MAT 121 – Exam2 – Spring 2019 Take Home Part

Instructor: Dr. Francesco Strazzullo **Student** _____

I certify that I did not receive third party help in *completing* this test (sign)_____

Instructions. This is an open book test. Each exercise is worth 10 points. Do not approximate, unless otherwise indicated. When approximating, use four decimal places. You cannot use a CAS to justify your answers, but only to perform computations. Use the appropriate units of measure in your answer, when applicable.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Consider the factored polynomial

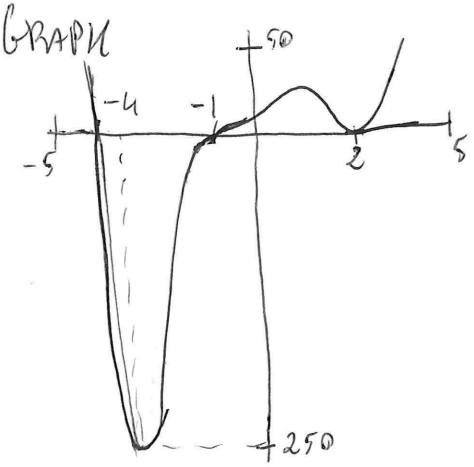
$$f(x) = (x+1)^3(x-2)^2(x+4).$$

Step 1. Determine the degree and y-intercept (write the y -intercept as an ordered pair).
Step 2. Determine the x-intercept(s) at which f crosses the axis. If there are none, state "none".
Step 3. Determine the zero(s) of f at which it "flattens out". If there are none, state "none".
Step 4. Determine the turning points of f (specifying if maximum or minimum, and approximate if needed).

1) DEG = 3+2+1 = 6 ; 2) MULTIPLICITY IS ODD: $(-1,0)$, $(-4,0)$
Y-INTERCEPT = $(0, f(0) = 1^3(-2)^2 \cdot 4 = 16)$

3) MULTIPLICITY $\geq 2 \Rightarrow$ $x = -1$, $x = 2$

4) GRAPH



RELATIVE MINIMA: $(-3.386, -241.9426)$

$(2, 0)$

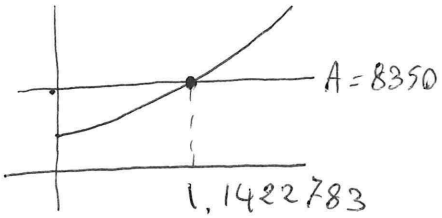
RELATIVE MAXIMUM: $(.886, 40.677)$

2. Jacob invests \$8,000 in an account that compounds weekly with 3.75% annual interest rate. How long would it take Jacob to earn \$350 in interest? Solve graphically, sketching your graph and rounding to one decimal place.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$, $I = A - P$; $P = 8000$, $r = \frac{3.75}{100}$, WEEKLY $n = 52$

$A = 8000\left(1 + \frac{.0375}{52}\right)^{52t}$

$I = 350 \Rightarrow A = P + I = 8350$



$t = 1.1$

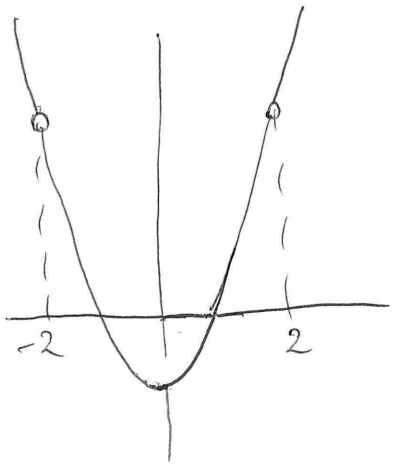
(NOTE: $A(1.1) < 8350 \Rightarrow t = 1.2$)

3. Use polynomial long division to rewrite the following rational function in the form $f(x) = q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder. Then write the equations of any asymptote.

$f(x) = \frac{2x^4 - 9x^2 + 4}{x^2 - 4} = 2x^2 - 1$ THEN $f(x)$ HAS NONE V.A. (HOLES AT $x = \pm 2$)

$Y = 2x^2 - 1$ IS THE NON-VERTICAL ASYMP.

$$\begin{array}{r|l} & 2x^2 - 1 \\ \hline x^2 - 4 & 2x^4 - 9x^2 + 4 \\ & 2x^4 - 8x^2 \\ \hline & -x^2 + 4 \\ & -x^2 + 4 \\ \hline & 0 \end{array}$$



MAT 121 – Exam2 – Spring 2019

Instructor: Dr. Francesco Strazzullo **Name**_____

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test: only a textbook can be used, or a cheat-sheet approved by your instructor. Personal notebooks cannot be used. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used. **SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

4. Solve the rational inequality. Write your answer in interval notation (use two decimal places if needed).

$3x - 2 \geq \frac{2}{x-4}$

$\frac{(3x-2)(x-4) - 2}{x-4} \geq 0$

$\frac{3x^2 - 14x + 6}{x-4} \geq 0$

REST VALUES: $x-4=0 \Rightarrow x=4$

ZERO'S: $3x^2 - 14x - 10 = 0 \Rightarrow$

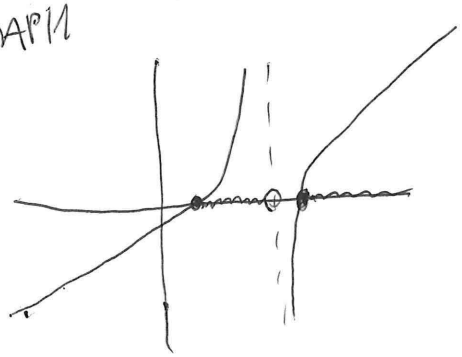
$\Rightarrow x = \frac{14 \pm \sqrt{124}}{2(3)} = \frac{7 \pm \sqrt{31}}{3} \approx .48 \approx 4.19$

TABLE

	$-\infty$		$\frac{7-\sqrt{31}}{3}$		4		$\frac{7+\sqrt{31}}{3}$		∞
x		0		1		4.1		5	
$f(x)$		-1.5	0	1.2	0	-9.2	0	11	
		−		+		−		+	

SOLUTION = $\left[\frac{7-\sqrt{31}}{3}, 4\right) \cup \left[\frac{7+\sqrt{31}}{3}, \infty\right)$

GRAPH



HONOR SECTION $\frac{3x-2}{x-1} \geq \frac{2x+1}{x-4}$

$\frac{(3x-2)(x-4) - (2x+1)(x-1)}{(x-1)(x-4)} \geq 0$

$\frac{3x^2 - 14x + 8 - 2x^2 + x + 1}{(x-1)(x-4)} \geq 0$

$\frac{x^2 - 13x + 9}{(x-1)(x-4)} \geq 0$ REST VALUES: $x = 1, 4$

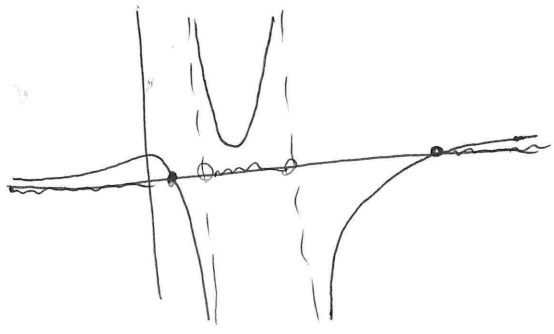
ZERO'S: $x = \frac{13 \pm \sqrt{133}}{2} \approx .73 \approx 12.27$

TABLE

	$-\infty$		$\frac{13-\sqrt{133}}{2}$		1		4		$\frac{13+\sqrt{133}}{2}$		∞
x		0		.8		2		5		13	
$f(x)$		2.25	0	-1.19	∄	6.5	∄	-7.25	0	.08	
		+		−		+		−		+	

SOLUTION = $\left(-\infty, \frac{13-\sqrt{133}}{2}\right] \cup (1,4) \cup \left[\frac{13+\sqrt{133}}{2}, \infty\right)$

GRAPH



5. Kryptonite is a radioactive isotope. After t years, a 50-gram sample of kryptonite decays according to the model

$$K(t) = 50(3^{-0.2t}).$$

Step 1. How much of a 50-gram sample of kryptonite would remain after 7 years? Round to 3 decimal places.

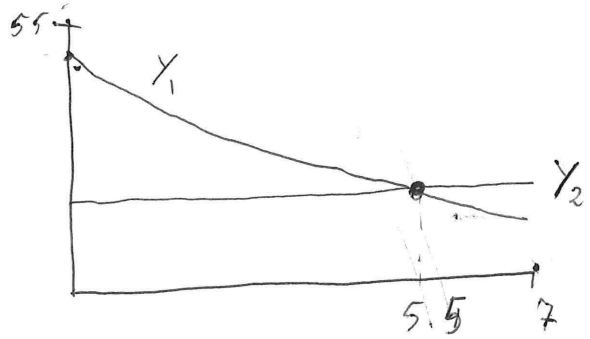
Step 2. Superman would be harmed by 15 or more grams of kryptonite. How long would it take for the remaining amount of a 50-gram sample of kryptonite to be harmless to Superman? Round to 1 decimal place.

1) $K(7) = 50(3^{-.2(7)}) \approx 10.74$ GRAMS

2) AFTER $K(t) = 15$, SUPERMAN WOULD BE SAFE

SOLVE GRAPHICALLY

$\underbrace{50(3^{-0.2t})}_{Y_1} = \underbrace{15}_{Y_2}$



TI-84: 2ND + TRACE + 5

ABOUT 5.5 YEARS (TO BE SAFE, 5.6 YEARS)

6. Consider the factored polynomial

$$f(x) = (x-3)^2(x+3)^2(x-1).$$

Step 1. Determine the zero(s) of f. If there are none, state "none".

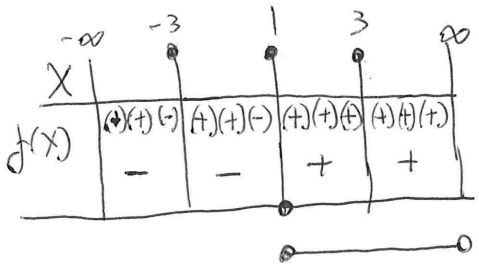
Step 2. Determine the set, in interval notation, on which f is positive. If there are none, state "none".

Step 3. Using technology, determine the turning points of f (specifying if maximum or minimum, and approximating if needed).

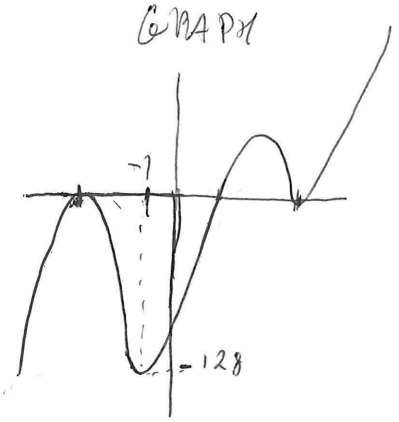
HONOR only. Determine the set, in interval notation, on which f is decreasing. If there are none, state "none".

1) ZEROS : $X = 3, -3, 1$

2) TABLE



GRAPH



$f(x) > 0$ ON $(1,3) \cup (3,\infty)$

3) RELATIVE MINIMA : $(-1,-128)$; $(3,0)$

RELATIVE MAXIMA: $(-3,0)$; $(1.8, 26.542)$

4) FROM GRAPH AND TURNING POINTS:

$f(x)$ DECREASING ON $(-3,-1) \cup (1.8,3)$

7. Use polynomial long division to find factor the following polynomial function $f(x)$, knowing that $x = 3 + i$ is a complex root of multiplicity 1, then state all the roots of $f(x)$.

$f(x) = 6x^4 - 35x^3 + 52x^2 + 22x - 20$

I) IF $3+i$ IS A ROOT THEN ALSO $\overline{3+i} = 3-i$ IS A ROOT WITH SAME MULTIPLICITY (F.T.A), THEREFORE $(x-(3+i))(x-(3-i))$ DIVIDES $f(x)$

$(x-(3+i))(x-(3-i)) = x^2-(3-i)x-(3+i)x+(3+i)(3-i)$

$= x^2-6x+9-i^2 = x^2-6x+10$

II)

$$\begin{array}{r|l} & 6x^2+x-2 \\ \hline x^2-6x+10 & 6x^4-35x^3+52x^2+22x-20 \\ & 6x^4-36x^3+60x^2 \\ \hline & x^3-8x^2+22x-20 \\ & x^3-6x^2+10x \\ \hline & -2x^2+12x-20 \\ & -2x^2+12x-20 \\ \hline & 0 \quad \checkmark \end{array}$$

III) THEN

$f(x) = (x-(3+i))(x-(3-i))(6x^2+x-2)$

IV)

$6x^2+x-2$

$-12 \mid -1 \mid -2 \mid -3$

$\quad\;\; 12 \mid 6 \mid 4$

SUM = 1

$6x^2+4x-3x-2$

$2x(3x+2)-1(3x+2)$

$(3x+2)(2x-1)$

V) THEN

$f(x) = (x-(3+i))(x-(3-i))(3x+2)(2x-1)$

$x = -\frac{2}{3}$ $\quad$ $x = \frac{1}{2}$

VI) ROOTS: $x = 3+i,\ 3-i,\ -\frac{2}{3},\ \frac{1}{2}$