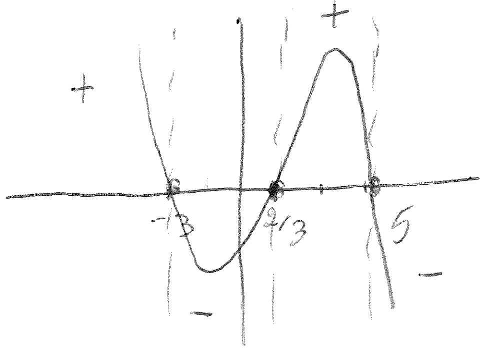
MAT 121 – Exam 2 – Spring 2017

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test: only a textbook can be used, or a cheat-sheet approved by your instructor. Personal notebooks cannot be used. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the polynomial inequality $(x - 5)(x + 3)(2 - 3x) \leq 0$. Write your answer in interval notation.



ZEROS: $x = 5, -3, \frac{2}{3}$

SOLUTION: $\left[-3, \frac{2}{3}\right] \cup [5, +\infty)$

2. Use polynomial long division to rewrite the following rational function in the form $f(x) = q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder. Then write the equation of the non-vertical asymptote.

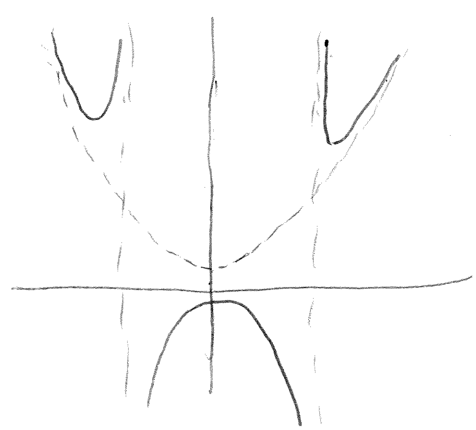
$$f(x) = \frac{5x^4 - 8x^3 - 5x^2 + 4x + 5}{2x^2 - 18}$$

USING TECH OR BY HANDS:

$$\begin{array}{r|l} & \frac{5}{2}x^2 - 4x + 20 \\ \hline 2x^2 - 18 & 5x^4 - 8x^3 - 5x^2 + 4x + 5 \\ & -5x^4 \qquad\quad + 45x^2 \\ \hline & -8x^3 + 40x^2 + 4x + 5 \\ & 8x^3 \qquad\quad - 72x \\ \hline & 40x^2 - 68x + 5 \\ & -40x^2 \qquad + 360 \\ \hline & -68x + 365 \end{array}$$

$$f(x) = \underbrace{\frac{5}{2}x^2 - 4x + 20}_{\text{NON V.A.}} + \frac{-68x + 365}{2x^2 - 18}$$

$Y = q(x) = \frac{5}{2}x^2 - 4x + 20$



3. Consider the factored polynomial

$$f(x) = (x-5)^3(x+1)^4(x-2).$$

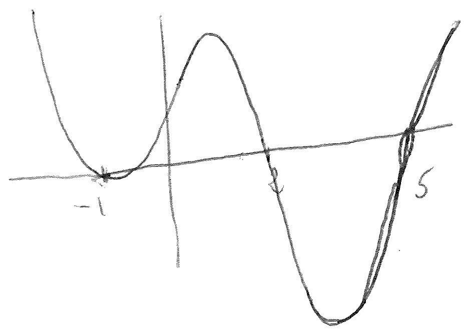
Step 1. Determine the degree and y-intercept (write the y-intercept as an ordered pair).
Step 2. Determine the x-intercept(s) at which f crosses the axis. If there are none, state "none".
Step 3. Determine the zero(s) of f at which it "flattens out". If there are none, state "none".

1) DEGREE $= 3+4+1 = 8$

NOTATION: $f(x) = a(x-c_1)^{k_1} \cdots (x-c_m)^{k_m}$

2) k_j ODD: $x = 5, 2 \Rightarrow (5,0), (2,0)$

3) $k_j > 1$: $x = 5, -1 \Rightarrow (5,0), (-1,0)$



4. Solve the rational inequality. Write your answer in interval notation (use two decimal places if needed).

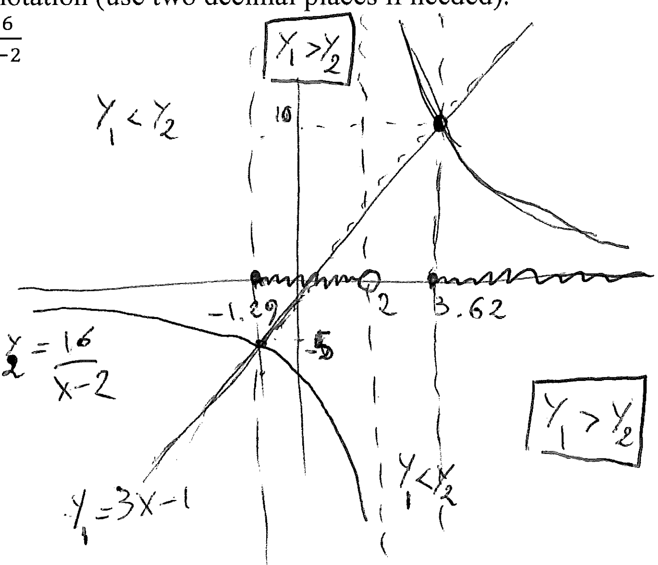
$$3x - 1 > \frac{16}{x-2}$$

V.A. $X = 2$; INTERSECTION POINTS: $3X - 1 = \dfrac{16}{X-2} \Rightarrow$

$\Rightarrow (3x-1)(x-2) = 16 \Rightarrow$

$\Rightarrow 3X^2 - 7X - 14 = 0$

$X = \dfrac{7 \pm \sqrt{217}}{6} \approx \begin{cases} -1.29 \\ 3.62 \end{cases}$



SOLUTION: $\left[\frac{7-\sqrt{217}}{6}, 2\right) \cup \left[\frac{7+\sqrt{217}}{6}, \infty\right)$

5. The half-life of gold-194 is approximately 1.4 days.

Step 1. Determine a so that $A(t) = A_0 a^t$ describes the amount of gold-194 left after t days, where A_0 is the amount at time $t = 0$. Round to 6 decimal places.

Step 2. How much of a 10 gram sample of gold-194 would remain after 4 days? Round to 3 decimal places.

Step 3. How much of a 10 gram sample of gold-194 would remain after 2 days? Round to 3 decimal places.

1) $A_0 a^{1.4} = \frac{1}{2} A_0 \Rightarrow \left(a^{1.4}\right)^{\frac{1}{1.4}} = \left(\frac{1}{2}\right)^{1/1.4} \Rightarrow a \approx 0.609507$

$A = 10\,(.609507)^t$

2) $A(4) = 1.38$ g

3) $A(2) = 3.71$ g

6. In an effort to control vegetation overgrowth, 149 rabbits are released in an isolated area free of predators. After 2 years, it is estimated that the rabbit population has increased to 596. Assuming exponential population growth, what will the population be after another 9 months? Round to the nearest rabbit.

$P = P_0 a^t$, $P_0 = 149$, t IN MONTHS: 2 YEARS $\rightarrow t = 24$

$149\, a^{24} = 596 \Rightarrow \left(a^{24}\right)^{\frac{1}{24}} = \left(\frac{596}{149}\right)^{\frac{1}{24}} \Rightarrow a = 4^{\frac{1}{24}} = 2^{\frac{1}{12}} \approx 1.059463$

$P = 149\; 2^{\frac{1}{12}t} \approx 149\,(1.059463)^t$

"ANOTHER 9 MONTHS" $\Rightarrow t = 24 + 9 = 33$.

$P(33) = 149\left(2^{\frac{33}{12}}\right) \approx 1002$ RABITS

7. Jose hopes to earn \$600 in interest in 4.3 years time from \$6,000 that he has available to invest. To decide if it's feasible to do this by investing in an account that compounds quarterly, he needs to determine the annual interest rate such an account would have to offer for him to meet his goal. What would the annual rate of interest have to be? Round to two decimal places.

QUARTERLY $\Rightarrow n=4$; $P=6000$; $t=4.3$; $A=600+6000$

$$A=P\left(1+\frac{r}{n}\right)^{nt} \Rightarrow 6000\left(1+\frac{r}{4}\right)^{4\cdot 4.3}=6600 \Rightarrow$$

$$\Rightarrow \left(1+\frac{r}{4}\right)^{17.2}=\frac{11}{10} \Rightarrow 1+\frac{r}{4}=(1.1)^{\frac{1}{17.2}} \Rightarrow$$

$$\Rightarrow r=4\cdot\left(1.1^{\frac{1}{17.2}}-1\right)\approx 0.0222 \Rightarrow r=2.22\%$$

8. Solve the following logarithmic equation. Round your answer to 4 decimal places.
$\log_5(x-2)+\log_5(2x+3)=2$

$$5^{\log_5((x-2)(2x+3))}=5^2 \Rightarrow 2x^2-x-6=25 \Rightarrow$$

$$\Rightarrow 2x^2-x-31=0 \Rightarrow x=\frac{1\pm\sqrt{249}}{4}\approx\begin{cases}-3.6949\\ 4.1949\end{cases}$$

NOTE THAT $\log_5(-3.69-2)$ IS UNDEFINED

ONLY ONE SOLUTION:

$$x=\frac{1+\sqrt{249}}{4}\approx 4.1949$$