MAT 221 - Fall 2014 - Exam 2

Instructor: Dr. Francesco Strazzullo Name KEY

I did not receive third party help in completing this test. *Signature*__________

Instructions. You are expected to use a graphing calculator or software to complete some problems. Files can be downloaded and uploaded to the Eagleweb Coursework page for this assignment. Upload files or sketch any graph that you use or tables of input/output, **approximating up to the fourth decimal place.** Each problem is worth 10 points, unless otherwise specified. Total points = 80.
SHOW YOUR WORK NEATLY, PLEASE. (no work = no points)

1. Compute the first derivatives of the following functions. Show your work, applying the correct derivation rule, and give a simplified expression. Each part is worth 10 points.

a. $y = \ln(\sin x)$

$\frac{d}{dx}[\ln u] = \frac{u'}{u} \rightarrow y' = \frac{\cos x}{\sin x} = \cot x$

$u = \sin x \rightarrow u' = \cos x$

NOTE: DOMAIN OF y IS $\sin x > 0 \Rightarrow 2k\pi < x < (2k+1)\pi$

b. $s = \arcsin(1 - 2t)$

$\left.\begin{array}{l}\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \\ u = 1-2t \rightarrow u' = -2\end{array}\right] \rightarrow \frac{ds}{dt} = \frac{-2}{\sqrt{1-(1-2t)^2}} = \frac{-2}{\sqrt{1-(4t^2-4t+1)}}$

$\rightarrow \frac{ds}{dt} = \frac{-2}{\sqrt{4t-4t^2}} = \frac{-2}{\sqrt{4}\sqrt{t-t^2}} = \frac{-1}{\sqrt{t-t^2}}$

20)

c. $y = 3^{x^2+x}$

$$\frac{d}{dx}\left[a^u\right] = \ln a \cdot a^u \cdot u' \quad u = x^2+x \to u' = 2x+1 \quad \to y' = \ln 3\,(2x+1)\cdot 3^{x^2+x}$$

d. $z = w^{\cos(2w)}$

NO FORMULA. USE LOGARITHMIC DIFFERENTIATION:

$$z = w^{\cos(2w)} \to \ln z = \ln\left(w^{\cos(2w)}\right) \to \ln z = \cos(2w)\cdot\ln w$$

IMPLICIT DIFFERENTIATION: $\frac{d}{dw}\left[\ln z\right] = \frac{d}{dw}\left[\cos(2w)\cdot\ln w\right] \to$

$$\to \frac{z'}{z} = -2\sin(2w)\cdot\ln w + \cos(2w)\cdot\frac{1}{w} \to \text{MULTIPLY BY } z = w^{\cos(2w)}$$

PRODUCT RULE

$$\frac{d}{dw}\left[\cos(2w)\right] = 2\cdot(-\sin(2w))$$

$$\to \frac{dz}{dw} = w^{\cos(2w)}\left(\frac{\cos(2w)}{w} - 2\ln w\cdot\sin(2w)\right)$$

20

2. Write the equation of the tangent line to graph of $x^2 + 3yx - 4y^3 = \ 2$ at the point $(1, -1)$.

$$L(x) = y\Big|_{\substack{x=a\\y=b}} + y'\Big|_{\substack{x=a\\y=b}} \cdot (x-a) \ , \quad a=1 \ , \quad b=-1 \ :$$

IMPLICIT DIFFERENTIATION: $\frac{d}{dx}\left[x^2+3yx-4y^3\right] = \frac{d}{dx}\left[-2\right] \rightarrow$

$$\rightarrow 2x + 3(y'x + y\cdot 1) - 12y^2\cdot y' = 0$$

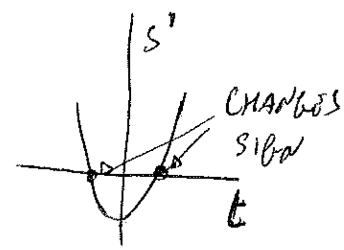
$$(3x - 12y^2)y' = -2x - 3y \rightarrow y' = \frac{2x+3y}{12y^2-3x}$$

$$y'\Big|_{\substack{x=1\\y=-1}} = \frac{2+3(-1)}{12\cdot 1 - 3(1)} = \frac{-1}{9} = -\frac{1}{9}$$

$$L(x) = -1 - \frac{1}{9}(x-1) = -\frac{1}{9}x - \frac{8}{9}$$

3. A particle moves along a straight line with equation of motion $s = t^3 - 24t + 4$. Find the value of t at which the particle reverses its direction.

THE PARTICLE REVERSES ITS DIRECTION ONLY IF ITS DERIVATIVE (SPEED) CHANGES SIGN.



$$s' = 3t^2 - 24 = 3(t^2 - 24)$$

$$s' = 0 \Rightarrow t = \pm\sqrt{8} = \pm 2\sqrt{2} \approx \pm 2.8284$$

$$t > 0 \Rightarrow t = 2\sqrt{2}.$$

AFTER ABOUT 2.8 SECONDS THE PARTICLE REVERSES DIRECTION.

20

4. Find the equation of the normal line at the point $\left(\sqrt{2}-1, \frac{3\sqrt{2}}{2}-2\right)$ to the curve with parametric equations $x = 2\cos t - 1$, $y = 3\sin t - 2$, for $0 \le t \le \frac{\pi}{2}$. Use symbolic notation.

IF $y'\big|_{x=a, y=b} = m$ IS DEFINED THEN N IS A VERTICAL LINE IF $m = 0$

AND $N(x) = b + \frac{-1}{m}(x-a)$ IF $m \neq 0$. NOTE $a = f(t_0)$, $b = g(t_0)$, $m = y'\big|_{t_0}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\cos t}{-2\sin t} = -\frac{3}{2}\cot t$$

$$\frac{dy}{dt} = 3\cos t$$

$$\frac{dx}{dt} = -2\sin t$$

TO FIND t_0, SOLVE: $\sqrt{2}-1 = 2\cos t - 1 \Rightarrow \cos t = \frac{\sqrt{2}}{2} \Rightarrow t = \frac{\pi}{4}$

BECAUSE $0 \le t \le \frac{\pi}{2}$.

CHECK: $y\left(\frac{\pi}{4}\right) = 3\sin\left(\frac{\pi}{4}\right) - 2 = 3\frac{\sqrt{2}}{2} - 2$ ✓

$$y'\big|_{\frac{\pi}{4}} = -\frac{3}{2}\cot\left(\frac{\pi}{4}\right) = -\frac{3}{2} \neq 0$$

$$N(x) = \frac{3}{2}\sqrt{2} - 2 + \frac{2}{3}\left(x - (\sqrt{2}-1)\right)$$

$$= \frac{2}{3}x + \left(\frac{3}{2} - \frac{2}{3}\right)\sqrt{2} - 2 + \frac{2}{3}$$

$$N(x) = \frac{2}{3}x + \frac{5}{6}\sqrt{2} - \frac{4}{3}$$

10

5. In the last column of the following table it is reported the unemployment rate in Georgia on the month of May in selected years (check the files on Ealeweb).

X	Year	Period	labor force	employment	unemployment	unemployment rate
1	2004	May	4440938	4235167	205771	4.6
2	2005	May	4600673	4357662	243011	5.3
3	2006	May	4705966	4481678	224288	4.8
4	2007	May	4795144	4581612	213532	4.5
5	2008	May	4855568	4574010	281558	5.8
6	2009	May	4779197	4317559	461638	9.7
7	2010	May	4707772	4237853	469919	10.0
8	2011	May	4738272	4269802	468470	9.9
9	2012	May	4771128	4336469	434659	9.1
10	2013	May	4782639	4382537	400102	8.4
11	2014	May	4780418	4435028	345390	7.2

Source: http://data.bls.gov/timeseries/LASST130000000000003

a. Use this table to approximate the rate of change of the unemployment rate in May 2007 and in May 2013. Use units. USE X = 0 IN MAY 2003 AND Y = UNEMPL. RATE. (May 2007: X = 4; May 2013: X = 10)

SECANT: $y'(4) \approx \frac{4.5-4.8}{4-3} = -.3$ OR $y'(4) \approx \frac{5.8-4.5}{5-4} = 1.3$

$y'(10) \approx \frac{8.4-9.1}{10-9} = -.7$ OR $y'(10) \approx \frac{7.2-8.4}{11-10} = -.8$

UNIT: % PER YEAR

MIDPOINT: $y'(4) \approx \frac{5.8-4.8}{5-3} = .5$ AND $y'(10) \approx \frac{7.2-9.1}{11-9} = -.95$

b. Compute the 7[th] degree polynomial regression that models the unemployment rate as a function of the year, **considering years from May 2003**: report the correlation coefficient (round to 4 decimal places).

$$Y = -.0003X^7 + .0099X^6 - .1424X^5 + .9692X^4 - 3.0265X^3 + 3.1925X^2 + 1.7354X + 1.8418$$

$R^2 = .9751$

c. Use the model from part b. to estimate the rate of change of the unemployment rate in May 2007 and in May 2013, then compare these to your results from part a. Which ones should you consider a better estimate?

$y'(4) = .8926$ % PER YEAR

$y'(10) = .2183$ % PER YEAR

THOSE IN PART a. COULD NOT BE GOOD ESTIMATES BECAUSE WE NEEDED CLOSER VALUES OF X TO BOTH 4 AND 10, FOR INSTANCE AT LEAST APRIL AND JUNE 2007.

THE CORRELATION COEFFICIENT R^2 IS VERY GOOD, THEREFORE THOSE COMPUTED HERE ARE VERY GOOD ESTIMATES OF THE RATE OF CHANGE OF THE UNEMPLOYMENT RATE IN GEORGIA.