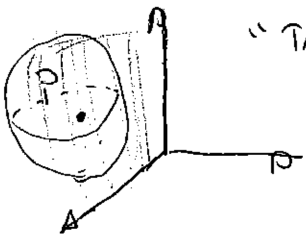
MAT 421 - Exam1 – Fall 2013

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Complete 10 out of the following 20 exercises. Each exercise is worth 10 points.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find an equation of the sphere with center $P\ (6,-2,4)$ that is tangent to the xz-plane.

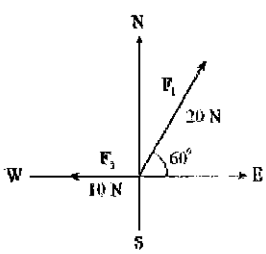


"TANGENT TO XZ-PLANE" $\Rightarrow$ RADIUS PARALLEL TO THE Y-AXIS IS PERPENDICULAR to XZ-PLANE $\Rightarrow R = |P_y| = |-2| = 2$

EQ: $(x-6)^2 + (y-(-2))^2 + (z-4)^2 = 2^2$

$$(x-6)^2 + (y+2)^2 + (z-4)^2 = 4$$

2. For the given forces $\mathbf{F}_1$ and $\mathbf{F}_2$, compute the magnitude of the resulting force and its direction.



$$\vec{F}_1 = 20 < \cos 60°, \sin 60° > = 20 < \frac{1}{2}, \frac{\sqrt{3}}{2} >$$
$$= 10 < 1, \sqrt{3} >$$
$$\vec{F}_2 = -10\,\vec{i} = 10 < -1, 0 >$$

$$\vec{F}_1 + \vec{F}_2 = 10 < 1 + (-1), \sqrt{3} + 0 > = 10 < 0, \sqrt{3} > = 10\sqrt{3}\,\vec{j}$$

$$\| \vec{F}_1 + \vec{F}_2 \| = 10\sqrt{3}$$

DIRECTION of $\vec{F}_1 + \vec{F}_2$: NORTH (VERTICAL, 90°).

3. Determine all values for a such that the vectors $\mathbf{x} = 2\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{y} = \mathbf{i}+2\mathbf{j}+a\mathbf{k}$ will form a 60° angle.

$$\frac{1}{2} = \cos 60^\circ = \frac{\vec{X}\cdot\vec{Y}}{\|\vec{X}\|\cdot\|\vec{Y}\|} = \frac{2+2+2a}{\sqrt{2^2+1^2+2^2}\cdot\sqrt{1+2^2+a^2}} = \frac{2(2+a)}{3\sqrt{5+a^2}} \Rightarrow$$

$$\Rightarrow \sqrt{5+a^2} = \frac{4}{3}(2+a) \Rightarrow 5+a^2 = \frac{16}{9}(2+a)^2 \Rightarrow 45+9a^2 =$$

$$= 64+64a+16a^2 \Rightarrow 7a^2+64a+19=0 \Rightarrow a = \frac{-64\pm\sqrt{64^2-4(7)(19)}}{2(7)} \Rightarrow$$

$$\Rightarrow a = \frac{-32\pm 9\sqrt{11}}{7} \approx -8.84, -.31. \ \boxed{\text{CHECK}}\ \underline{a=-8.84}: \sqrt{5+a^2} \approx 9.12 \text{ AND}$$

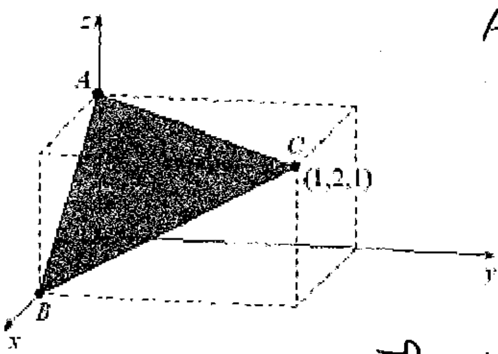
$\frac{4}{3}(2+a) \approx -9.12$ NOT A SOLUTION BUT A -60° ANGLE ; $\boxed{a = \frac{-32+9\sqrt{11}}{7}} \approx -.31: \sqrt{5+a^2} \approx 2.26 \approx \frac{4}{3}(2+a)$ ✓

4. If $\mathbf{b} = \langle 1,0,1\rangle$, $\mathbf{c} = \langle 0,-1,1\rangle$, and $\mathbf{a} = \langle 2,-3,z\rangle$, find a value for z which guarantees that $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are coplanar.

$$\text{COPLANAR} \Rightarrow a\cdot(b\times c)=0 \ ; \ b\times c = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & -1 & 1\end{vmatrix} = \left\langle \begin{vmatrix} 0 & 1 \\ -1 & 1\end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ 0 & 1\end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & -1\end{vmatrix}\right\rangle$$

$$= \langle 1,-1,-1\rangle \Rightarrow \langle 2,-3,z\rangle\cdot\langle 1,-1,-1\rangle = 2+3-z \Rightarrow 5-z=0 \Rightarrow z=5$$

5. Find an equation of the plane passing through the points A, B, and C shown below.



$A(0,0,1)$ AND $B(1,0,0)$.

A DIRECTIONAL VECTOR IS $\vec{BA}\times\vec{BC} = \vec{N}$

$\vec{BA} = A-B = \langle -1,0,1\rangle$ AND $\vec{BC} = C-B = \langle 0,2,1\rangle$

$$\vec{N} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 0 & 2 & 1\end{vmatrix} = \left\langle \begin{vmatrix} 0 & 1 \\ 2 & 1\end{vmatrix}, -\begin{vmatrix} -1 & 1 \\ 0 & 1\end{vmatrix}, \begin{vmatrix} -1 & 0 \\ 0 & 2\end{vmatrix}\right\rangle = \langle -2,1,-2\rangle$$

PLANE: $\langle x,y,z\rangle\cdot\vec{N} = d \Rightarrow$ (PLUG A): $d=-2$

EQ: $-2x+y-2z=-2$ OR $2x-y+2z=2$

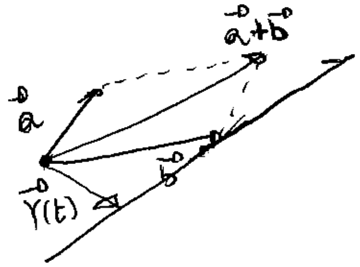
6. Let **a** = 2**i** + **j** – **k** and **b** = **i** + 2**j** + 3**k**. Find an equation of the line parallel to **a** + **b** and passing through the tip of **b**.

"TIP OF $\vec{b}$" $= (1, 2, 3)$.

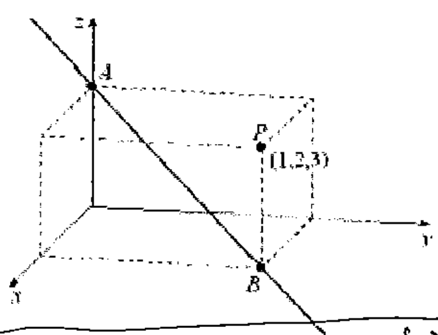
$$\vec{r}(t) = \vec{b} + t(\vec{a} + \vec{b})$$

$$= \langle 1, 2, 3 \rangle + t \langle 3, 3, 2 \rangle$$

$$\vec{r}(t) = \langle 1 + 3t, 2 + 3t, 3 + 2t \rangle$$



7. Find parametric equations and symmetric equations of the line passing through the points A and B shown below.



$A(0, 0, 3)$ AND $B(1, 2, 0)$

$$\vec{r}(t) = \vec{OA} + t\,\vec{AB}$$

$$\vec{r}(t) = \langle 0, 0, 3 \rangle + t \langle 1, 2, -3 \rangle$$

$$\langle x, y, z \rangle = \langle t, 2t, 3 - 3t \rangle$$

$$\boxed{\text{PARAMETRIC EQUATIONS: } \begin{cases} x = t \\ y = 2t \\ z = 3 - 3t \end{cases}}$$

$\left(\text{NOTE: } \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}\right)$

SOLVE ALL EQUATIONS FOR t: $z = 3 - 3t \Rightarrow z - 3 = -3t \Rightarrow t = \frac{3 - z}{3} = \frac{z - 3}{-3}$

$$\boxed{\text{SYMMETRIC EQ: } x = \frac{y}{2} = \frac{z - 3}{-3}}$$

$\left(\text{NOTE: } \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}\right)$

8. Let L be the line given by $x = 2 - t$, $y = 1 + t$, and $z = 1 + 2t$. L intersects the plane $2x + y - z = 1$ at the point $P = (1, 2, 3)$. Find parametric equations for the line through P which lies in the plane and is perpendicular to L.

"DIRECTIONAL VECTOR OF PLANE": $\vec{n} = \langle 2, 1, -1 \rangle$

"DIRECTIONAL VECTOR OF L": $\vec{v} = \langle -1, 1, 2 \rangle$ (FROM COEFFICIENTS OF t IN THE PARAMETRIC EQUATIONS).

"DIRECT. VECT OF PERP. LINE" $= \vec{n} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ -1 & 1 & 2 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} \right\rangle$

$= \langle 2+1, -(4-1), 2+1 \rangle = 3\langle 1, -1, 1 \rangle$. ONE CAN USE $\langle 1, -1, 1 \rangle$

PARAMETRIC EQUATIONS: $\begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3 + t \end{cases}$ OR $\begin{cases} x = 1 + 3t \\ y = 2 - 3t \\ z = 3 + 3t \end{cases}$

9. Let $f(x, y) = \sqrt{4 - x^2 - 2y^2}$

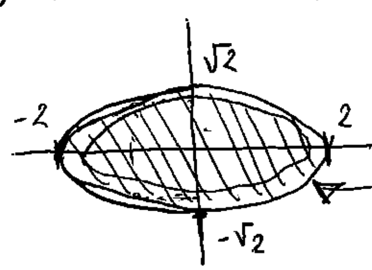
(a) Evaluate $f(-1,-1) = \sqrt{4 - 1 - 2(1)} = 1$

(b) Find the domain of f.

(c) Find the range of f.

(b) $4 - x^2 - 2y^2 \geq 0 \Rightarrow x^2 + 2y^2 \leq 4 \Rightarrow \boxed{\frac{x^2}{4} + \frac{y^2}{2} \leq 1}$

DOMAIN: "INSIDE OF ELLIPSES"

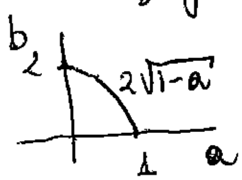


SOLID BOUNDARY LINE (B.L.)

(c) $f(x,y) \geq 0 \Rightarrow$ RANGE $\subseteq [0, +\infty)$

IF $0 \leq a \leq 1$ THEN $\frac{x^2}{4} + \frac{y^2}{2} = a$ IS AN ELLIPSES INSIDE OUR B.L. THEREFORE (x, y) IS IN OUR DOMAIN. $D = \left\{ (x,y) \mid \frac{x^2}{4} + \frac{y^2}{2} = a,\ 0 \leq a \leq 1 \right\}$

THEN $b = f(x,y) = \sqrt{4 - 4\left(\frac{x^2}{4} + \frac{y^2}{2}\right)} = \sqrt{4}\sqrt{1-a} = 2\sqrt{1-a}$ FOR $0 \leq a \leq 1$



WHICH SHOWS $b = f(x,y) \leq 2 \Rightarrow$ RANGE $= [0, 2]$

10. Find rectangular and spherical equations for the surface whose equation in cylindrical coordinates is $\theta = \frac{\pi}{4}$. Describe the surface.

CYLINDRICAL TO RECTANGULAR $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \Rightarrow \begin{cases} x = \frac{\sqrt{2}}{2} r \\ y = \sqrt{2}/2\, r \\ z = z \text{ IS FREE} \end{cases} \Rightarrow \boxed{x = y}$ VERTICAL PLANE THROUGH MAIN DIAGONAL OF XY-PLANE.

SPHERICAL TO RECTANGULAR $\begin{cases} x = \rho\sin\varphi\cos\theta \\ y = \rho\sin\varphi\sin\theta \\ z = \rho\cos\varphi \end{cases}$

CYLINDRICAL TO SPHERICAL $\begin{cases} r = \rho\sin\varphi \\ \theta = \theta \\ z = \rho\cos\varphi \end{cases} \Rightarrow \begin{cases} r = r \text{ IS FREE} \\ \theta = \frac{\pi}{4} \\ z = z \text{ IS FREE} \end{cases} \Rightarrow \boxed{\theta = \frac{\pi}{4}}$

11.Use the given data:

Los Angeles: Latitude $\overbrace{34.05°\text{N}}^{\alpha}$ and Longitude $\overbrace{118.25°\text{W}}^{\beta}$;
Hawaii: Latitude 21.3°N and Longitude 157.83°W.

$\theta = 90 - \alpha$
$\varphi = 360 - \beta$

Find the distance from Los Angeles to Hawaii (Assume the radius of earth is 3960 miles.)

IN SPHERICAL COORDINATES. LA $\equiv P_1(3960, 55.95, 241.75)$
AND HAWAII $\equiv P_2(3960, 68.7, 202.17)$ $\boxed{(\rho, \theta, \varphi)}$

IN RECTANGULAR COORDINATES: $P_i(\rho\sin\varphi_i\cos\theta_i, \rho\sin\varphi_i\sin\theta_i, \rho\cos\varphi_i)$

DISTANCE $= \text{arc}(P_1P_2) = \rho\cdot\alpha$, WHERE $\cos\alpha = \frac{\vec{OP_1}\cdot\vec{OP_2}}{\rho^2} = \langle \sin(241.75)\cdot\cos(55.95), \sin(241.75)\sin(55.95), \cos(241.75)\rangle \cdot \langle \sin(202.17)\cos(68.7), \sin(202.17)\sin(68.7), \cos(202.17)\rangle \approx .7625 \Rightarrow \alpha \approx .7036$ (RADIANS) $\Rightarrow$

α IN RADIANS
$\cos\alpha = \frac{\vec{OP_1}\cdot\vec{OP_2}}{\rho^2}$

$\Rightarrow \text{arc}(P_1P_2) \approx 2786$ MILES
(OR 2548 APPROXIMATING $\cos\alpha \approx .8$)

12. Show that the curve with vector equation $\mathbf{r}(t) = 2\cos^2 t\ \mathbf{i} + \sin(2t)\ \mathbf{j} + 2\sin t\ \mathbf{k}$ is the curve of intersection of the surfaces $\underbrace{(x-1)^2 + y^2 = 1}_{S_1}$ and $\underbrace{x^2 + y^2 + z^2 = 4}_{S_2}$. Use this fact to sketch the curve.

$$r(t) \equiv \begin{cases} x = 2\cos^2 t \\ y = \sin(2t) \\ z = 2\sin t \end{cases}$$

NEED TO PROVE THESE COORDINATE FUNCTIONS SATISFY BOTH S_1 AND S_2

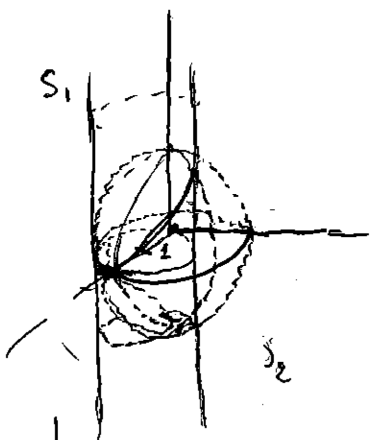
S_1: $(2\cos^2 t - 1)^2 + (\sin(2t))^2 = (\cos(2t))^2 + (\sin(2t))^2 = 1$ ✓

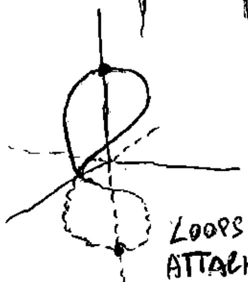
S_2: $(2\cos^2 t)^2 + (\sin(2t))^2 + (2\sin t)^2 = (1 + \cos(2t))^2 + (\sin(2t))^2 +$

$+ 4\sin^2 t = 1 + 2\cos(2t) + \cos^2(2t) + \sin^2(2t)$

$+ 4\sin^2 t = 1 + 2(2\cos^2 t - 1) + 1 + 4\sin^2 t$

$= 2 + 4\cos^2 t - 2 + 4\sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$ ✓





LOOPS ATTACHED TO THE Z-AXIS AT (0,0,2) AND (0,0,−2) AND TO THE X-AXIS AT (2,0,0)

13. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t^2\mathbf{i} + 4t^3\mathbf{j} - t^2\mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$$\vec{r}(t) = \int \vec{r}\,'(t)\,dt + \vec{C} = \left\langle \frac{t^3}{3}, t^4, -\frac{t^3}{3} \right\rangle + \vec{C}$$

$$\langle 1, 1, 2 \rangle = \vec{r}(1) = \left\langle \frac{1}{3}, 1, -\frac{1}{3} \right\rangle + \vec{C} \Rightarrow$$

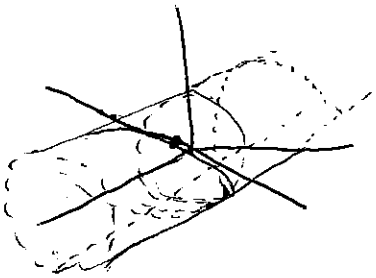
$$\Rightarrow \vec{C} = \left\langle 1 - \frac{1}{3}, 1 - 1, 2 - \left(-\frac{1}{3}\right) \right\rangle = \left\langle \frac{2}{3}, 0, \frac{7}{3} \right\rangle \Rightarrow$$

$$\Rightarrow \vec{r}(t) = \left\langle \frac{t^3}{3} + \frac{2}{3}, t^4, \frac{7}{3} - \frac{t^3}{3} \right\rangle$$

14. Find parametric equations of the tangent line to the curve $\mathbf{r}(t) = \langle t, \sqrt{2}\cos t, \sqrt{2}\sin t\rangle$ at $\left(\frac{\pi}{4}, 1, 1\right)$. Then sketch the curve and its tangent line.

"DIRECTIONAL VECTOR OF TANG. LINE" $= \vec{r}\,'(t) = \langle 1, -\sqrt{2}\sin t, \sqrt{2}\cos t\rangle$

AT GIVEN POINT $t = \frac{\pi}{4}$, THEN $\vec{d} = \vec{r}\,'\left(\frac{\pi}{4}\right) = \langle 1, -1, 1\rangle$



$L:\ \vec{x} = \vec{p} + t\vec{d} = \langle \frac{\pi}{4}, 1, 1\rangle + t\langle 1, -1, 1\rangle$

PARAMETRIC: $\begin{cases} x = \frac{\pi}{4} + t \\ y = 1 - t \\ z = 1 + t \end{cases}$

15. Find the unit tangent and the unit normal to the graph of the vector function $\mathbf{r}(t) = \langle t^2 - 2, 2t - t^3\rangle$ at $t = 1$.

$\vec{T}(t) = \frac{1}{\|\vec{r}\,'(t)\|}\vec{r}\,'(t) = \frac{1}{\sqrt{4t^2+(2-3t^2)^2}}\langle 2t, 2-3t^2\rangle = \frac{1}{\sqrt{4-8t^2+9t^4}}\langle 2t, 2-3t^2\rangle$

ONE CAN USE A CAS

$\vec{N}(t) = \frac{1}{\|\vec{T}\,'\|}\vec{T}\,' = \frac{1}{\|\vec{T}\,'\|}\Big(\left(-\frac{1}{2}\right)(36t^3-16t)(4-8t^2+9t^4)^{-3/2}\langle 2t, 2-3t^2\rangle +$

$+ (4-8t^2+9t^4)^{-\frac{1}{2}}\langle 2, -6t\rangle\Big) = \frac{1}{\|\vec{T}\,'\|}\Big((4-8t^2+9t^4)^{-\frac{3}{2}}\langle (16t-36t^3)t +$

$+ (4-8t^2+9t^4)(2), (8t-18t^3)(2-3t^2) - 6t(4-8t^2+9t^4)\rangle\Big)$

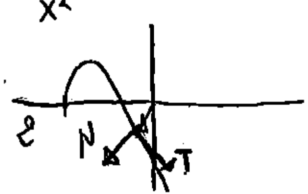
$\vec{T}\,'(1) = (5)^{-3/2}\langle -20+10, (-10)(-1)-30\rangle = 5^{-1/2}(-2)\langle 1, 2\rangle$

$\|\vec{T}\,'(1)\| = \frac{2}{\sqrt{5}}\sqrt{5} = 2 \Rightarrow \vec{N}\,'(1) = \frac{1}{\sqrt{5}}\langle -1, -2\rangle$

$\vec{T}(1) = \frac{1}{\sqrt{5}}\langle 2, -1\rangle$

$y = -tx \Rightarrow t = -\frac{y}{x} \Rightarrow$

$\Rightarrow x = \frac{y^2}{x^2} - 2 \Rightarrow x^3 - y^2 + 2 = 0$



16. At what point does the curve $\mathbf{r}(t) = \left\langle \sqrt{2}t, e^{t}, e^{-t} \right\rangle$ have minimum curvature? What is the minimum curvature?

$\vec{r}\,'(t) = \langle \sqrt{2}, e^{t}, -e^{-t} \rangle$, $\vec{r}\,''(t) = \langle 0, e^{t}, e^{-t} \rangle$

$\|\vec{r}\,'\| = \sqrt{2 + e^{2t} + e^{-2t}}$, $\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} i & j & k \\ \sqrt{2} & e^{t} & -e^{-t} \\ 0 & e^{t} & e^{-t} \end{vmatrix} = \langle 2, -\sqrt{2}e^{-t}, \sqrt{2}e^{t} \rangle$

$\|\vec{r}\,' \times \vec{r}\,''\| = \sqrt{4 + 2e^{-2t} + 2e^{2t}} = \sqrt{2}\sqrt{2 + e^{2t} + e^{-2t}}$

$\kappa = \dfrac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^{3}} = \dfrac{\sqrt{2}}{2 + e^{2t} + e^{-2t}} \Rightarrow \kappa' = \dfrac{-\sqrt{2}\left(2e^{2t} - 2e^{-2t}\right)}{\left(2 + e^{2t} + e^{-2t}\right)^{2}}$

$\kappa' = 0 \Rightarrow e^{2t} - e^{-2t} = 0 \Rightarrow e^{4t} - 1 = 0 \Rightarrow e^{4t} = 1 \Rightarrow 4t = \ln 1 = 0 \Rightarrow t = 0.$

1ST DERIVATIVE TEST:

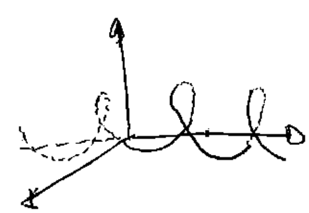
	-1	0	1
κ'	+		−
κ	↗		↘

MAX CURVATURE FOR $t = 0$, AT $r(0) = \langle 0, 1, 1 \rangle$ ONE HAS $\kappa(0) = \dfrac{\sqrt{2}}{4}$

17. Find the center of the osculating circle of the curve described by $x = 4\sin t$, $y = 3t$, $z = 4\cos t$ at $(0,0,4)$.

$\vec{r}(t) = \langle 4\sin t, 3t, 4\cos t \rangle$ FOR $t = 0$

ELIX ON A RIGHT CIRCULAR CYLINDER WITH AXIS THE Y-AXIS AND RADIUS 4



$\vec{r}\,'(t) = \langle 4\cos t, 3, -4\sin t \rangle$

$\|\vec{r}\,'(t)\| = \sqrt{(4\cos t)^{2} + (-4\sin t)^{2} + 3^{2}} = 5 \Rightarrow \vec{T}(t) = \frac{1}{5}\langle 4\cos t, 3, -4\sin t \rangle$

$\vec{T}\,'(t) = \frac{1}{5}\langle -4\sin t, 0, -4\cos t \rangle$, $\|\vec{T}\,'(t)\| = \frac{1}{5}\sqrt{(-4\sin t)^{2} + (4\cos t)^{2}} = \frac{4}{5}$

$\Rightarrow \vec{N}(t) = \frac{1}{4}\langle -4\sin t, 0, -4\cos t \rangle = \langle -\sin t, 0, -\cos t \rangle.$

$\vec{N}(0) = \langle 0, 0, -1 \rangle = \frac{1}{R}\overrightarrow{PC}(0)$, WHERE $P(0) = (0,0,4)$ AND $C(0)$ IS THE CENTER OF THE OSCULATING CIRCLE, WHOSE RADIUS IS $R = \frac{1}{\kappa(0)}$.

$\kappa(0) = \dfrac{\|\vec{T}\,'(0)\|}{\|\vec{r}\,'(0)\|} = \dfrac{4}{25} \Rightarrow C - P = \langle 0, 0, -\frac{25}{4} \rangle \Rightarrow C = \left(0, 0, -\frac{9}{4}\right)$
$= (0, 0, -2.25)$

18. Let **a** (t), **v** (t), and **r** (t) denote the acceleration, velocity, and position at time t of an object moving in the xy-plane. Find **r** (t), given that $\mathbf{a}(t) = \langle e^{2t} + 2t, e^{2t} - 3 \rangle$, $\mathbf{v}(0) = \left\langle \frac{3}{2}, \frac{7}{2} \right\rangle$ and $\mathbf{r}(0) = \left\langle \frac{5}{4}, \frac{9}{4} \right\rangle$.

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle \frac{1}{2}e^{2t} + t^2, \frac{1}{2}e^{2t} - 3t \rangle + \vec{c}$; PLUG $t = 0$: $\langle \frac{3}{2}, \frac{7}{2} \rangle =$

$= \vec{V}(0) = \langle \frac{1}{2}, \frac{1}{2} \rangle + \vec{c} \Rightarrow \vec{c} = \langle 1, 3 \rangle \Rightarrow \vec{V}(t) = \langle \frac{1}{2}e^{2t} + t^2 + 1, \frac{1}{2}e^{2t} - 3t$

$+3\rangle \Rightarrow \vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{1}{4}e^{2t} + \frac{t^3}{3} + t, \frac{1}{4}e^{2t} - \frac{3}{2}t^2 + 3t \rangle + \vec{c} \Rightarrow$

$\Rightarrow \langle \frac{5}{4}, \frac{9}{4} \rangle = \vec{r}(0) = \langle \frac{1}{4}, \frac{1}{4} \rangle + \vec{c} \Rightarrow \vec{c} = \langle 1, 2 \rangle \Rightarrow$

$\Rightarrow \vec{r}(t) = \langle \frac{1}{4}e^{2t} + \frac{1}{3}t^3 + t + 1, \frac{1}{4}e^{2t} - \frac{3}{2}t^2 + 3t + 2 \rangle$

19. Find a parametric representation for the surface consisting of that part of the cylinder $x^2 + z^2 = 1$ that lies between the planes $y = -1$ and $y = 3$.

Y IS "FREE" ALONG THE CYLINDER SO IT IS A PARAMETER, WHILE ALONG EACH CIRCLE WE CAN USE THE TRIGONOMETRIC UNIT CIRCLE EQUATIONS.

$$S \equiv \begin{cases} x = \cos\theta \\ Y = Y \\ z = \sin\theta \end{cases}, \quad -1 \leq Y \leq 3, \quad 0 \leq \theta \leq 2\pi$$

20. Find the direction vector of the line which is the intersection of the planes $x + y + z = 1$ and $x + y - z = 2$.

SUCH LINE WILL BE PERPENDICULAR TO BOTH DIRECTIONAL VECTORS:

$$\vec{u} = \vec{n}_1 \times \vec{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 1, -1 \rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \langle -2, 2, 0 \rangle$$