70 + 10 EXTRA

MAT 121 - Exam1 – Spring 2015

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Instructions. Complete 7 out of the following 11 exercises, as indicated. Exercise 12 is for extra points. Each exercise is worth 10 points. You can use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

Complete 3 of the exercises 1-4

1. The frequency of vibrations F of a piano string varies directly as the square root of the tension T on the string and inversely as the length L of the string.

a) Express the mathematical model representing the relation between F, T, and L.

b) The middle A string has a frequency of 440 vibrations per second. Find the frequency of a string that has 1.25 times as much tension and is 1.2 times as long.

$$F = K_1\sqrt{T} \quad \text{AND} \quad F = \frac{K_2}{L} \quad \Rightarrow \quad \boxed{F = K\frac{\sqrt{T}}{L}} \quad \text{(a)}$$

b) GIVEN FREQUENCY: $440 = K\dfrac{\sqrt{T}}{L}$

OTHER STRING HAS LENGTH $= 1.2\,L$ AND TENSION $= 1.25\,T$

THEN:

$$F = K\frac{\sqrt{1.25T}}{1.2L} = K\frac{\sqrt{T}\cdot\sqrt{1.25}}{L\cdot 1.2} = \left(K\frac{\sqrt{T}}{L}\right)\cdot\frac{\sqrt{1.25}}{1.2}$$

$$= 440\cdot\frac{\sqrt{1.25}}{1.2} = \frac{110}{.3}(.5\sqrt{5}) = \frac{550}{3}\sqrt{5} \approx 409.9$$

ABOUT 410 VIBRATIONS PER SECOND.

2. Mark bought a painting at a flea-market. After one year the painting was estimated to be worth 1250$, while 7 years later the painting could be sold for \$2905.50. Assume the appreciation of the painting is linear. Write a linear equation giving the value V of Mark's painting.

X = YEARS AFTER PURCHASE , Y = VALUE IN DOLLARS.

YEAR ONE: $x = 1$, $Y = 1250$; SEVEN YEARS LATER: $X = 1+7 = 8$, $Y = 2905.5$

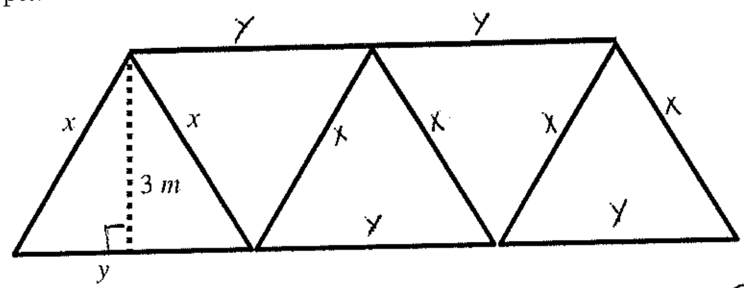
APPRECIATION RATE $= \dfrac{2905.5 - 1250}{8 - 1} = 236.50$ DOLLARS PER YEAR

LINEAR MODEL $Y = mx + b$ WITH $m = 236.5$: $Y = 236.5X + b$

PLUG POINT TO FIND b: $1250 = 236.5(1) + b \Rightarrow b = 1250 - 236.5 = 1013.5$

$$\boxed{Y = 236.5X + 1013.5}$$

3. A farmer has 196 meters of fencing and wants to build five identical pens for his prize-winning goats. The pens will be arranged as shown, each as an isosceles triangle with a fixed high of 3 meters. Determine the dimensions of a pen that will maximize its area.



PRIMARY EQUATION: $A = 5 \cdot \frac{1}{2}(3 \cdot Y)$

$A = \frac{15}{2}Y$

SECONDARY EQUATION, PERIMETER: $196 = 6X + 5Y$] ⇒

PITAGORIAN THEOREM: $X^2 = 3^2 + \left(\frac{Y}{2}\right)^2 \Rightarrow X = \sqrt{\frac{Y^2}{4} + 3^2}$] ⇒

$$\Rightarrow 5Y + 6\sqrt{\frac{Y^2}{4} + 9} = 196 \Rightarrow \left(6\sqrt{\frac{Y^2}{4}+9}\right)^2 = (196 - 5Y)^2 \Rightarrow$$

$$\Rightarrow 36\left(\frac{Y^2}{4} + 9\right) = 38416 - 1960Y + 25Y^2 \Rightarrow$$

$-9y^2 \quad -324 \qquad -324 \qquad -9y^2$

$$\Rightarrow 16Y^2 - 1960Y + 38092 = 0 \Rightarrow Y = \frac{1960 \pm \sqrt{1960^2 - 4(38092)(16)}}{2(16)}$$

$$Y = \frac{1960 \pm \sqrt{1403712}}{32} \approx \begin{cases} 98.3 \xrightarrow{\text{CHECK}} 5(98.3) + 6\sqrt{\frac{98.3^2}{4} + 9} = 786.9 \neq 196 \\ 24.2 \longrightarrow 5(24.2) + 6\sqrt{\frac{24.2^2}{4} + 9} = 195.8 \approx 196 \checkmark \end{cases}$$

ONLY ONE POSSIBLE CHOICE $Y = 24.2$ m AND $X = 12.5$ m GIVES MAX POSSIBLE AREA (NOTE: FIXED THE HIGHT THE STRUCTURE IS RIGID WITH FIXED PERIMETER. SWAPPING X AND 3 WOULD FREE THE STRUCTURE) OVER →

4. The total revenue R earned per day (in dollars) from a pet-sitting service is given by

$$R(p) = -12p^2 + 150p,$$

where p is the price charged per pet (in dollars). Find the price that will yield a maximum revenue. What is the maximum revenue?

R IS QUADRATIC IN p $\Rightarrow$ $R(p)$ PARABOLA DOWNWARD $\Rightarrow$

$\Rightarrow$ MAX AT VERTEX (h, k) WITH $h = -\frac{b}{2a}$, $k = R(h)$.

$h = \frac{-150}{2(-12)} = 6.25 \Rightarrow k = R(6.25) = -12(6.25)^2 + 150(6.25) = 468.75$

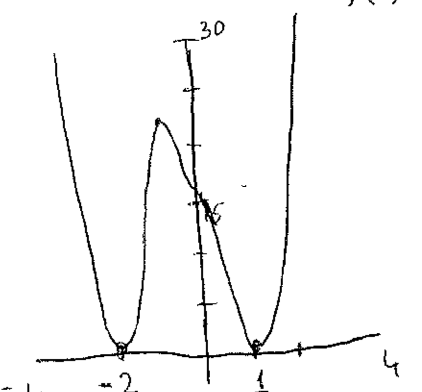
ONE COULD USE THE GRAPH ON A TI: $Y = -12X^2 + 150X$ $\boxed{2^{ND}} + \boxed{TRACE} + \boxed{MAXIMUM}$

A MAXIMUM REVENUE OF \$468.75 IS OBTAINED WITH A PRICE OF \$6.25 PER PET.

Complete 2 of the exercises 5-7

5. Using a graphing utility, graph and approximate the zeros and their multiplicity for the function

$$f(x) = x^6 + 2x^5 + x^4 + 4x^3 - 8x^2 - 16x + 16.$$



REAL ROOTS: $X = -2$, $X = 1$

BOUNCING $\Rightarrow$ MULTIPLICITY IS EVEN
NOTE THAT NONE LOOKS FLATTENED $\Rightarrow$ BOTH HAVE MULTIPLICITY 2

EXTRA: $(X - (-2))^2 \cdot (X-1)^2$ DIVIDES $f(x)$

$(x+2)^2(x-1)^2 = (x^2+4x+4)(x^2-2x+1) = x^4 - 2x^3 + x^2 + 4x^3 - 8x^2 + 4x + 4x^2 - 8x + 4$

$= x^4 + 2x^3 - 3x^2 - 4x + 4$

$x^2 + 4 = (x+2i)(x-2i)$

$$\begin{array}{r} x^2 + 4 \\ \overline{)\,x^6 + 2x^5 + x^4 + 4x^3 - 8x^2 - 16x + 16} \\ -x^6 - 2x^5 + 3x^4 + 4x^3 - 4x^2 \\ \hline 4x^4 + 8x^3 - 12x^2 - 16x + 16 \\ -4x^4 - 8x^3 + 12x^2 + 16x - 16 \\ \hline 0 \end{array}$$

✓

THEN

$f(x) = (x+2)^2(x-1)^2(x+2i)(x-2i)$

6. Using the factors $(x - 2i)$ and $(x + 2i)$, find the remaining factor(s) of

$$f(x) = x^4 + x^3 + 2x^2 + 4x - 8$$

and write the polynomial in fully factored form.

$f(x)$ IS DIVIDED BY $(x-2i)(x+2i) = x^2+4$

$$\begin{array}{r|l} & x^2 + x - 2 \\ \hline x^2+4 & x^4 + x^3 + 2x^2 + 4x - 8 \\ & -x^4 \qquad\quad -4x^2 \\ \hline & \quad x^3 - 2x^2 + 4x - 8 \\ & \quad -x^3 \qquad\quad -4x \\ \hline & \qquad -2x^2 \qquad -8 \\ & \qquad +2x^2 \qquad +8 \\ \hline & \qquad\qquad 0 \ \checkmark \end{array}$$

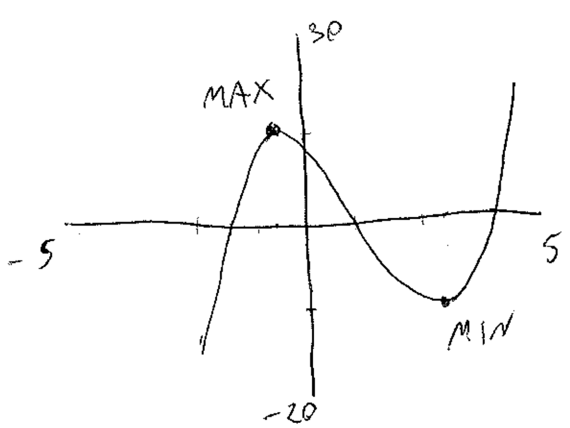
$$f(x) = (x-2i)(x+2i)(x^2+x-2)$$
$$= (x-2i)(x+2i)(x+2)(x-1)$$

7. Use a graphing utility to graph the function and approximate (to two decimal places) any relative minimum or relative maximum values.

$$f(x) = x^3 - 3x^2 - 6x + 8$$

ONE CAN USE THE "FUNCTION EXPLORER" TOOL ON GGB OR

$\boxed{2^{ND}}$ + $\boxed{TRACE}$



MIN: $(2.73, -10.39)$

MAX: $(-.73, 10.39)$

Complete 1 of the exercises 8-9

8. Use long division to simplify the rational expression

$$\frac{x^4 - 2x^2 + 3x - 1}{x^2 - 3x + 4} = \boxed{x^2+3x+3+\frac{-13}{x^2-3x+4}}$$

$$\frac{x^4}{x^2} \qquad \frac{3x^3}{x^2} \qquad \frac{3x^2}{x^2}$$

$$\begin{array}{r|l}
 & x^2 + 3x + 3 \\
\hline
x^2-3x+4 & x^4 - 2x^2 + 3x - 1 \\
 & -x^4 + 3x^3 - 4x^2 \quad \leftarrow -x^2(x^2-3x+4) \\
\cline{2-2}
 & 3x^3 - 6x^2 + 3x - 1 \\
 & -3x^3 + 9x^2 - 12x \quad \leftarrow -3x(x^2-3x+4) \\
\cline{2-2}
 & 3x^2 - 9x - 1 \\
 & -3x^2 + 9x - 12 \quad \leftarrow -3(x^2-3x+4) \\
\cline{2-2}
 & -13
\end{array}$$

9. Use long division to divide $(x^4 + 3x^3 + 2x + 2) \div (x^2 - 2) = x^2+3x+2+\frac{8x+6}{x^2-2}$

$$\frac{x^4}{x^2} \qquad \frac{3x^3}{x^2} \qquad \frac{2x^2}{x^2}$$

$$\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x^2-2 & x^4 + 3x^3 + 2x + 2 \\
 & -x^4 + 2x^2 \quad \leftarrow -x^2(x^2-2) \\
\cline{2-2}
 & 3x^3 + 2x^2 + 2x + 2 \\
 & -3x^3 + 6x \quad \leftarrow -3x(x^2-2) \\
\cline{2-2}
 & 2x^2 + 8x + 2 \\
 & -2x^2 + 4 \quad \leftarrow -2(x^2-2) \\
\cline{2-2}
 & 8x + 6
\end{array}$$

Complete 1 of the exercises 10-11

10. Find the inverse of the function $f(x) = \sqrt[5]{2x} - 3$.

I) SWAP X AND Y : $x = \sqrt[5]{2y} - 3$

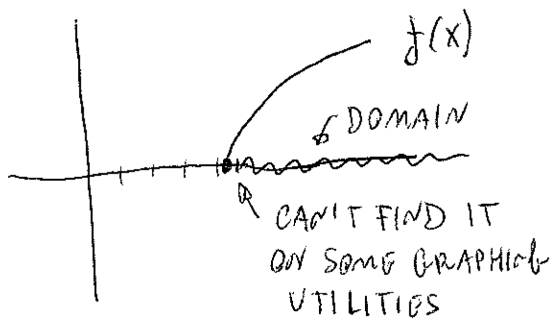
II) SOLVE FOR Y : $\left(\sqrt[5]{2y}\right)^5 = (x+3)^5 \Rightarrow 2y = (x+3)^5 \Rightarrow y = \frac{(x+3)^5}{2}$

III) CHECK $f^{-1}(x) = \frac{(x+3)^5}{2}$:

1) $f(f^{-1}(x)) = \sqrt[5]{\cancel{2}\,\frac{(x+3)^5}{\cancel{2}}} - 3 = \sqrt[\cancel{5}]{(x+3)^{\cancel{5}}} - 3 = (x+3) - 3 = x$ ✓

2) $f^{-1}(f(x)) = \frac{((\sqrt[5]{2x} - 3) + 3)^5}{2} = \frac{(\sqrt[5]{2x})^5}{2} = \frac{2x}{2} = x$ ✓

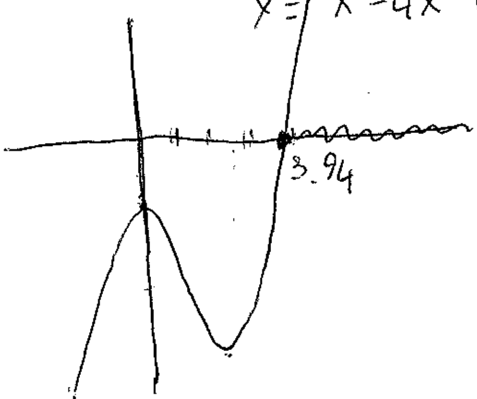
11. Find the interval notation of the domain of the function $f(x) = \sqrt[6]{x^3 - 4x^2 + x - 3}$.



SET UP THE DOMAIN ALGEBRAICALLY:

$x^3 - 4x^2 + x - 3 \geq 0$

$y = x^3 - 4x^2 + x - 3 \geq 0$ ABOVE X-AXIS : $x \geq 3.94$ APPROX.



DOMAIN OF $f(x)$: $[3.94, \infty)$

Extra points

12. Determine the equations of any asymptote of the function

$$f(x) = \frac{x^3 - 3x^2 + 2x - 4}{x + 3}$$

$$f(x) = \underbrace{q(x)}_{\text{NON-V.A.}} + \boxed{\frac{R(x)}{Q(x)}}$$

V.A. AMONG $Q(x) = 0$

x^3/x

$$
\begin{array}{r|l}
 & x^2 - 6x + 20 \\
\hline
x + 3 & x^3 - 3x^2 + 2x - 4 \\
 & -x^3 - 3x^2 \\
\hline
 & -6x^2 + 2x - 4 \\
 & 6x^2 + 18x \\
\hline
 & 20x - 4 \\
 & -20x - 60 \\
\hline
 & -64
\end{array}
$$

$$f(x) = x^2 - 6x + 20 + \frac{-64}{x+3}$$

$Y = x^2 - 6x + 20$ QUADRATIC ASYMPTOTE

$X + 3 = 0 \Rightarrow X = -3$ POSSIBLE VERTICAL ASYMPTOTE

$$\frac{R(-3)}{Q(-3)} = \frac{-64}{0} \neq 0 \Rightarrow X = -3 \text{ IS V.A.}$$