MAT 121 – Exam4 – Spring 2019

Instructor: Dr. Francesco Strazzullo **Name** Key

I certify that I did not receive third party help in *completing* this test (sign) _____

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use your own cheat sheet after I approve it, or the one on Eagleweb, or our book. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equation.

$$\sin^2(x) - 5\cos(x) = 2\cos(x) - 6$$

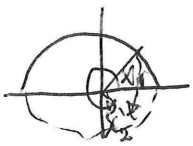
Please, **indicate all possible solutions**. Use the parameter k to represent any integer multiple of the function's period in the interval $[0, 2\pi)$. Write your answer in radians, as an exact answer when possible, else round to four decimal places. If there is no solution, please state "No Solution".

$\sin^2 x = 1 - \cos^2 x \Rightarrow \quad 1 - \cos^2 x - 5\cos x - 2\cos x + 6 = 0 \Rightarrow$

$\cos^2 x + 7\cos x - 7 = 0 \Rightarrow y^2 + 7y - 7 = 0 \Rightarrow y = \dfrac{-7 \pm \sqrt{49+28}}{2} =$

$\cos x = y$

$= \dfrac{-7 + \sqrt{77}}{2} \Rightarrow \cos x = y = \begin{cases} \dfrac{-7-\sqrt{77}}{2} < -1 & \text{NOT POSSIBLE} \\ \dfrac{-7+\sqrt{77}}{2} \approx .8875 \end{cases}$



$x_1 = \cos^{-1}\left(\dfrac{-7+\sqrt{77}}{2}\right) \approx .4789$

$x_2 = -x_1$ OR $x_2 = 2\pi - x_1 \approx 5.8042$

$x = x_1 + 2k\pi \approx .4789 + 2k\pi$

OR

$x = x_2 + 2k\pi \approx 5.8042 + 2k\pi$

2. Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equation.

$$\cos\left(2x - \frac{\pi}{6}\right) - 1 = \sin\left(\frac{\pi}{3} - 2x\right)$$

Please indicate **only the solutions in the interval $[0, 2\pi)$**. Write your answer in radians, as an exact answer when possible, else round to four decimal places. If there is no solution, please state "No Solution".

COFUNC.: $\sin\alpha = \cos\left(\frac{\pi}{2} - \alpha\right) \Rightarrow \sin\left(\frac{\pi}{3} - 2x\right) = \cos\left(\frac{\pi}{2} - \left(\frac{\pi}{3} - 2x\right)\right) = \cos\left(2x + \frac{\pi}{6}\right)$

SUM OF COSINE: $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

EQ. BECOMES: $\cos\left(2x - \frac{\pi}{6}\right) - 1 = \cos\left(2x + \frac{\pi}{6}\right) \Rightarrow \cos\left(2x - \frac{\pi}{6}\right) - \cos\left(2x + \frac{\pi}{6}\right) = 1$

$\Rightarrow -2\sin\left(\frac{2x - \frac{\pi}{6} + 2x + \frac{\pi}{6}}{2}\right) \cdot \sin\left(\frac{2x - \frac{\pi}{6} - \left(2x + \frac{\pi}{6}\right)}{2}\right) = 1 \Rightarrow$

$\Rightarrow \sin(2x) \cdot \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2} \Rightarrow -\frac{1}{2}\sin(2x) = -\frac{1}{2} \Rightarrow$

$\Rightarrow \sin(2x) = 1 \Rightarrow 2x = \frac{\pi}{2} + 2k\pi \Rightarrow$

$\Rightarrow x = \frac{\pi}{4} + k\pi$

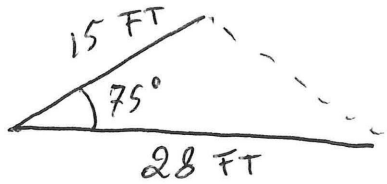
k	x
0	$\pi/4$
1	$5\pi/4$
2	$9\pi/4 > 2\pi$

ONLY ONES IN $[0, 2\pi)$

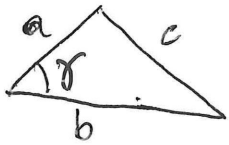
SOLUTIONS: $x = \frac{\pi}{4}, \frac{5}{4}\pi$.

$\approx .7854, 3.927$

3. Nick wants to build an herb garden in the backyard but is only using the area beyond the path running through the yard. How large would the garden be if it is triangular-shaped with two sides of length 15 feet, 28 feet, with a 75° angle between them? Round to the nearest hundredth.



$$\text{AREA} = \frac{1}{2}\, a b \cdot \sin \gamma$$



THEN $\text{AREA} = \frac{1}{2}(15)(28)\sin(75°) \approx 202.84 \text{ FT}^2$

4. Consider the following parametric equations:

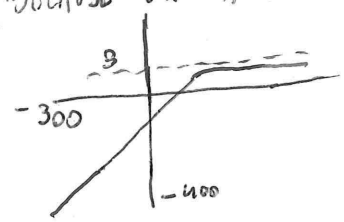
$$\begin{cases} x = 4 - t^5 \\ y = 3 - e^{2t} \end{cases}, -1 \le t \le 3$$

(a) Eliminate the parameter t, by writing your answer in simplest form solved for y .
(b) Determine the domain and range of the equation obtained by eliminating the parameter. Please write your answer in interval notation.
(c) Sketch the graph (copying it from a graphing utility).

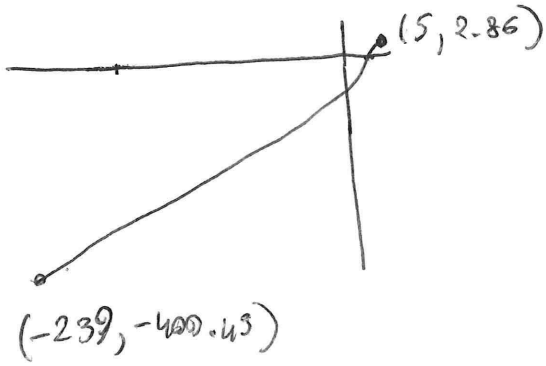
(a) $t = \sqrt[5]{4-x} \Rightarrow y = 3 - e^{2\sqrt[5]{4-x}}$

(b) DOMAIN $= (-\infty, \infty)$ (BECAUSE EXPO AND ODD-ROOT)

RANGE $= (-\infty, 3)$



(c) THE ORIGINAL PARAMETRIC FUNCTION :



5. Use any convenient method to solve the following system of equations. Indicate the number of solutions to this system. State the solution, if one exists, as an ordered triple, and if there are infinitely many solutions, express the solution set in terms of one of the variables. Leave all fractional answers in fraction form.

$$\begin{cases} 2x + y - 3z &= -2 \\ 3x - y + 2z &= -1 \\ x - 2y + z &= 4 \end{cases}$$

AU, MATRIX $\left[\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 3 & -1 & 2 & -1 \\ 1 & -2 & 1 & 4 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3/4 \\ 0 & 1 & 0 & -11/4 \\ 0 & 0 & 1 & -3/4 \end{array}\right] \Rightarrow$

SYST $\Rightarrow \begin{cases} X = -3/4 \\ Y = -11/4 \\ Z = -3/4 \end{cases} \rightarrow$ UNIQUE SOLUTION $\left(-\frac{3}{4}, -\frac{11}{4}, -\frac{3}{4}\right)$

6. Use Gauss-Jordan elimination (applying matrices) to solve the following system of equations.

$$\begin{cases} 2w + x - y &= 1 \\ w - x + 2z &= 2 \\ 2w + 3x - z &= 4 \\ w - 3x + y + 3z &= -1 \end{cases}$$

Indicate the number of solutions to this system. State the solution, if one exists, as an ordered quadruple, and if there are infinitely many solutions, express the solution set in terms of one of the variables.

AUG. MATRIX $\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 2 & 2 \\ 2 & 3 & 0 & -1 & 4 \\ 1 & -3 & 1 & 3 & -1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \rightarrow$

$\xrightarrow{\text{SYST}} \begin{cases} W = -1 \\ X = 3 \\ Y = 0 \\ Z = 3 \end{cases} \Rightarrow$ UNIQUE SOLUTION $(-1, 3, 0, 3)$

7. The following are the winning times for the goats' 800-ft race in Waleska, for selected years:

x	Year	Time (sec)	x	Year	Time (sec)
0	1990	24.2	8	1998	20.6
1	1991	21.9	12	2002	20.8
2	1992	21.7	16	2006	20.2
4	1994	21.5	18	2008	19.9
6	1996	20.8	26	2016	19.1

Consider x to be the number of years after 1990, and y to be the winning time. Use technology to complete the following steps.

(a) Compute the quadratic and the exponential models that are the best fit for these data (Round your answer to five decimal places and report their ***correlation coefficients***).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate the winning times in 2015 and 2017. Round to the hundredth of second.

(a) QUADR.: $Y = .00534x^2 - .26797x + 22.75025$, $R^2 = .78561$

EXPO: $Y = 22.35568e^{-.00657x}$, $R^2 = .73349$ TI. → .75727

(b) QUADRATIC IS BETTER, WITH LARGER R^2.

(c) 2015 → $X = 25$, $Y = 19.39$ SEC.

2017 → $X = 27$, $Y = 19.41$ SEC

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SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

8. Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equation.

$$\sin^2(2x) - 4\cos(2x) = 2\cos(2x) - 5$$

Please indicate **only the solutions in the interval $[0, \pi]$**. Write your answer in radians, as an exact answer when possible, else round to four decimal places. If there is no solution, please state "No Solution".

PYTHAG.: $\sin^2(2x) = 1 - \cos^2(2x) \Rightarrow 1 - \cos^2(2x) - 6\cos(2x) + 5 = 0$

$\Rightarrow \cos^2(2x) + 6\cos(2x) - 6 = 0$ SUBS $Y = \cos(2x)$;

$Y^2 + 6Y - 6 = 0 \Rightarrow Y = \dfrac{-6 \pm \sqrt{36+24}}{2} = \dfrac{-6 \pm 2\sqrt{15}}{2} \Rightarrow$

$\Rightarrow \cos(2x) = Y = \begin{cases} -3-\sqrt{15} < -1 & \text{NOT POSSIBLE} \\ -3+\sqrt{15} \approx .873 \end{cases}$

$2X = \cos^{-1}(-3+\sqrt{15}) + 2k\pi \Leftrightarrow X = \frac{1}{2}\cos^{-1}(-3+\sqrt{15}) + k\pi = X_1$

OR

$2X = -\cos^{-1}(-3+\sqrt{15}) + 2k\pi \Leftrightarrow X = -\frac{1}{2}\cos^{-1}(-3+\sqrt{15}) + k\pi = X_2$

k	X_1	X_2
0	.2548	<0
1	$>\pi$	2.8868

IN $[0, \pi]$ WE HAVE THE TWO SOLUTIONS $X = \frac{1}{2}\cos^{-1}(-3+\sqrt{15}) \approx .2548$

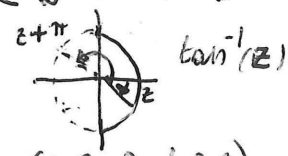
OR $X = \pi - \frac{1}{2}\cos^{-1}(-3+\sqrt{15}) \approx 2.8868$

9. Express the polar coordinates of the point with cartesian coordinates $(-2, \sqrt{3})$.

GIVEN (x, y). COMPUTE (r, θ) WHERE $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$

OR $\theta = \frac{\pi}{2}, \frac{3}{2}\pi$. $r = \sqrt{(-2)^2 + (\sqrt{3})^2} = \sqrt{7}$; $0 \le \theta < 2\pi \Rightarrow$

$\Rightarrow \theta = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \pi \approx 2.4279$



POL. COORD: $\left(\sqrt{7}, \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \pi\right) \approx (2.6458, 2.4279) \xrightarrow[\text{3 DECIMALS}]{} (2.646, 2.428)$

10. Use any convenient method to solve the following system of equations. Indicate the number of solutions to this system. State the solution, if one exists, as an ordered quadruple, and if there are infinitely many solutions, express the solution set in terms of one of the variables. Leave all fractional answers in fraction form.

$$\begin{cases} w - 2x + y - 3z &= -2 \\ 3w + 2x - y + z &= 0 \\ x + y + z &= 1 \end{cases}$$

AUG. MATRIX $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & -2 \\ 3 & 2 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/2 & -1/2 \\ 0 & 1 & 0 & 7/6 & 5/6 \\ 0 & 0 & 1 & -1/6 & 1/6 \end{array}\right] \rightarrow$

SYST $\Rightarrow \begin{cases} w - \frac{1}{2}z = -1/2 \rightarrow w = \frac{1}{2}z - \frac{1}{2} \\ x + \frac{7}{6}z = 5/6 \rightarrow x = -\frac{7}{6}z + 5/6 \\ y - \frac{1}{6}z = 1/6 \rightarrow y = \frac{1}{6}z + 1/6 \end{cases}$

INFINITELY MANY SOLUTIONS "Z IS FREE"

SOL. SET $= \left\{\left(\frac{1}{2}z - \frac{1}{2}, -\frac{7}{6}z + \frac{5}{6}, \frac{1}{6}z + \frac{1}{6}, z\right) \text{ FOR } z \text{ IN } \mathbb{R}\right\}$