MAT 121 – Final – Spring 2016

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Name KEY

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use your own cheat sheet after I approve it, or the one on Eagleweb. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Consider the following equation of a line.

$$1 - \frac{6y + 4x}{2} = 4$$

Find the equation, in slope-intercept form, for the line which is **perpendicular** to this line and passes through the point $(6, 2)$.

SLOPE OF GIVEN LINE: $-6y - 4x = 6 \Rightarrow y = -\frac{2}{3}x - 1 \Rightarrow$ PERPENDICULAR LINE HAS SLOPE $m = -\left(-\frac{2}{3}\right)^{-1} = \frac{3}{2}$. $\Rightarrow y = \frac{3}{2}x + b$ THEN

SEEKING: $y = mx + b$

PLUG POINT $x = 6, y = 2$: $2 = \frac{3}{2}(6) + b \Rightarrow b = -7$.

$$y = \frac{3}{2}x - 7$$

2. Solve the system of two linear inequalities graphically.

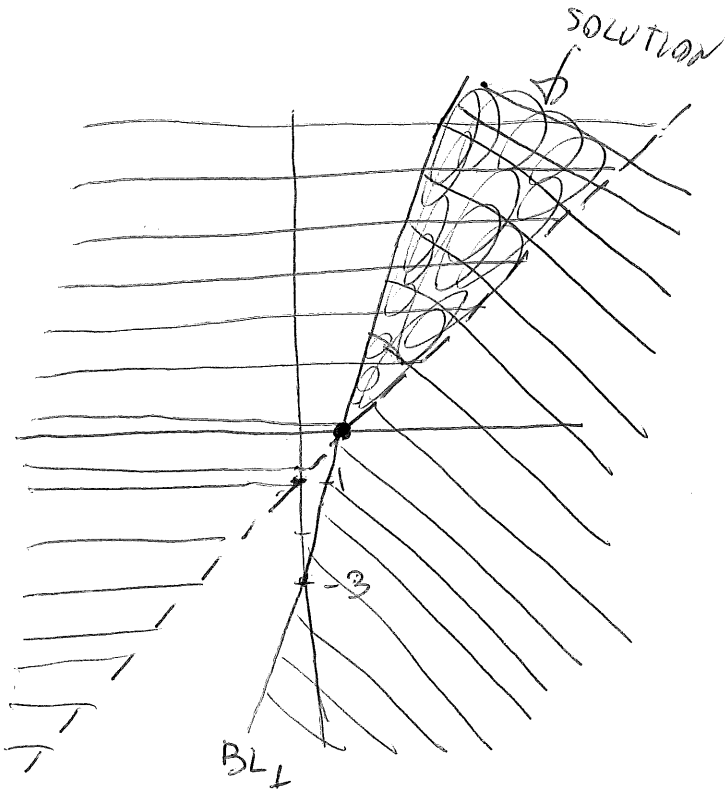
$y \leq 3x - 3$ and $y > x - 1$

BL_1: $y = 3x - 3$ SOLID, REGION BELOW.

BL_2: $y = x - 1$ DASHED, REGION ABOVE

CORNER POINT $(1, 0)$.

SOLUTION IS OVERLAPPING.

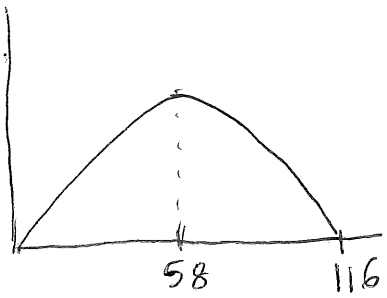


3. A small cruising ship that can hold up to 68 people provides three-day excursions to groups of 46 or more. If the group contains 46 people, each person pays \$70. The cost per person for all members of the party is reduced by \$1 for each person in excess of 46. Find the size of the group that maximizes income for the owners of the ship.

X = # OF PEOPLE ≥ 46

p = "PRICE PER PERSON" $= 70 - 1\cdot(x-46) = 116 - x$

"INCOME" = "REVENUE" $= x \cdot p = 116x - x^2$



MAX AT VERTEX (h, k), WITH

$$h = -\frac{b}{2a} = -\frac{116}{-2} = 58$$

58 PERSONS MAXIMIZE REVENUE

4. Solve the polynomial inequality $\sqrt{(x-5)(x+3)(4-x)} > 0$. Write your answer in interval notation.

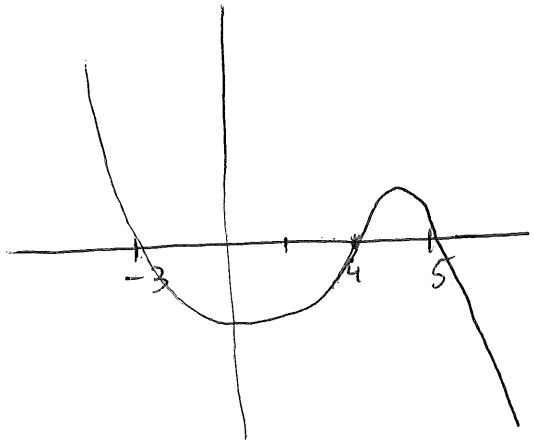
	$-\infty$	-3	4	5	∞
$x-5$	$-$	$-$	$-$	$+$	
$x+3$	$-$	$+$	$+$	$+$	
$4-x$	$+$	$+$	$-$	$-$	
y	$+$	$-$	$+$	$-$	

$x-5>0 \Rightarrow x>5$

$x+3>0 \Rightarrow x>-3$

$4-x>0 \Rightarrow x<4$

SOLUTION $(-\infty, -3) \cup (4, 5)$



5. Use polynomial long division to rewrite the following rational function in the form $f(x) = q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

$$f(x) = \frac{3x^4 - 12x^3 + 18x^2 + x + 6}{3x^2 - 3}$$

(a) describe the domain of $f(x)$ in interval notation,
(b) determine the equations of any asymptote, and
(c) sketch the graph of $f(x)$ (you can use a graphing utility and copy the graph here).

(a) $3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \Rightarrow$ DOMAIN: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b)

$$\begin{array}{r|l} & x^2 - 4x + 7 \\ \hline 3x^2 - 3 & 3x^4 - 12x^3 + 18x^2 + x + 6 \\ & -3x^4 \quad\quad + 3x^2 \\ \hline & -12x^3 + 21x^2 + x + 6 \\ & 12x^3 \quad\quad - 12x \\ \hline & 21x^2 - 11x + 6 \\ & -21x^2 \quad\quad + 21 \\ \hline & -11x + 27 \end{array}$$

$$f(x) = \underbrace{x^2 - 4x + 7}_{q(x)} + \frac{\overbrace{27 - 11x}^{r(x)}}{3x^2 - 3}$$

NON-VERTICAL ASYMPTOTE:

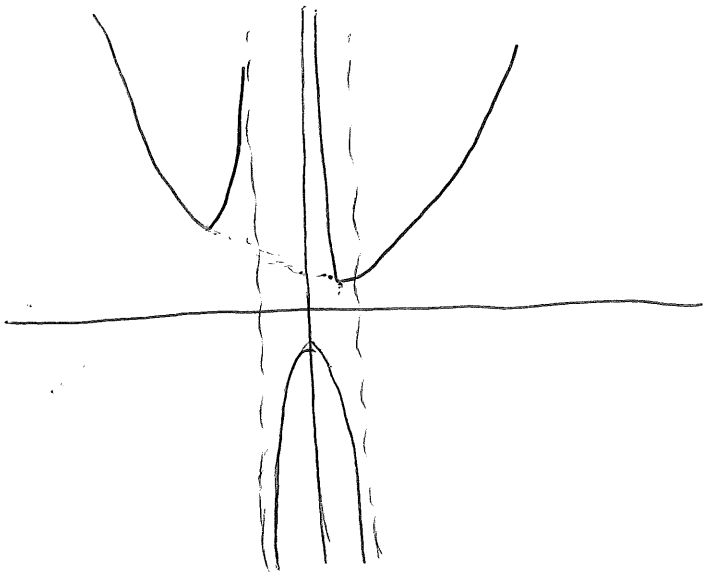
$y = q(x)$

$y = x^2 - 4x + 7$

V.A. POSSIBLE $x = 1, x = -1$

1) $\frac{r(-1)}{q(-1)} = \frac{38}{0} \rightarrow$ V.A.

2) $\frac{r(1)}{q(1)} = \frac{16}{0} \rightarrow$ V.A.



6. Chester hopes to earn $1500 in interest in 3.3 years time from $15,000 that he has available to invest. To decide if it's feasible to do this by investing in an account that compounds quarterly, he needs to determine the annual interest rate such an account would have to offer for him to meet his goal. What would the annual rate of interest have to be? Round to two decimal places.

SIMPLE INVESTMENT COMPOUNDED n-TIMES: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

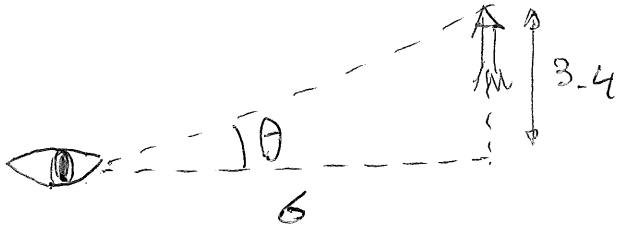
HERE $n = 4$, $t = 3.3$, $P = 15000$, $A = 1500 + 15000 = 16500$ $\Rightarrow$

$\Rightarrow 15000\left(1 + \frac{r}{4}\right)^{4\cdot(3.3)} = 16500$ POWER EQUATION:

$\left(1 + \frac{r}{4}\right)^{\frac{66}{5}} = \frac{11}{10} \Rightarrow 1 + \frac{r}{4} = \left(\frac{11}{10}\right)^{\frac{5}{66}} \Rightarrow r = 4(1.0072 - 1)$

$\Rightarrow r \approx 0.029$ OR 2.9%

7. Ethan is watching a satellite launch from an observation spot 6 miles away. Find the angle of elevation from Ethan to the satellite, which is at a height of 3.4 miles. Write your answer in degrees rounded to two decimal places.



$\tan(\theta) = \frac{3.4}{6} \Rightarrow$

$\Rightarrow \theta = \tan^{-1}\left(\frac{17}{30}\right) \Rightarrow$

MULTIPLY BY $\frac{180}{\pi}$ IF IN RAD

$\Rightarrow \theta \approx 29.54^\circ$

8. Determine the amplitude, period, and phase shift of the following trigonometric function.

$y = -4 + 8\cos(5x - 3\pi)$

AMPLITUDE $= 8$

PERIOD $= \frac{2\pi}{b} = \frac{2}{5}\pi$

SHIFT TO THE RIGHT OF $\frac{c}{b}$ UNITS: $\frac{c}{b} = \frac{3\pi}{5}$

9. Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{cases} -3x + 2y + 2z = 15 \\ x + 6y - 4z = 15 \\ 4x + 4y - 6z = 0 \end{cases}$$

AUGMENTED MATRIX $= \begin{pmatrix} -3 & 2 & 2 & | & 15 \\ 1 & 6 & -4 & | & 15 \\ 4 & 4 & -6 & | & 0 \end{pmatrix}$ $\xrightarrow{\text{RREF}}$ $\begin{pmatrix} 1 & 0 & -1 & | & -3 \\ 0 & 1 & -1/2 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

$$\Rightarrow \begin{cases} X - Z = -3 \Rightarrow X = Z - 3 \\ Y - \frac{1}{2}Z = 3 \Rightarrow Y = \frac{1}{2}Z + 3 \end{cases}$$

SOLUTION SET $= \left\{ \left(Z - 3, \frac{1}{2}Z + 3, Z\right) \text{ FOR } Z \text{ IN } \mathbb{R} \right\}$

10. The table below gives the violent crime rate (per 100,000 people) for a particular state every five years from 1970 to 2010.

Year	1970	1975	1980	1985	1990	1995	2000	2005	2010
Violent Crime Rate	4.9	5.1	6	7.4	9	10.5	11.7	12.4	12.2

Consider x to be the number of years after 1960, and y to be the violent crime rate during that year.

(a) Use technology to compute the power, the exponential and the trigonometric regression models for this data.

(b) Use the best model among those of the previous step to estimate the violent crime rate during 2009 (rounded to the first decimal place).

(a) POWER: $Y = 0.8891\, X^{0.6792}$, $R^2 = 0.9581$

EXPONENTIAL: $Y = 3.7265\, e^{0.0267x}$, $R^2 = 0.914$

SINE: $Y = 8.6291 + 3.7468 \sin(0.0876x - 2.5301)$, $R^2 = .9999$

NOTE THE PERIOD $\frac{2\pi}{.0876}$ IS ABOUT 72 YEARS.

(b) THE BEST MODEL IS THE TRIGONOMETRIC (OR SINE) ONE

INTERPOLATION FOR 2009 IS $X = 49$ AND $Y(49) = 12.3\%$