MAT 102 - Exam1 - Fall 2014

80/80

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Name KEY

Instructions. Each problem is worth 10 points. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded to two decimal places. You might need some of the following formulas:

- $d = vt$
- $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- $(A \pm B)^2 = A^2 \pm 2AB + B^2$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$
- $A^2 - B^2 = (A - B)(A + B)$
- $h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$, with $g = 32\frac{ft}{sec^2} \approx 9.8\frac{m}{sec^2}$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Simplify the following radical expression

$$\sqrt[5]{64\,t^8\,s^{32}}$$

$$\sqrt[5]{32\cdot 2}\cdot\sqrt[5]{t^{5+3}}\cdot\sqrt[5]{s^{6\cdot 5+2}} = 2\sqrt[5]{2}\cdot t\sqrt[5]{t^3}\cdot s^6\cdot\sqrt[5]{s^2}$$

$$= 2ts^6\sqrt[5]{2t^3s^2}$$

2. Solve the following quadratic equation.

$$12x^2 - 21 = -19x$$

$$+19x \quad +19x$$

$$12x^2 + 19x - 21 = 0$$

FIRST: STANDARD FORM ✓

SECOND: SPLIT $19x = 28x - 9x$ THEN

$$12x^2 + 28x - 9x - 21 = 0$$

PRODUCT $= 12\cdot(-21) = -252 = -2\cdot 2\cdot 3\cdot 3\cdot 7$

PAIRS OF DIVISOR: $\pm(2, -126), \pm(3, -84), \pm(4, -63), \pm(6, -42), \pm(9, -28)$

SUM $= 19$ ← $28 + (-9)$

$$4x(3x+7) - 3(3x+7) = 0$$

$$(3x+7)(4x-3) = 0$$ (CHECK BY DISTRIBUTIVE)

THIRD: ZERO FACTOR PROPERTY

$3x + 7 = 0 \rightarrow 3x = -7 \rightarrow x = -\frac{7}{3}$

$4x - 3 = 0 \rightarrow 4x = 3 \rightarrow x = \frac{3}{4}$

3. During a show two airplanes cross their trajectories and immediately after they start flying at the same altitude. One travels due north at an average speed of 310 miles per hour, and the other travels due south at an average speed of 320 miles per hour. After how many seconds from their crossing will the two planes be about 7 miles apart?

DISTANCE - RATE - TIME: $d = rt$

STARTING AT THE SAME TIME IT WILL TAKE t "HOURS".

@ 320 mph, S ← ▲ → $d_S = 320t$

@ 310 mph, ▲ → N $d_N = 310t$

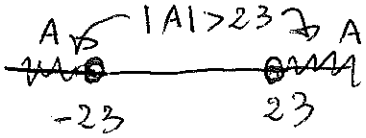
$d_S + d_N = 7 \rightarrow 320t + 310t = 7 \rightarrow 630t = 7 \rightarrow$

$\rightarrow t = \frac{7}{630}$ HOURS $= \frac{7}{630} \cdot 60 \cdot 60$ SECONDS $= 40$ SECONDS

4. Consider the following absolute value inequality.

$$|6x - 13| > 23$$

(a) Describe the solution set using interval notation, and **(b)** graph the solution set.



$|A| > 23$ IS THE COMPOUND INEQUALITY $A < -23$ OR $A > 23$. THEN

$6x - 13 < -23 \mapsto 6x < -10 \rightarrow x < -\frac{10}{6} \rightarrow x < -\frac{5}{3}$

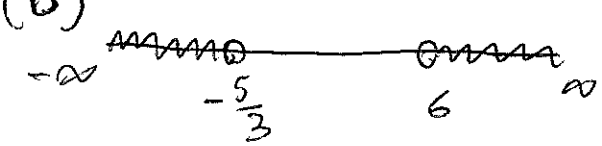
(+13, +13)

OR

$6x - 13 > 23 \mapsto 6x > 36 \rightarrow x > \frac{36}{6} \rightarrow x > 6$

(+13, +13)

(b)



(a) $(-\infty, -\frac{5}{3}) \cup (6, \infty)$

5. On a free kick, during a soccer game, the ball is kicked (from the ground) upward at a speed of 5 meters per second. Considering this ball a free falling object, how long does it take this ball to be 3 meters above the ground?

FREE FALLING OBJECT LAW: $h = -\frac{9.8}{2}t^2 + V_0 t + h_0$ (IMS)

$V_0 = \underbrace{+5}_{\text{UPWARD}}$ AND $h_0 = 0$ (GROUND). ASKED FOR $h = 3$

$\rightarrow -4.9t^2 + 5t = 3 \rightarrow 4.9t^2 - 5t + 3 = 0$ QUADRATIC FORMULA

$$t = \frac{5 \pm \sqrt{5^2 - 4(4.9)(3)}}{2 \cdot (4.9)} = \frac{5 \pm \sqrt{8.4}}{9.8}$$

$+0.806 \approx 0.8$ SECONDS

$-0.214 \approx 0.2$ SECONDS

THE BALL IS 3 METERS ABOVE THE GROUND TWICE, AFTER .2 SECONDS AND AFTER .8 SECONDS.

6. Solve the following rational equation.

$$\frac{x^2+2x}{x+2} + \frac{1}{x+2} = \frac{x+20}{x^2+7x+10}$$

$$\frac{x(x+2)}{x+2} + \frac{1}{x+2} = \frac{x+20}{(x+2)(x+5)}$$

RESTRICTED VALUES FOR

$x+2=0 \rightarrow x=-2$

$x+5=0 \rightarrow x=-5$

LCD $= (x+2)(x+5)$. MULTIPLY BOTH SIDES BY THE LCD:

$$(x^2+2x)(x+5) + (x+5) = x+20 \rightarrow x^3+5x^2+2x^2+10x+x+5 = x+20$$

$$-x-20 \quad -x-20$$

$$\rightarrow x^3 + 7x^2 + 10x - 15 = 0$$

↳ SIGNS ARE NOT UNIFORM (NOT IN PAIRS) THEREFORE FACTORING TECHNIQUES (USING RATIONAL NUMBERS) LIKE FACTORING BY GROUPING CAN'T BE APPLIED. WE CAN'T FIND SOLUTIONS.

7. Mark can move all the boxes out of his storage unit, but it would take him 3 hours more than it takes Chuck moving all the boxes out by himself. Offering him a football thicket, Mark recruits Chuck to help him move all the boxes out of his storage unit and together they spend 2 hours. How long would it take Mark to move all the boxes out of his storage unit alone?

	TIME	WORK RATE
MARK	$x+3$	$\frac{1}{x+3}$
CHUCK	x	$\frac{1}{x}$
TOGETHER	2	$\frac{1}{2}$

$\frac{1}{x+3} + \frac{1}{x} = \frac{1}{2}$

LCD: $(x+3)x \cdot 2$

$\rightarrow 2x + 2(x+3) = x(x+3) \rightarrow 2x + 2x + 6 = x^2 + 3x \rightarrow$

$-4x \quad -6 \qquad -4x - 6$

$\rightarrow x^2 - x - 6 = 0 \rightarrow (x-3)(x+2) = 0$ $\begin{cases} x+2=0 \rightarrow x=-2 \\ x-3=0 \rightarrow x=3 \end{cases}$

$x = -2$ NOT POSSIBLE IN THIS CONTEXT.

$x = 3$ (CHUCK TAKES 3 HOURS) $\rightarrow$ MARK WOULD SPEND $(3+3=)$ 6 HOURS.

8. Solve the following radical equation.

$$\sqrt{3x-2} + 3 = 4x$$

ISOLATE THE ROOT: $\sqrt{3x-2} = 4x-3$

SQUARE BOTH SIDES: $3x-2 = (4x-3)^2 \rightarrow 3x-2 = 16x^2 - 24x + 9 \rightarrow$

$-3x+2 \qquad -3x+2$

$\rightarrow 16x^2 - 27x + 11 = 0$

PRODUCT $= 11 \cdot 16 = 11 \cdot 2^4 \rightarrow$ PAIR $(-11, -16)$

SUM $= -27$

$16x^2 - 16x - 11x + 11 = 0 \rightarrow 16x(x-1) - 11(x-1) = 0 \rightarrow$

$\rightarrow (x-1)(16x-11) = 0$ $\begin{cases} x-1=0 \rightarrow x=1 \\ 16x-11=0 \rightarrow x=11/16 \end{cases}$

CHECK:

$x=1$: $\sqrt{3\cdot 1 - 2} + 3 = \sqrt{1} + 3 = 4(1)$ ✓

$x = \frac{11}{16}$: $\sqrt{3\frac{11}{16} - 2} + 3 = \sqrt{\frac{33-32}{16}} + 3 = \sqrt{\frac{1}{16}} + 3 = \frac{1}{4} + 3 = \frac{13}{4}$

$4\left(\frac{11}{16}\right) = \frac{11}{4}$ NOT EQUAL

$x = \frac{11}{16}$ IS NOT A SOLUTION.