Math 102 - Spring 2013 - Test 4

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Instructions. Only calculators are allowed on this examination. **Each problem is worth 10 points.** *Always use the appropriate wording and units of measure in your answers (when applicable).* You might need the following formulas:

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left((1+i)^n - 1\right), \quad A = \frac{R}{i}\left(1-(1+i)^{-n}\right).$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Use a calculator to find the value of the following expressions.

   (a) $1.5^{1.8} \approx 2.075$

   (b) $\log_{.34} 4.65 \approx -1.425$

   $= \dfrac{\log 4.65}{\log(.34)}$

2. Solve the following logarithmic equation and check for *extraneous solutions*

$\log_5(2x+1) + \log_5(4-x) - 2$

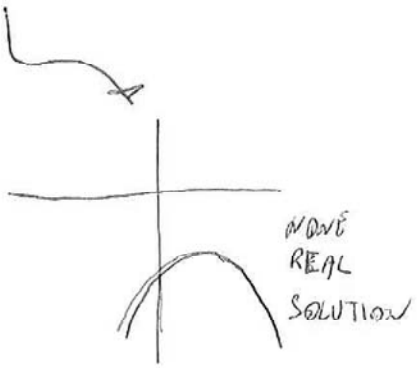
$\log_5\left((2x+1)(4-x)\right) = 2$

$5^{\log_5\left((2x+1)(4-x)\right)} = 5^2 \Rightarrow (2x+1)(4-x) = 25 \rightarrow$

$\rightarrow 8x - 2x^2 + 4 - x - 25 = 0 \rightarrow -2x^2 + 7x - 21 = 0$

$x = \dfrac{-7 \pm \sqrt{49 - 4(-2)(-21)}}{2(-2)}$

$x = \dfrac{-7 \pm \sqrt{-119}}{-4} = \dfrac{7 \pm i\sqrt{119}}{4}$

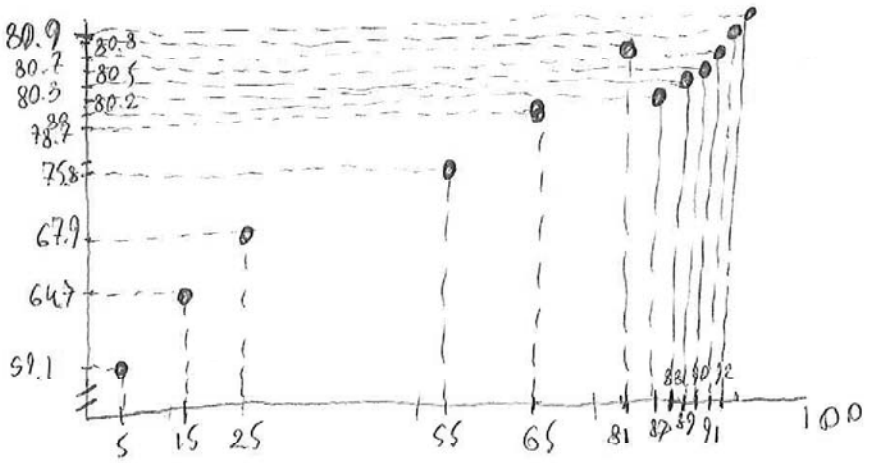


THESE ARE BOTH "EXTRANEOUS" SOLUTIONS. THE LOGARITHMIC EQUATION DOES NOT HAVE SOLUTIONS.

3. The table below gives the life expectancy for people in the United States for selected birth years from 1925 to 2012.

X	Year	Life Span (years)	X	Year	Life Span (years)		Year	Life Span (years)
5	1925	59.1	65	1985	78.7	89	2009	80.3
15	1935	64.7	81	2001	80.7	90	2010	80.5
25	1945	67.9	87	2007	80	91	2011	80.8
55	1975	75.8	88	2008	80.2	92	2012	80.9

(a) Consider x to be the birth year, counting from 1920, and y to be the expected life span, then draw a scatter-plot of the given data.



(b) Using your calculator, find both the quadratic and the logarithmic models which best fit the data. Report your answer to the third decimal place. Report and use the *correlation coefficients* to say which of these models is the best fit for the given data.

QUAD: $Y = -.003X^2 + .496X + 57.144$, $R^2 = .9965$

LOG (OR LN REG): $Y = 44.38 + 8.009 \ln X$, $R^2 = .9792$

QUADRATIC MODEL IS BEST FIT.

(c) Use the unrounded best model to compute examples of interpolation and extrapolation from the given data.

INTERPOLATION: 2005 $\rightarrow X = 85$, $Y \approx 80.4$ YEARS

EXTRAPOLATION: 2013 $\rightarrow X = 93$, $Y \approx 80.6$ YEARS

4. Your new employer offers you two options for an early contract-termination benefit:

   (a) a one time payment of \$60,000, or

   (b) a monthly payment of \$2,500 for two years.

   If you can assume a constant inflation rate of 4.75%, which option is more valuable for you when (and if) your contract is terminated early? Why?

a) THIS IS JUST A LUMP SUM WITH VALUE \$60,000

b) THIS IS AN ANNUITY (OVER TWO YEARS): WE NEED TO COMPUTE ITS PRESENT VALUE. $A = \frac{R}{i}\left(1-(1+i)^{-n}\right)$

"MONTHLY PAYMENTS" → $K=12$; $R = 2500$; r = APR = INFLATION = .0475 ; $t=2$

→ $i = \frac{r}{K} = \frac{.0475}{12}$; $n = K \cdot t = 12 \cdot 2 = 24$

$A = \frac{2500}{(.0475/12)}\left(1-\left(1+\frac{.0475}{12}\right)^{-24}\right) = \$57{,}130.43$.

NOTE TO b) TOTAL PAYMENTS = $R \cdot n$ = $2500(24) = 60{,}000$

THEREFORE OPTION (a) IS MORE VALUABLE

5. You are shopping for a simple investment of \$5000 that you expect to cash in three years. They offer you two options:

   (a) a daily compounded account at 3.75%;

   (b) a continuously compounded account at 4%.

   Compute which option is the best deal for you.

FUTURE VALUES: $t=3$, $P = 5000$

(a) $S = P\left(1+\frac{r}{K}\right)^{Kt}$ FOR: "DAILY → $K = 365$" ; $r = .0375$

$= 5000\left(1+\frac{.0375}{12}\right)^{12\cdot 3} = \$5594.38$

(b) CONT. COMP.: $S = Pe^{rt}$ FOR $r = .04$

$= 5000\, e^{.04(2)} = \$5638.48$

OPTION (a) IS BETTER.