

Instructor: Dr. Francesco Strazzullo

Name _____

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test: only a textbook can be used, or a cheat-sheet approved by your instructor. Personal notebooks cannot be used. You can also use a graphing calculator. When solving a problem graphically sketch the graph you used.

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the following quadratic equation. If needed, write your answer as a fraction reduced to lowest terms.

$$46x + 7x^2 = 21$$

$$7x^2 + 46x - 21 = 0 \quad -21 \cdot 7 = 7^2 \cdot 3 \quad \begin{matrix} +7^2 = 49 \\ -3 \end{matrix} \quad \begin{array}{r} \text{sum} \\ 46 \end{array}$$

$$7x^2 + 49x - 3x - 21 = 7x(x+7) - 3(x+7) = (7x-3)(x+7)$$

$$\Rightarrow x = \frac{3}{7} \text{ or } -7$$

2. Find the slope-intercept form of the line which passes through the point (10, 8) and is **perpendicular** to

$$\frac{4}{-4} - \frac{5y+4x}{3} = \frac{6}{-4} \quad \Rightarrow \quad 5y+4x = -\frac{2}{3} \quad \Rightarrow \quad y = -\frac{4}{5}x - \frac{2}{15} \quad \Rightarrow \quad \text{slope} = -\frac{4}{5}$$

PERP. $\Rightarrow m = -(-\frac{4}{5})^{-1} = \frac{5}{4} \Rightarrow y = \frac{5}{4}x + b$ PLUG-POINT:

$$8 = \frac{5}{4}(10) + b \Rightarrow b = -\frac{9}{2} \Rightarrow y = \frac{5}{4}x - \frac{9}{2}$$

$\begin{matrix} -25/2 & 42 & -25/2 \end{matrix}$

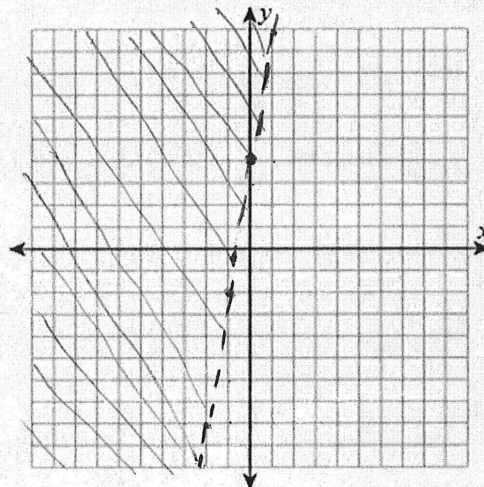
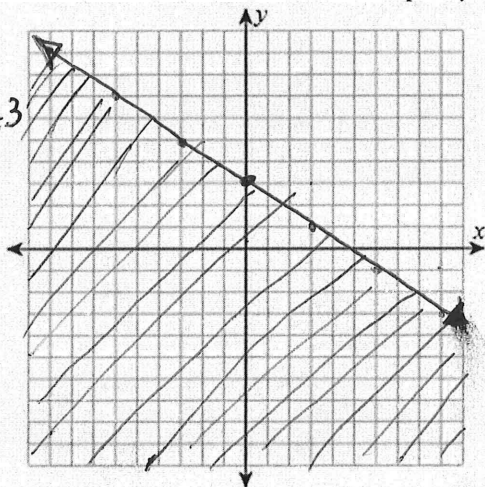
3. Solve the following system of two linear inequalities graphically.

$$3y + 2x \leq 9 \text{ and } y > 6x + 4$$

Graph the solution set of the **first** linear inequality.

Graph the solution set of the **second** linear inequality.

1st
 $y \leq -\frac{2}{3}x + 3$
 \downarrow
 BOLD BL,
 REGION
 BELOW

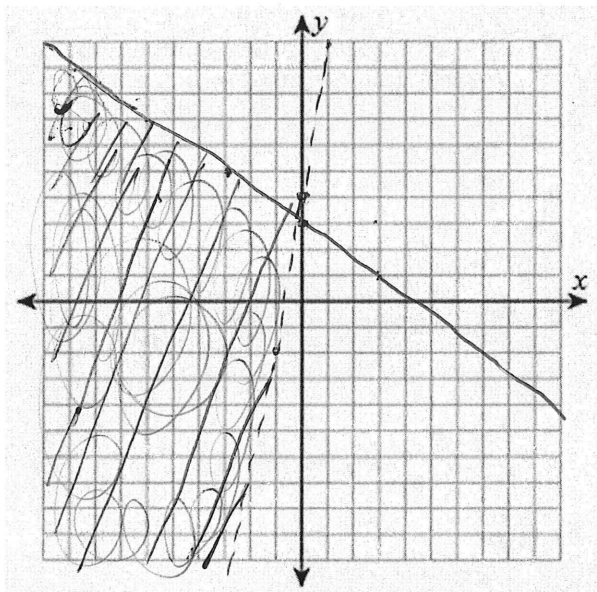


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 DASHED BL,
 REGION
 ABOVE

Graph the overall solution set.

ANS

INTERSECTION
 (OVERLAP)

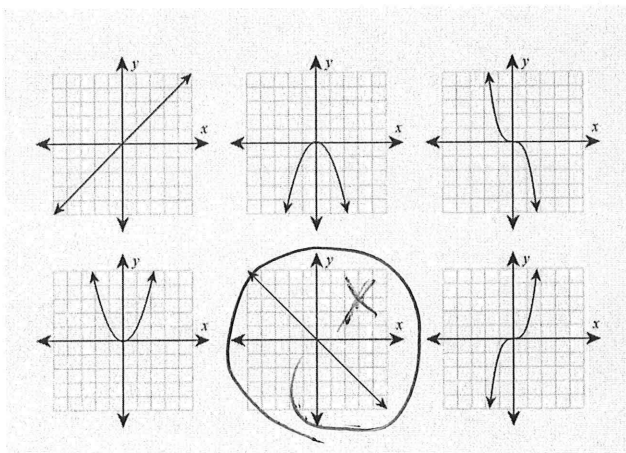


4. Consider the following function.

$$q(x) = \begin{cases} -\frac{2}{3}x & \text{if } x < -2 \\ \frac{5}{8}\sqrt[3]{x} & \text{if } x \geq -2 \end{cases}$$

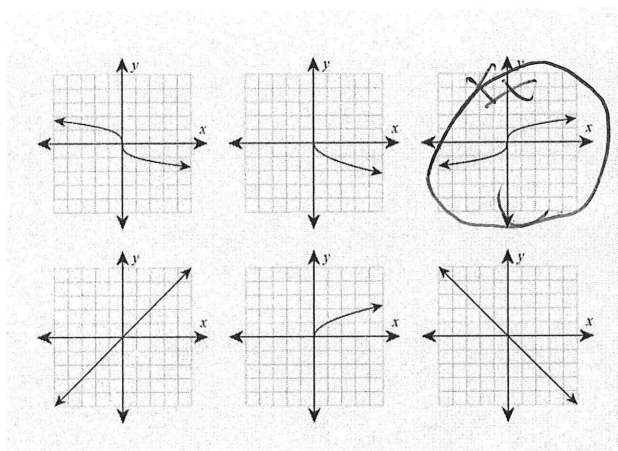
Step 1. Cross-out the general shape and direction of the graph of this function on the interval $(-\infty, -2)$.

LINEAR
NEGATIVE SLOPE
(DECREASING)



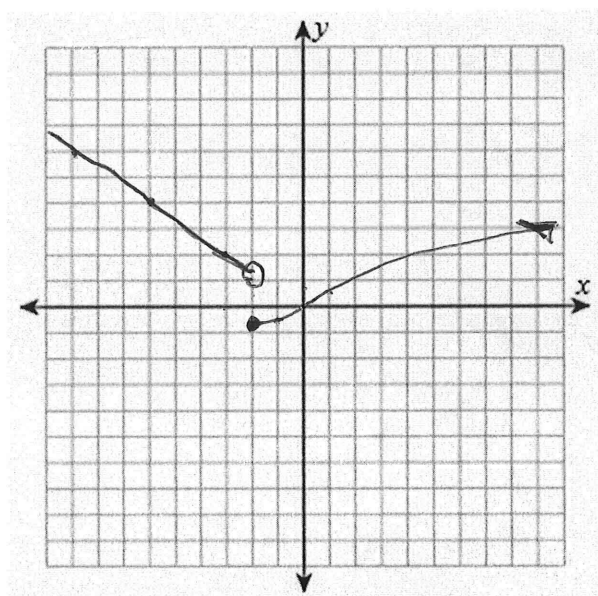
Step 2. Cross-out the general shape and direction of the graph of this function on the interval $[-2, \infty)$.

ODD ROOT,
POSITIVE COEFFICIENT



Step 3. Sketch the graph of $q(x)$, using bullet-points or open circles if needed.

$$\begin{aligned} f(0) &= \frac{5}{8}\sqrt[3]{0} = 0 \\ f(-2) &= \frac{5}{8}\sqrt[3]{-2} \\ &= -\frac{5\sqrt[3]{2}}{8} \approx -0.787 \end{aligned}$$

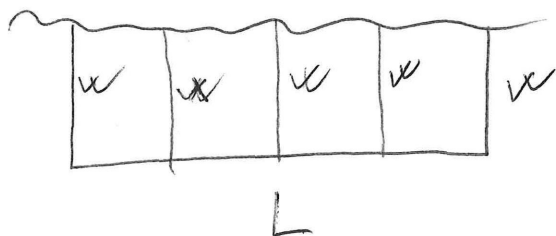


5. Suppose that y varies directly as the square root of x , and inversely as z , and that $y = 25$ when $x = 289$ and $z = 32$. What is y when $x = 81$ and $z = 64$? (Round your answer to the nearest hundredth.)

$$y = k \frac{\sqrt{x}}{z} \quad \text{AND} \quad 25 = k \frac{\sqrt{289}}{32} \Rightarrow k = \frac{25 \cdot 32}{17} \approx 47.0588$$

$$y = \frac{800}{17} \frac{\sqrt{x}}{z} \Rightarrow y = \frac{800}{17} \frac{\sqrt{81}}{64} = \frac{225}{34} \approx 6.62$$

6. The back of Alisha's property is a creek. Alisha would like to enclose 4 identical rectangular areas, using the creek as one common side and fencing for the other sides, to create various pastures. If there is 360 feet of fencing available, what is the maximum possible area of each pasture?



$$L + 5W = 360 \Rightarrow L = 360 - 5W$$

$$\begin{aligned} \text{MAX: } A &= LW = (360 - 5W)W \\ &= 360W - 5W^2 \end{aligned}$$

$$\text{MAX AT VERTEX } (h, k), \quad W = -\frac{b}{2a} = -\frac{360}{2(-5)} = 36 \text{ FT}$$

$$\text{Looking for } K = A(W) = (360 - 5(36))36 = 6480 \text{ FT}^2 \quad \text{OR} \quad \text{SQ FT}$$

$$\text{EACH PASTURE'S AREA} = \frac{K}{4} = 1620 \text{ FT}^2$$

7. The revenue function for a bicycle shop is given by $R(x) = x \cdot p(x)$ dollars where x is the number of units sold and $p(x) = 300 - 0.4x$ is the unit price. Find the maximum revenue.

$$R(x) = x(300 - 0.4x) = 300x - .4x^2 \quad \text{MAX AT}$$

$$\text{VERTEX } (h, k): h = -\frac{b}{2a} = -\frac{300}{2(-.4)} = 375 \text{ UNITS}$$

$$\text{MAX REVENUE } K = R(375) = 56250 \text{ DOLLARS}$$

8. Assuming $h \neq 0$, find the simplest form for the difference quotient of $f(x) = 4x^2 - 3x + 2$

$$\text{DIFF. QDOT.} = \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} f(x+h) &= 4(x+h)^2 - 3(x+h) + 2 = 4(x^2 + 2hx + h^2) - 3x - 3h + 2 \\ &= 4x^2 + 8hx + 4h^2 - 3x - 3h + 2 \end{aligned}$$

$$f(x+h) - f(x) = 8hx + 4h^2 - 3h = h(8x + 4h - 3)$$

$$\text{D.Q.} = \frac{h(8x + 4h - 3)}{h} \stackrel{h \neq 0}{=} 8x + 4h - 3$$