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Math 102 - Spring 2013 - Test 2

KEY

Instructor: Dr. Francesco Strazzullo Name_____

Instructions. Only calculators are allowed on this examination. Each problem is worth 10 points. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

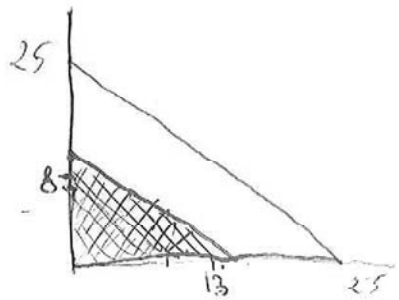
1. You want to order some pizza for your friends. You have a budget of \$95. There are medium pizzas at \$7 each and large ones at \$11.50 each. The delivery can carry at most 25 boxes.

   (a) Set up a system of linear inequality describing the constraints for your order.

   (b) List three possible orders. $X = \#$ MEDIUM, $Y = \#$ LARGE

TOTALS: $X+Y \leq 25$

BUDGET: $7X + 11.5Y \leq 95$



I) $(13,0)$ CHECK: $\begin{cases} 13+0 = 13 \leq 25 \checkmark \\ 7(13)+0 = 91 \leq 95 \checkmark \end{cases}$

II) $(4,3)$ CHECK: $\begin{cases} 4+3 = 7 \leq 25 \checkmark \\ 7(4)+11.5(3) = 62.5 \leq 95 \checkmark \end{cases}$

III) $(0,8)$ CHECK: $\begin{cases} 0+8 = 8 \leq 25 \checkmark \\ 0+11.5(8) = 92 \leq 95 \checkmark \end{cases}$

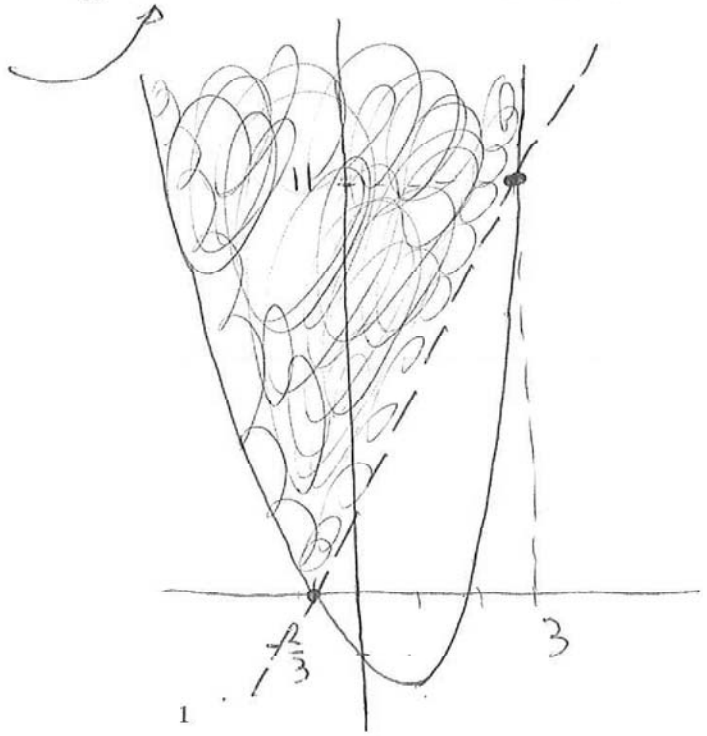
2. Graph the solution set of the following system of inequalities, labeling the corner point and highlighting the contour.

$$\begin{cases} 2y \geq 3x^2 - x - 2 \\ 6x - 2y < -4 \end{cases}$$

$-2y < -6x - 4$

$\frac{-2y}{-2} > \frac{-6x}{-2} - \frac{4}{-2}$

$$\begin{cases} y \geq \frac{3}{2}x^2 - \frac{1}{2}x - 1 & \text{(SOLID B.L.) ABOVE} \\ y > 3x + 2 & \text{(DASHED B.L.) ABOVE} \end{cases}$$



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3. You found a bag of dollar bills and you return it to the police. Later on, an officer tells you that the bag contained three kinds of bills: an x amount of \$1 bills, a y amount of \$20 bills, and a z amount of \$50 bills. The bag contained 225 bills worth \$4790. Moreover, there were twice as many \$20 bills as the sum of \$1 bills and \$50 bills. Write a system of three linear equations in three variables modeling the bag content, then find out how many bills of each type there were.

TOTAL BILLS: $X+Y+Z=225$

TOTAL VALUE: $1X+20Y+50Z=4790$

"TWICE AS MANY \$20 BILLS AS THE SUM OF THE OTHER": $2(X+Z)=Y \rightarrow 2X-Y+2Z=0$

SYSTEM:
$$\begin{cases} X+Y+Z=225 \\ X+20Y+50Z=4790 \\ 2X-Y+2Z=0 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 225 \\ 1 & 20 & 50 & 4790 \\ 2 & -1 & 2 & 0 \end{array}\right] \xrightarrow{\text{RREF}}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 150 \\ 0 & 0 & 1 & 35 \end{array}\right] \rightarrow \begin{cases} X=40 \\ Y=150 \\ Z=35 \end{cases}$$

THERE WERE:

40 \$1 BILLS

150 \$20 BILLS

35 \$50 BILLS

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4. Solve the following systems of linear equations.

(a) $\begin{cases} x - 3y + 2z + w = 1 \\ 2x \quad - z \quad = 4 \\ x + 2y + 5z + w = 7 \\ \quad y - z + 2w = 1 \end{cases}$ MATRIX $\begin{bmatrix} 1 & -3 & 2 & 1 & 1 \\ 2 & 0 & -1 & 0 & 4 \\ 1 & 2 & 5 & 1 & 7 \\ 0 & 1 & -1 & 2 & 1 \end{bmatrix}$ RREF [2nd] + [x^{-1}] [MATH] + [B]

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 77/34 \\ 0 & 1 & 0 & 0 & 15/17 \\ 0 & 0 & 1 & 0 & 9/17 \\ 0 & 0 & 0 & 1 & 11/34 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 2.26 \\ 0 & 1 & 0 & 0 & .88 \\ 0 & 0 & 1 & 0 & .53 \\ 0 & 0 & 0 & 1 & .32 \end{bmatrix}$ SYSTEM → SOLUTION

$\rightarrow \begin{cases} X = 77/34 \\ Y = 15/17 \\ Z = 9/17 \\ W = 11/34 \end{cases}$

(b) $\begin{cases} x \quad - 3z = 6 \\ x + 2y + z = 2 \end{cases}$ MATRIX $\begin{bmatrix} 1 & 0 & -3 & 6 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ RREF $\begin{bmatrix} 1 & 0 & -3 & 6 \\ 0 & 1 & 2 & -2 \end{bmatrix}$

$\rightarrow \begin{cases} X - 3z = 6 \\ Y + 2z = -2 \\ Z = z \end{cases} \rightarrow \begin{cases} X = 3z + 6 \\ Y = -2z - 2 \\ Z = z \end{cases}$

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5. Solve the equation $9x^2 + 15x - 14 = 0$. (Show your work)

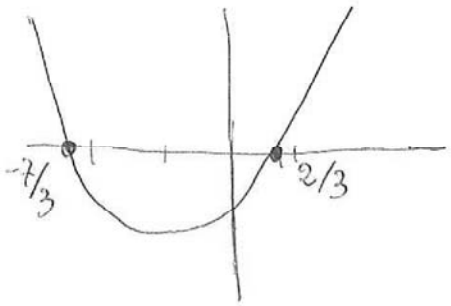
QUADRATIC EQUATION OR FACTOR: $(3X-2)(3X+7)=0 \rightarrow$

$3X-2=0 \rightarrow 3X=2 \rightarrow X=2/3$

$3X+7=0 \rightarrow X=-7/3$

OR $X = \dfrac{-15 \pm \sqrt{15^2-4(9)(-14)}}{2 \cdot 9}$

GRAPH



6. A ball is kicked from the 7th row of a stadium, 90 feet above the ground, at a speed of 60 feet per second. The height h of the ball depends on the time t after the kick, according to the quadratic model $h = -16t^2 + 60t + 90$.

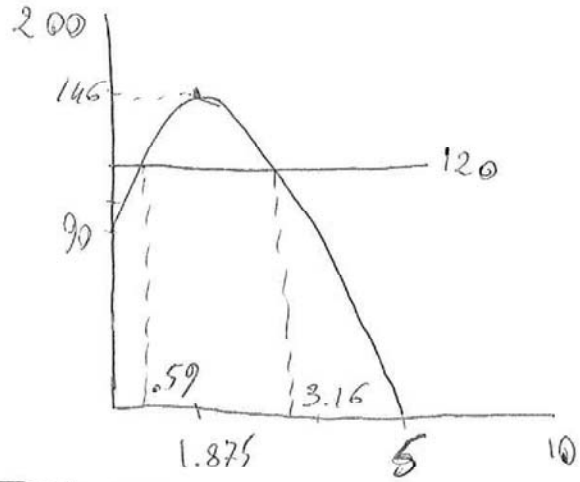
   (a) How long does it take the ball to reach its highest altitude?

   (b) After how many seconds will the ball be 120 feet above the ground?

(a) MAX WITH TI: [2ND] + [TRACE] + [4]

FORMULA VERTEX: $-\dfrac{b}{2a} = \dfrac{-60}{2(-16)} = \dfrac{15}{8} = 1.875$

IT TAKES ABOUT 1.9 SECONDS.



(b) SET UP EQUATION: $h(t) = 120$

$Y_1 = -16t^2 + 60t + 90$

$Y_2 = 120$

→ [2ND] + [TRACE] + [5]

↑ INTERSECT

$t = .59$ SECONDS

$t = 3.16$ SECONDS

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