MAT321 – Spring 2019 –Exam2

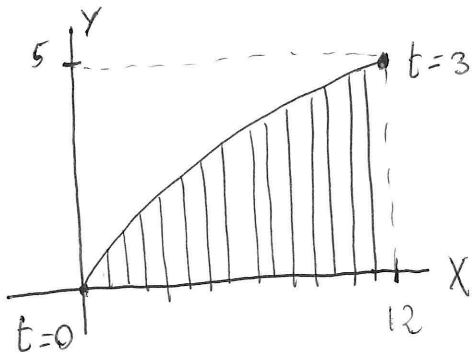
Instructor: Dr. Francesco Strazzullo **Name** KEY

I certify that I did not receive third party help in *completing* this test (sign) ____________

Instructions. Technology and open book are allowed on this exam. Each problem is worth 10 points. If you use notes or formula sheets, make a reference. Do not approximate, unless otherwise indicated. When approximating, use four decimal places. You cannot use a CAS to justify your answers, but only to perform computation, unless otherwise indicated. Use the appropriate units of measure in your answer, when applicable.

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1.Find the area of the region bounded by the curve $x = t^2 + t, y = t - \cos t + 1,\ 0 \leq t \leq 3$, and the *x*-axis.



$$\text{AREA} = \int_0^{12} y\,dx =$$

$$dx = x'dt = (2t+1)dt$$

$$= \int_0^3 (t+1-\cos t)(2t+1)\,dt =$$

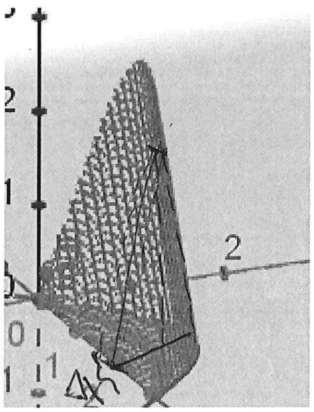
$$= \int_0^3 2t^2+2t-2t\cos t+t+1-\cos t\,dt =$$

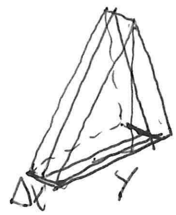
BY PARTS $u = t$, $dv = \cos t\,dt$

$$= \left[\frac{2}{3}t^3+\frac{3}{2}t^2+t-\sin t\right]_0^3 - 2\int_0^3 t\cos t\,dt$$

$$= 18+\frac{27}{2}+3-\sin 3 - 2\left[t\sin t+\cos t\right]_0^3 = \frac{67}{2}-\sin 3-2(3\sin 3+\cos 3-1) = \frac{73}{2}-7\sin 3-2\cos 3 \approx 37.4921$$

2. The base of a certain solid is a plane region $\boldsymbol{R} = \{(x, y) |\ 0 \le y \le \sin x, 0 \le x \le \pi\}$. Each cross-section of the solid perpendicular to the x-axis is an isosceles triangle with base lying in $\boldsymbol{R}$ and height three times the base. Find the volume of this solid.





BASE $= Y$; HEIGHT $= 3Y$

$$A(x) = \frac{1}{2}(Y)(3Y) = \frac{3}{2}Y^2$$

$$= \frac{3}{2}\sin^2 x$$

$$V = \int_0^{\pi} A(x)\,dx = \int_0^{\pi} \frac{3}{2}\sin^2 x\,dx$$

$$\int \sin^2 x\,dx = \sin x(-\cos x) - \int -\cos x\,(\cos x\,dx) = -\sin x\cos x + \int \cos^2 x\,dx \Rightarrow$$

$u = \sin x$
$dv = \sin x\,dx$

$$\Rightarrow \int \sin^2 x\,dx = -\frac{1}{2}\sin(2x) + \int 1 - \sin^2 x\,dx \Rightarrow$$

$$\Rightarrow 2\int \sin^2 x\,dx = -\frac{1}{2}\sin(2x) + x + C \Rightarrow$$

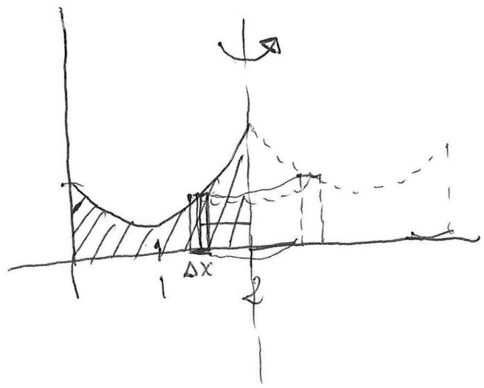
$$\Rightarrow \int \sin^2 x\,dx = \frac{1}{4}\left(2x - \sin(2x)\right) + C$$

THEN

$$V = \frac{3}{2}\cdot\frac{1}{4}\left[2x - \sin(2x)\right]_0^{\pi} = \frac{3}{8}(2\pi - 0) = \frac{3}{4}\pi$$

$$\approx 2.3562$$

3. Find the volume of the solid obtained by rotating about the line $x = 2$ the region bounded by $y = 0, x = 0, x = 2$, and $y = e^x - 3x + 1$.



CYLINDRICAL SHELLS: $V = \int_a^b 2\pi\, h(x)\, r(x)\, dx$

$h(x) = e^x - 3x + 1$; $r(x) = 2 - x$ $\Rightarrow$

$$\Rightarrow V = 2\pi \int_0^2 (e^x - 3x + 1)(2 - x)\, dx =$$

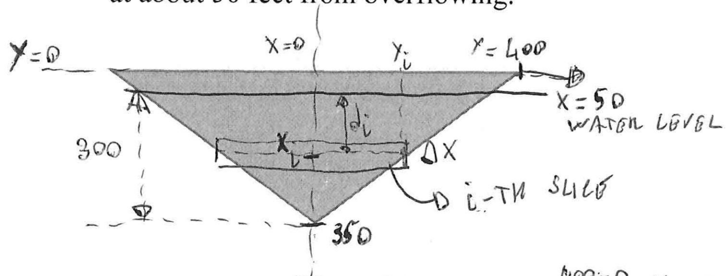
$$= 2\pi \int_0^2 2(e^x - 3x + 1) - \underbrace{xe^x}_{\text{BY PARTS KNOW } e^x(x-1)} + 3x^2 - x\, dx$$

$$= 2\pi \left[2\left(e^x - \frac{3}{2}x^2 + x\right) - e^x(x-1) + x^3 - \frac{1}{2}x^2 \right]_0^2$$

$$= 2\pi \left[2e^2 \cancel{-12} \cancel{+4} - e^2 \cancel{+8} - 2 - 2e^0 + e^0(-1) \right]$$

$$= 2\pi (e^2 - 5) \approx 15.0109$$

4. Find the total hydraulic force on a dam in the shape of an isosceles triangle (like in the figure below), if the top rim of the dam is 800 feet and the water has filled the basin, to a depth of 300 feet, with the water level at about 50 feet from overflowing.



δ = WEIGHT DENSITY OF WATER = $62.4 \ \text{lb}/\text{FT}^3$

PRESSURE ON i-TH SLICE = $P_i = d_i \cdot \delta$

DEPTH OF i-TH SLICE = $d_i = X_i - 50$

W_i = WEIGHT ON i-TH SLICE = $A_i P_i$

$Y_i = 400 + \frac{400-0}{0-350} X_i \Rightarrow A_i$ = AREA OF i-TH SLICE = $2Y_i \Delta X = 2(400 - \frac{8}{7}X_i)\Delta X$

THEN HYD. FORCE = $\sum_{i=1}^{\infty} W_i = \int_{50}^{350} \underset{\frac{\text{lb}}{\text{FT}^3}}{62.4} \cdot \underset{\text{FT}}{(X-50)} \cdot \underset{\text{FT}}{2\left(400 - \frac{8}{7}X\right)} \underset{\text{FT}}{dx} =$ ✓

$$= 124.8 \int_{50}^{350} -\frac{8}{7}X^2 + \frac{3200}{7}X - 20000 \, dx$$

$$= 124.8 \left[-\frac{8}{21}X^3 + \frac{1600}{7}X^2 - 20000X \right]_{50}^{350}$$

$$= 124.8 \, (5142857.143)$$

$$= 641{,}828{,}571.4 \text{ lb}$$

MAT321 – Spring 2019 – Exam2 – InClass

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5. Find the arc length of the curve $\begin{cases} x = e^{-5t+2} \\ y = t^2 \end{cases}$ from $t = 0$ to $t = 2$. (*Once you apply the formula and simplify its terms, you can use technology to perform the computation.*)

HONOR: curve $\begin{cases} x = e^{-5t^2+2} \\ y = t^2 \end{cases}$

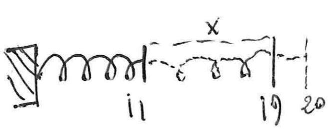
$$L = \int_a^b \sqrt{(x')^2+(y')^2}\,dt = \int_0^2 \sqrt{(-5e^{-5t+2})^2 + (2t)^2}\,dt = \int_0^2 \sqrt{25e^{-10t+4}+4t^2}\,dt$$

$$\approx 10.8045$$

HONOR: $$L = \int_0^2 \sqrt{\left(-10t\,e^{-5t^2+2}\right)^2+(2t)^2}\,dt = \int_0^2 \sqrt{100t^2e^{-10t^2+4}+4t^2}\,dt$$

$$\approx 10.6031$$

6. A force of 15 pounds is required to stretch a spring from its natural length of 11 inches to a length of 19 inches (By Hooke's Law the force is directly proportional to the stretching). How much work is done in stretching the spring to a length of 20 inches?



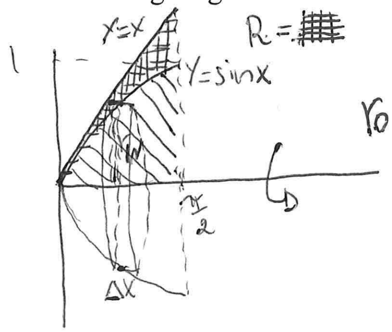
$x = 19 - 11 = 8$ WHEN $F = 15\text{ lb}$, AND $F = kx \Rightarrow$

$\Rightarrow 15 = k \cdot 8 \Rightarrow k = 15/8 \Rightarrow F = \frac{15}{8}x$

$$W = \int_0^9 \frac{15}{8}x\,dx = \frac{15}{8}\left[\frac{1}{2}x^2\right]_0^9 = \frac{15}{16}9^2 = \frac{1215}{16} \text{ IN}-\text{lb}$$

$$\approx 75.9375$$

7. The plane region $\boldsymbol{R} = \left\{(x, y) \mid \sin x \le y \le x, 0 \le x \le \frac{\pi}{2}\right\}$ is rotated around the x-axis, generating the solid S. Assuming lengths are measured in feet, compute the volume of S.



WASHER METHOD $A_i = \pi\left(r_{OUT}^2 - r_{IN}^2\right)$ $\Big] \Rightarrow$

$r_{OUT} = x$; $r_{IN} = \sin x$

$$\Rightarrow V = \pi \int_0^{\frac{\pi}{2}} x^2 - \sin^2 x \, dx \overset{\#2}{\underset{6}{=}}$$

$$= \pi \left[\frac{x^3}{3} - \frac{1}{2}x + \frac{1}{4}\sin(2x) \right]_0^{\frac{\pi}{2}}$$

$$= \pi \left(\frac{\pi^3}{24} - \frac{\pi}{4} \right) = \frac{1}{4}\left(\frac{\pi^4}{6} - \pi^2 \right) \quad \text{FT}^3$$

$$\approx 1.5913$$

8. A rod with uniform density (mass/unit length) $\delta(x)=2+\cos(x)$ lies on the x-axis between $x=0$ and $x=\pi$. Find the mass and center of mass of the rod.

HONOR: A rod with uniform density (mass/unit length) $\delta(x)=2+\ln(x)$ lies on the x-axis between $x=1$ and $x=3$. Find the mass and center of mass of the rod.

$$m = \int_a^b \delta(x)\,dx \quad ; \quad \bar{x} = \frac{1}{m}\int_a^b x\delta(x)\,dx$$

$$m = \int_0^\pi 2+\cos x\,dx = \Big[2x+\sin x\Big]_0^\pi = 2\pi \approx 6.2832 \quad \text{UNIT MASS}$$

$$\bar{x} = \frac{1}{2\pi}\int_0^\pi 2x+\underbrace{x\cos x}_{\text{BY PARTS}}\,dx = \frac{1}{2\pi}\Big[x^2+x\sin x+\cos x\Big]_0^\pi$$

BY PARTS $u=x$; $dv=\cos x\,dx$

$$= \frac{1}{2\pi}\left(\pi^2-1-1\right) = \frac{\pi^2-2}{2\pi} \approx 1.2525 \quad \text{UNIT LENGTH}$$

HONOR:
$$m = \int_1^3 2+\ln x\,dx = \Big[2x+x(\ln x-1)\Big]_1^3 = 6+3\ln 3-3-2+1$$
$$= 2+\ln 27 \quad \text{UNIT MASS}$$
$$\approx 5.2958$$

$$\bar{x} = \frac{1}{2+\ln 27}\int_1^3 2x+x\ln x\,dx =$$

$$= \frac{1}{2+\ln 27}\left[\frac{3}{4}x^2+\frac{x^2}{2}\ln x\right]_1^3 = \frac{1}{2+\ln 27}\left(\frac{27}{4}+\frac{9}{2}\ln 3-\frac{3}{4}\right) \approx 2.0665 \quad \text{UNIT LENGTH}$$

$$\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$