Mat321 – Spring 2018 –Exam1-Home

Instructor: Dr. Francesco Strazzullo Name KEY

I certify that I did not receive third party help in *completing* this test (sign)________________

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SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1.A particle travels along a line. Its velocity in meters per second is given by $v(t) = \cos(3\pi t) - t^3$. Find

(a) the displacement during the [1, 7] time frame;

(b) the distance traveled by the particle from $t = 2$ to $t = 4$.

$$(a) = \int_1^7 v(t)\,dt = \left[\frac{1}{3\pi}\sin(3\pi t) - \frac{t^4}{4}\right]_1^7 = \cancel{\frac{1}{3\pi}\left(\sin(21\pi) - \sin(3\pi)\right)} - \frac{1}{4}(7^4 - 1)$$

$$= -600 \text{ METERS}$$

$$(b)\ v(t) < 0 \text{ ON } [2;4]:\ \text{DIST} = \int_2^4 |v(t)| = \int_2^4 t^3 - \cos(3\pi t) = \left[\frac{1}{4}t^4 - \cancel{\frac{\sin(3\pi t)}{3\pi}}\right]_2^4$$

$$= 60 \text{ METERS}$$

2.A virus is spreading, infecting at a rate modeled by

$$I'(t) = 235\frac{(1 + 2t^3)}{(t^2 + 2)^2}$$

individuals per day, where $I(t)$ is the number of new people infected t days after the first case has been recorded. What is the net-change in new cases recorded during the first three weeks? (Once you setup this problem you can use technology to perform computations.)

FIRST THREE WEEKS $\Rightarrow$ $0 \leq t \leq 21$

$$I(21) - I(0) = \int_0^{21} I'(t)\,dt = 235\int_0^{21} \frac{1+2t^3}{(t^2+2)^2} \underset{\text{TECH.}}{\approx} 1100.41$$

THE NET CHANGE IS OF ABOUT 1100 PEOPLE.

3.Compute

$$\frac{d}{dx}\int_1^{x^2}\sqrt{3t^3+2}\,dt$$

$g(x)=\int_1^{x^2}\sqrt{3t^3+2}\,dt$ ASKED FOR $g'(x)$ $\Rightarrow$ CHAIN RULE

FTC: $H(x)=\int_1^{x}\sqrt{3t^3+2}\,dt \Rightarrow H'(x)=\sqrt{3x^3+2}$

$g'(x)=\frac{d}{dx}\left[H(x^2)\right]=2x\cdot H'(x)=2x\sqrt{3x^6+2}$

4.Use the Substitution Method to evaluate the following integral:

$$\int_2^4 x\sqrt{2x+4}\,dx = \int_8^{12}\left(\tfrac{1}{2}u-2\right)\sqrt{u}\cdot\tfrac{1}{2}du=\frac{1}{2}\int_8^{12}\tfrac{1}{2}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\,du=$$

$u=2x+4 \Rightarrow du=2dx \Rightarrow dx=\frac{1}{2}du$

$\downarrow$

$x=\frac{1}{2}u-2$ ENDPOINTS $\begin{cases} x=2 \Rightarrow u=8 \\ x=4 \Rightarrow u=12 \end{cases}$

$$=\frac{1}{2}\left[\frac{1}{5}u^{\frac{5}{2}}-\frac{4}{3}u^{\frac{3}{2}}\right]_8^{12}=\frac{1}{2}\left[u^{\frac{3}{2}}\left(\frac{1}{5}u-\frac{4}{3}\right)\right]_8^{12}$$

$$=\frac{1}{2}\left[12\sqrt{12}\left(\frac{1}{5}12-\frac{4}{3}\right)-8\sqrt{8}\left(\frac{1}{5}8-\frac{4}{3}\right)\right]=$$

$$=12\sqrt{3}\left(\frac{16}{15}\right)-32\sqrt{2}\left(\frac{1}{15}\right)=\frac{32}{15}\left(6\sqrt{3}-\sqrt{2}\right)$$

≈ 19.15

5.Use Integration by Parts to evaluate the following integral:

$$\int \underbrace{x^2 \cos x \, dx} = x^2 \sin x - \underbrace{\int 2x \sin x \, dx}_{\text{BY PARTS}}$$

$u = x^2 \to du = 2x\,dx$

$dv = \cos x\,dx \to v = \int dv = \sin x$

$u = 2x \to du = 2dx$

$dv = \sin x\,dx \Rightarrow v = \int dv = -\cos x$

$$= x^2 \sin x + 2x\cos x + \int -2\cos x\,dx =$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$

6. Use the Partial Fractions Method to evaluate the following integral:

$$\int \frac{3x+2}{(3x-2)(x+4)} dx =$$

$$\frac{3x+2}{(3x-2)(x+4)} = \frac{A}{3x-2} + \frac{B}{x+4} \Rightarrow A(x+4) + B(3x-2) = 3x+2$$

$x = -4 \Rightarrow -14B = -10 \Rightarrow B = 5/7$

$x = 2/3 \Rightarrow \frac{14}{3} A = 4 \Rightarrow A = 6/7$

$$= \int \frac{6/7}{3x-2} + \frac{5/7}{x+4} dx = \frac{1}{7}\left(\frac{6}{3}\int \frac{3dx}{3x-2} + 5\int \frac{dx}{x+4}\right) =$$

$$= \frac{1}{7}\left(2\ln|3x-2| + 5\ln|x+4|\right) + C$$

$$= \ln \sqrt[7]{|3x-2|^2 |x+4|^5} + C$$

7. Determine whether the following integral is convergent or divergent, and evaluate it if it is convergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a \to 0^+} \int_a^1 \frac{\ln x}{\sqrt{x}}\,dx \Rightarrow$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}} \ln x\,dx \underset{P}{=} 2x^{\frac{1}{2}} \ln x - \int 2x^{-\frac{1}{2}}\,dx =$$

BY PARTS

$u = \ln x \Rightarrow du = x^{-1}dx$

$dv = x^{-\frac{1}{2}}dx \Rightarrow v = \int dv = 2x^{\frac{1}{2}}$

$$= 2x^{\frac{1}{2}} \ln x - 4x^{\frac{1}{2}} + C = 2x^{\frac{1}{2}}(\ln x - 2) + C$$

$$\Rightarrow \int_0^1 x^{-\frac{1}{2}} \ln x\,dx = \lim_{a \to 0^+} 2\left[x^{\frac{1}{2}}(\ln x - 2)\right]_a^1 =$$

$$= 2 \lim_{a \to 0^+} \left(-2 - a^{\frac{1}{2}}(\ln a - 2)\right) = 2(-2 - 0) = \boxed{-4}$$

$$\lim_{a \to 0^+} a^{\frac{1}{2}}(\ln a - 2) \underset{0 \cdot \infty}{=} \lim_{a \to 0^+} \frac{\ln a - 2}{a^{-\frac{1}{2}}} \underset{H.R.}{\overset{\frac{\infty}{\infty}}{=}}$$

$$= \lim_{a \to 0^+} \frac{a^{-1}}{-\frac{1}{2}a^{-\frac{3}{2}}} = \lim_{a \to 0^+} -2a^{\frac{1}{2}} = 0$$

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Evaluate the following

8) $\dfrac{d}{dx}\displaystyle\int_1^{x^3} \sec(2t-1)\,dt \quad = g'(x)$

$g(x) = \displaystyle\int_1^{x^3} \sec(2t-1)\,dt$

$H(x) = \displaystyle\int_1^{x} \sec(2t-1)\,dt \overset{FTC}{\Rightarrow} H'(x) = \sec(2x-1) \Rightarrow$

$\Rightarrow g(x) = H(x^3)$ AND $g'(x) = 3x^2 H'(x^3) =$

$= 3x^2 \sec(2x^3-1)$

9) $\int_1^2 \frac{1}{(3x-6)^2}\,dx$ $= \int_{-3}^{0} u^{-2}\frac{1}{3}du = \frac{1}{3}\lim_{b\to 0^-}\int_{-3}^{b} u^{-2}du =$

$u = 3x-6 \Rightarrow du = 3dx \Rightarrow dx = \frac{1}{3}du$

$\Rightarrow \begin{cases} x=1 \Rightarrow u=-3 \\ x=2 \Rightarrow u=0 \end{cases}$

$= \frac{1}{3}\lim_{b\to 0^-}\left[-u^{-1}\right]_{-3}^{b} = \frac{1}{3}\lim_{b\to 0^-}\left(-\frac{1}{b}-\frac{1}{3}\right) = \frac{1}{3}\left(\infty-\frac{1}{3}\right) = \infty$

10) $\int x\ln x\,dx$ $= \frac{1}{2}x^2\ln x - \int\frac{1}{2}x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

$u = \ln x \Rightarrow u' = x^{-1}dx$

$dv = x\,dx \Rightarrow v = \int dv = x^2/2$

$= \frac{1}{4}x^2(2\ln x - 1) + C$