f(-2) = f(2) = f(3) = 0$;
   - INC. (b) $f'(x) > 0$ for $-2 < x < 0$ and $2 < x < 3$;
   - DEC. (c) $f'(x) < 0$ for $0 < x < 2$;
   - CONC. D. (d) $f''(x) < 0$ for $-2 < x < 2$;
   - CONC. UP (e) $f''(x) > 0$ for $2 < x < 3$;



2. For $f(x) = 3x^4 - x^2 + 2x - 1$, compute $f'(x)$ and $f''(x)$.

$$f'(x) = \frac{d}{dx}\left[f(x)\right] = \frac{d}{dx}\left[3x^4 - x^2 + 2x - 1\right] = \frac{d}{dx}\left[3x^4\right] - \frac{d}{dx}\left[x^2\right] +$$

$$+ \frac{d}{dx}\left[2x\right] - \frac{d}{dx}\left[1\right] = 3\frac{d}{dx}\left[x^4\right] - 2x^{2-1} + 2\frac{d}{dx}\left[x\right] - 0 =$$

$$\overset{*}{=} 3\left(4x^{4-1}\right) - 2x + 2\left(1x^{1-1}\right) = 12x^3 - 2x + 2$$

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[12x^3 - 2x + 2\right] = \frac{d}{dx}\left[12x^3\right] - \frac{d}{dx}\left[2x\right] +$$

$$+ \frac{d}{dx}\left[2\right] = 12\frac{d}{dx}\left[x^3\right] - 2\frac{d}{dx}\left[x\right] + 0 \overset{*}{=} 12\left(3x^{3-1}\right) - 2\left(1 \cdot x^{1-1}\right)$$

$$= 36x^2 - 2$$

$\boxed{\text{* YOU COULD JUST SHOW THIS EQUALITY, THAT IS START FROM THIS STEP}}$

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3. Differentiate the following functions (each problem is worth 10 points):

(a) $h(x) = \frac{x^3+2}{x^2-1}$ QUOTIENT RULE: $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f'g - fg'}{g^2}$

$f(x) = x^3 + 2 \rightarrow f'(x) = 3x^2 + 0 = 3x^2$

$g(x) = x^2 - 1 \rightarrow g'(x) = 2x - 0 = 2x$

$$h'(x) = \frac{3x^2(x^2-1) - (x^3+2)(2x)}{(x^2-1)^2} = \frac{3x^4 - 3x^2 - 2x^4 - 4x}{(x^2-1)^2}$$

$$= \frac{x^4 - 3x^2 - 4x}{(x^2-1)^2}$$

(b) $g(t) = e^t \sqrt[3]{t}$ PRODUCT RULE: $\frac{d}{dt}[uv] = u'v + uv'$

$u = e^t \rightarrow u' = e^t$

$v = \sqrt[3]{t} = t^{\frac{1}{3}} \rightarrow u' = \frac{1}{3} t^{\frac{1}{3}-1} = \frac{1}{3} t^{-\frac{2}{3}} = \frac{1}{3t^{2/3}} = \frac{1}{3\sqrt[3]{t^2}}$

$$g'(t) = e^t \sqrt[3]{t} + e^t \frac{1}{3\sqrt[3]{t^2}} = e^t \left(\frac{3t+1}{3\sqrt[3]{t^2}}\right)$$

NOT NEEDED

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4. The revenue R for the production t months after the new *Epad* is released depends on the number x of units produced and sold and the price per units p after t months, according to the formula:

$$R = p(t)\,x(t).$$

Six months after the release 100,000 units are produced and the production is increasing at a rate of 800 units per month, while the price is of \$400 per units and it is decreasing at a rate of 25 cents of dollars per month. Use the product rule and these figures to estimate the marginal revenue six months after the release of the new *Epad*. Is the revenue increasing at the time considered?

DATA: TIME $\rightarrow t = 6$ ALL RATES ARE WITH RESPECT TO t.

PRODUCTION $\rightarrow x(6) = 100{,}000$ UNITS ; $x'(6) = 800 \frac{\text{UNITS}}{\text{month}}$

PRICE $\rightarrow p(6) = 400$ \$/UNIT ; $p'(6) = -25 \frac{¢/\text{U.}}{\text{month}} = -.25 \frac{\$/\text{U}}{\text{M.}}$ DECREASING

PRODUCT RULE: $R' = p'x + p \cdot x'$

AT GIVEN TIME: MARGINAL REVENUE $= R'(6) = p'(6) \cdot x(6) +$

$+ p(6) \cdot x'(6) = -.25(100{,}000) + 400(800) = 295{,}000$ \$/month

AFTER SIX MONTHS THE REVENUE IS INCREASING OF \$ 295,000 PER MONTH.

5. The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function

$$n = f(t) = \frac{140}{1 + 6e^{-0.7t}}$$

where t is measured in hours. Estimate the rate of growth of this population after 2.5 hours (use the units of measure and the context).

"RATE OF GROWTH OF POPULATION" $= \frac{dn}{dt} = f'(t)$

"RATE OF GROWTH AFTER 2.5 HOURS" $= f'(2.5) = \left.\frac{dn}{dt}\right|_{t=2.5}$

PLUG $y = \frac{140}{1+6e^{-.7x}}$ IN YOUR CALCULATOR (OR SOFTWARE):

$\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{6} + (2.5) \rightarrow f'(2.5) = 24.489$ cells/hour

THIS POPULATION IS INCREASING AT A RATE OF ABOUT 25 CELLS PER HOUR AFTER 2.5 HOURS.

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