

Math 102 - Spring 2012 - Test 4

KEY

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Name _____

Instructions. Only calculators are allowed on this examination. **Each problem is worth 20 points.**

Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulae:

$$S = P \left(1 + \frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}((1+i)^n - 1), \quad A = \frac{R}{i}(1 - (1+i)^{-n}).$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the following logarithmic equation and check the solution(s)

$$2 \log_4 (x-2) = 2$$

TWO WAYS HERE:

$$\text{I) } \frac{2 \log_4 (x-2)}{2} = \frac{2}{2} \Rightarrow \log_4 (x-2) = 1 \Rightarrow x-2 = 4 \Rightarrow x = 6$$

$$\text{II) } \log_4 (x-2)^2 = 2 \Rightarrow (x-2)^2 = 4^2 \Rightarrow x^2 - 4x + 4 = 16 \Rightarrow x^2 - 4x - 12 = 0 \Rightarrow (x-6)(x+2) = 0$$

$$\Rightarrow x = 6 \text{ or } x = -2$$

CHECK: $x = 6$ PLUG: $2 \log_4 (6-2) = 2 \log_4 4 = 2 \cdot 1 = 2 \checkmark$

$x = -2$ PLUG: $2 \log_4 (-2-2)$ IS NOT DEFINED (Domain of log is $(0, +\infty)$)

2. Use a calculator to evaluate to the third decimal place the following expressions:

(a) $\log_3 5 = \frac{\log 5}{\log 3} \approx 1.465$

(b) $(1.02)^{3.4} = (1.02)^1 (1.02)^{2.4} \approx 1.07$

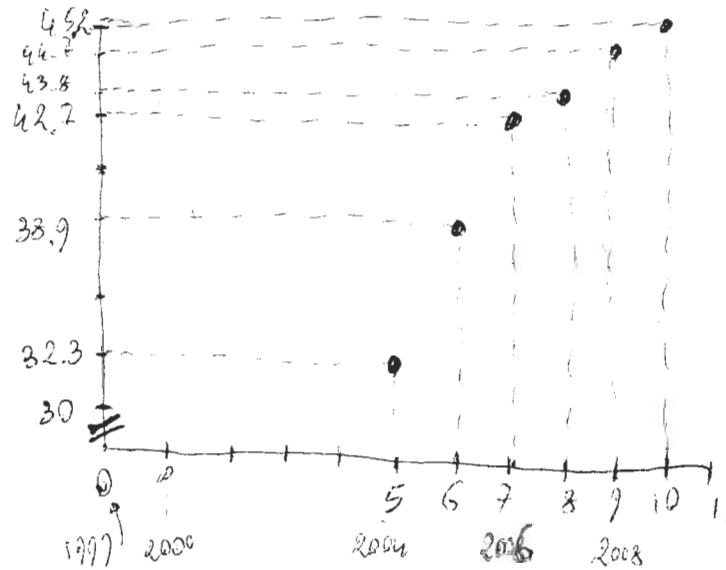
3. Because of the weakening of the U.S. dollar, U.S.-based corporations are generating a growing share of their sales overseas. The following table shows the percent of sales made abroad in selected fiscal years.

2000 $\rightarrow x=0$

Year	Percent (years)	x	Year	Percent (years)	x	Year	Percent (years)
2004	32.3	4	2006	42.7	6	2008	44.7
2005	38.9	5	2007	43.8	7	2009	45.2

- (a) Consider x the number of years from 2000 and y the percent of sales made abroad, then draw a scatter-plot of the given data.

YOU CAN TAKE
 $x=0$ IN 2000
 OR
 $x=1$ IN 2000
 HERE I CHOSE
 WHAT MOST OF
 YOU DID CHOOSE:
 $x=1$ IN 2000.
 THIS CORRESPONDS
 TO A LEFT SHIFT
 FOR THOSE WHO
 CHOSE $x=0$ IN 2000



- (b) Using your calculator, find both the quadratic and the logarithmic models which best fit the data. Report your answer to the third decimal place. Report and use the correlation coefficients to say which of these models is the best fit for the given data.

QUAD REG: $y = -.752x^2 + 13.648x - 16.614$, $R^2 = .982$

LOG REG: $y = 5.716 + 17.885 \ln x$, $R^2 = .879$

BEST FIT IS QUAD REG., WHOSE CORRELATION COEFFICIENT R^2 IS BIGGER.

- (c) Use the (unrounded) best model to compute the expected percent of sales made abroad in 2011.

FOR OUR CHOICE 2011 $\rightarrow x = 11 + 1 = 12$

PLUG $x=12$ IN QUAD. MODEL: $y = 38.91\%$ (or 38.907%)

4. You are shopping for a simple investment of \$4000 that you expect to cash in 3 years. They offer you two options:

- (a) a monthly compounded account at 4.5%;
- (b) a quarterly compounded account at 5.25%.

Compute which option is the best deal for you.

FUTURE VALUE OF A SIMPLE INVESTMENT, IN BOTH CASES DISCRETELY COMPOUNDED: $S = P \left(1 + \frac{r}{k}\right)^{kt}$

IN BOTH CASE: $P = 4000$; $t = 3$

$$a) k=12, r = \frac{4.5}{100} = 0.045 \rightarrow S = 4000 \left(1 + \frac{0.045}{12}\right)^{12 \cdot 3} = \$4586.99$$

$$b) k=4, r = \frac{5.25}{100} = 0.0525 \rightarrow S = 4000 \left(1 + \frac{0.0525}{4}\right)^{4 \cdot 3} = \$4677.53$$

OPTION b IS BEST FOR ME.

5. You are offered the following two retirement plans:

- (a) a one time payment of \$200,000, or
- (b) a monthly payment of \$2,000 for ten years.

If you can assume a constant inflation rate of 4.5%, which option is more valuable when you retire? Why?

(a) IT IS JUST 200,000 (CASH) WITHOUT ANY OTHER INTEREST.

(b) PERIODIC PAYMENTS \rightarrow ANNUITY. WE MUST CHECK THE CURRENT

BUYING POWER OF THIS ANNUITY \rightarrow PRESENT VALUE:

$$A = \frac{R}{i} \left(1 - (1+i)^{-n}\right) = \frac{2000}{0.00375} \left(1 - (1.00375)^{-120}\right) = 192,978.65$$

$$\left. \begin{array}{l} t = 10; r = \frac{4.5}{100} = 0.045; \\ R = 2000; k = 12 \end{array} \right\} \rightarrow \left[\begin{array}{l} i = \frac{r}{k} = \frac{0.045}{12} = 0.00375 \\ n = t \cdot k = 10 \cdot 12 = 120 \end{array} \right]$$

MONTHLY

OPTION a HAS MORE VALUE. NOTE THAT ACTUAL PAYMENTS $= R \cdot n = 240,000$. LOOSE VALUE BECAUSE OF INFLATION.