# Math 102-040 - Fall 2009 - Test 1

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**Instructions.** Only calculators are allowed on this examination. Point values of each problem are **10.**
**Always use the appropriate wording and units of measure in your answers (when applicable).**
**SHOW YOUR WORK NEATLY, PLEASE** (no work, no credit).

1. A company charting its profits notices that the relationship between the number of units sold, $x$, and the profit, $P$, is linear. If 150 units sold results in \$2750 and 375 units sold results in \$5000 profit, write the equation that models its profit.

LINEAR MODEL: $P = mx + b$ OR $P - P_1 = m(X - X_1)$, WHERE WE HAVE TWO POINTS $(X_1, P_1) = (150, 2750)$ AND $(X_2, P_2) = (375, 5000)$.

THE SLOPE IS $m = \dfrac{P_2 - P_1}{X_2 - X_1} = \dfrac{5000 - 2750}{375 - 150} = 10$.

I) WE CAN WRITE $P = 10x + b$; PLUG ONE POINT: $P_1 = 10X_1 + b$, THAT IS $2750 = 10 \cdot 150 + b$ AND SOLVE FOR $b$: $b = 2750 - 1500 = 1250$. THEREFORE $P = 10x + 1250$  5

OR:

II) $P - 2750 = 10 \cdot (X - 150)$ OR $P - 5000 = 10(X - 375)$

2. The total U.S. population during 1945 to 1985, for selected years, is shown in the table below, with the population given in thousands.

| Year | 1945 | 1950 | 1960 | 1965 | 1970 | 1985 |
|---|---|---|---|---|---|---|
| Population | 145,531 | 120,171 | 184,952 | 205,473 | 217,226 | 239,748 |

(a) Find the average annual rate of change in population during 1945–1985, with the appropriate units.

"AVERAGE RATE OF CHANGE" = "SLOPE" = $\dfrac{239{,}748 - 145{,}531}{1985 - 1945}$ ← THOUSAND PEOPLE / ← YEARS

$= 2355.425$ THOUSAND PEOPLE/YEAR

DURING 1945-1985 THE POPULATION IS INCREASING AT AN AVERAGE RATE OF CHANGE OF ABOUT 2,355,425 PEOPLE PER YEAR.

(b) Use the slope from part (a) and the population from 1945 to write the equation of the line associated with 1945 and 1985.

POINT SLOPE FORM: $P - P_1 = m(X - X_1)$, WHERE $P$ IS POPULATION IN THOUSAND PEOPLE AND $X$ IS THE NUMBER OF YEARS AFTER 1945

$P - 145{,}531 = 2355.425(X - 0)$

$P = 2355.425X + 145{,}531$

3. The total U.S. population can be modeled for the year 1960–1995 by the function $p = 2670x + 192,750$, where $p$ is in thousand of people and $x$ is in years from 1960. During what year does the model estimate the population to be 286, 200, 000 people?

POPULATION OF 286,200,000 PEOPLE MEANS $p = 286,200$

IT IS ASKED TO SOLVE THE EQUATION:

$$\begin{array}{l} 286,200 = 2670x + 192,750 \\ -192,750 \qquad\qquad -192,750 \end{array} \mapsto \frac{2670x}{2670} = \frac{93,450}{2670} \mapsto$$

$$\to x = \frac{93,450}{2670} = 35 \mapsto \text{YEAR} = 1960 + 35 = 1995.$$

THIS MODEL ESTIMATES THE POPULATION TO BE OF 286,200,000 PEOPLE IN 1995.

4. The total U.S. cigarette advertising and promotional expenditures can be modeled by the equation $y = 401.245x - 2349.85$, $y$ is measured in millions of dollars and $x$ is the number of years from 1977. If this model remains accurate, what are the expected advertising and promotional expenditures in 2017?

IT IS ASKED TO COMPUTE THE VALUE OF $Y$ IN 2017, WHICH IS FOR $X = 2017 - 1977 = 40$.

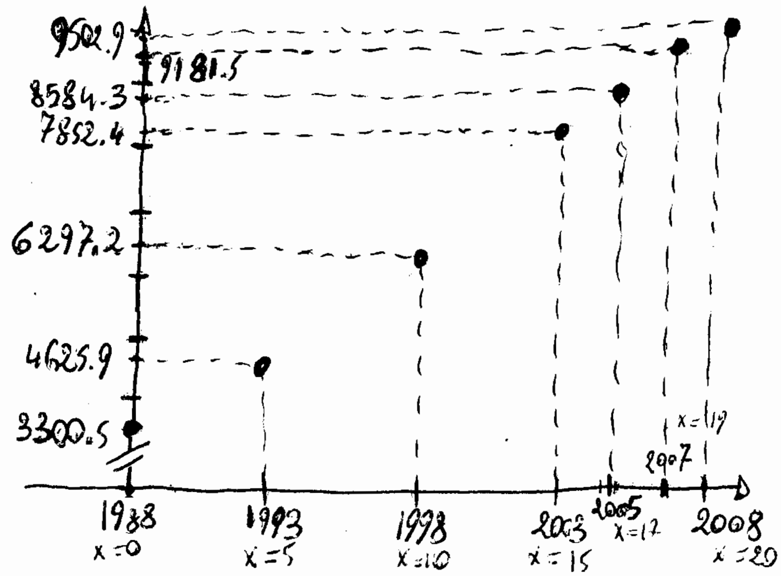
$$Y = 401.245 \cdot 40 - 2349.85 = 13699.950$$

THE EXPECTED ADVERTISING AND PROMOTIONAL EXPEDITURES IN 2017 ARE OF 13699.950 MILLIONS OF DOLLARS.

5. The sum of the personal consumption expenditures in the United States, in billions of dollars, for selected years from 1988 to 2008 is show in the following table.

| Year | 1988 | 1993 | 1998 | 2003 | 2005 | 2007 | 2008 |
|---|---|---|---|---|---|---|---|
| Personal Consumption ($ billions) | 3300.5 | 4625.9 | 6297.2 | 7852.4 | 8584.3 | 9181.5 | 9502.9 |

(a) Make a scatter plot of the data, with $x$ equal to the number of years past 1988 and $y$ equal to the billions of dollars spent. (Clearly report the coordinates of the points)




(b) Using your calculator, find the linear model which is the best fit for the data.

WE CAN SET $X = 0$ FOR 1988, SO THAT $X = 17$ IN 2005; WITH THIS CHOICE WE HAVE THE LINEAR REGRESSION

$$Y = 314.7443844\,X + 3182.3832279$$

(c) Use the unrounded model to estimate the U.S. personal consumption for 2009.

ACCORDING TO THE MODEL IN PART (b), WE PLUG $X = 2009 - 1988 = 21$ TO OBTAIN

$Y = 9792.0154$, THAT IS, THE ESTIMATED US PERSONAL CONSUMPTION FOR 2010 IS OF ABOUT 9.8 TRILLION DOLLARS (9,792,015 MILLION DOLLARS).

6. Suppose the percent of males who enrolled in college within 1 year of high school graduation is given by $y = -.12x + 64.2$ and the percent of females enrolled in college within 1 year of high school graduation is given by $y = .91x + 23.85$, where $x$ is the number of years after 1978. Use graphical methods to find the year these models indicate that the percent of females equaled the percent of males.

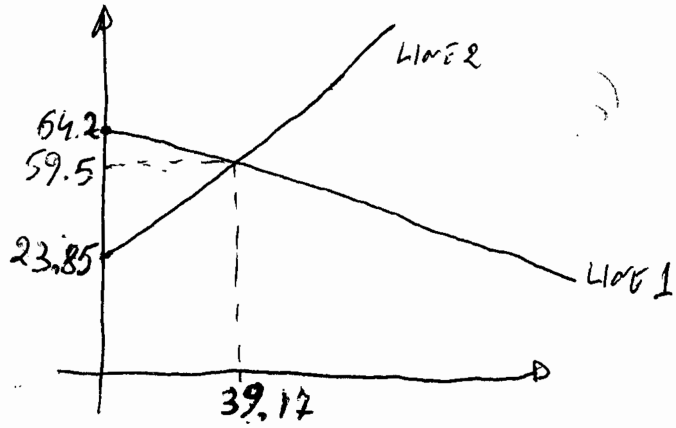
WE PLOT THE TWO LINES:

$\boxed{Y_1}$ LINE 1: $Y = -.12X + 64.2$

$\boxed{Y_2}$ LINE 2: $Y = .91X + 23.85$

IT IS ASKED TO FIND THE POINT OF INTERSECTION.

USE $\boxed{2ND} + \boxed{TRACE} + \boxed{5}$.




THE TWO PERCENTAGES ARE EQUAL TO ABOUT 59.5% WHEN $X = 39.17$, THAT IS AT THE BEGINNING OF $(1978 + 39 =)$ 2017.

7. A pharmacist wants to mix two solutions to obtain 350 *cc* of a solution that has 14% concentration of a certain medicine. If one solution has a 21% concentration of the medicine and the second has a 7% concentration, how much of each solution should he mix?

X IS CC OF SOLUTION WITH 21% CONCENTRATION

Y IS CC OF " " 7% " . THEN WE MUST SOLVE:

$$\begin{cases} \text{EQ I} & X + Y = 350 \\ \text{EQ II} & .21X + .07Y = .14 \cdot 350 \end{cases}$$

BY SUBSTITUTION, FROM EQ I: $Y = 350 - X$. WE PLUG THIS

IN EQ II: $.21X + .07 \cdot (350 - X) = 49 \rightarrow$

$\rightarrow .21X + .07 \cdot 350 - .07X = 49$

$\quad - 24.5 \qquad\qquad -24.5$

$\rightarrow \dfrac{.14X}{.14} = \dfrac{24.5}{.14} \rightarrow X = 175$

(CHECK) WE CAN HAVE ONE OF THE SOLUTIONS TO BE 175 CC.

PLUG IN EQ I: $Y = 350 - 175 = 175$.

THE PHARMACIST HAS TO MIX 175 CC OF THE SOLUTION AT 21% AND 175 CC OF THE SOLUTION AT 7% IN ORDER TO OBTAIN A 350 CC SOLUTION AT 14%

8. The median annual earnings for blacks ($B$) as a function of the median annual earnings for whites ($W$), both in thousand dollars, can be modeled by $B = .4325W + .3781$ using one set of data, and by $B = .72W - 14.113$ using more recent data. Find what median annual earnings by whites will result in both models giving the same median annual earnings for blacks.

IT IS ASKED FOR THE VALUE OF $W$ FOR WHICH THE TWO MODELS PRODUCE THE SAME VALUE OF B, THAT IS, THE SOLUTION OF THE SYSTEM:

$$\begin{array}{l} \text{EQ I} \\ \text{EQ II} \end{array} \begin{cases} B = .4325W + .3781 \\ B = .72W - 14.113 \end{cases}$$

NOTE: IT IS ASKED FOR THE VALUE OF W ONLY.

ELIMINATION

EQ II - EQ I: $0 = .72W - .4325W - 14.113 - .3781 \rightarrow .2875W - 14.4911 = 0$

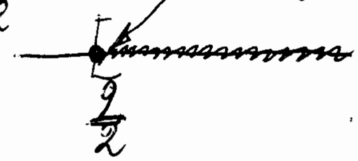
$\rightarrow .2875W = 14.4911 \rightarrow W = \dfrac{14.4911}{.2875} = 50.404$

A MEDIAN ANNUAL EARNING OF \$50,404 FOR WHITES WILL RESULT IN BOTH MODELS GIVING THE SAME M.A.E. FOR BLACKS.

9. Give the solution in interval notation for the inequality $2x + 3 \le 4x - 6$.

$$\begin{array}{l} 2x+3 \le 4x-6 \\ -4x-3 \quad -4x-3 \end{array} \rightarrow \frac{-2x}{-2} \ge \frac{-9}{-2} \rightarrow x \ge \frac{9}{2}$$

WE CAN HAVE $x = 9/2$

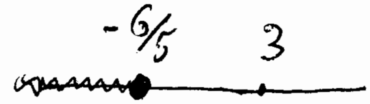


THE SOLUTION IS THE INTERVAL $\left[\frac{9}{2}, +\infty\right)$

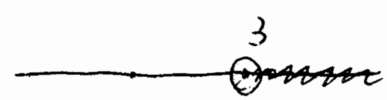
10. Solve the double inequality $5x + 2 \le -4$ and $3x - 4 > 5$.

SINCE WE HAVE "AND", WE MUST CONSIDER THE OVERLAPPING OF THE SOLUTION OF EACH INEQUALITY

I) $\begin{array}{l} \quad -2 \quad\; -2 \\ 5x+2 \le -4 \end{array} \rightarrow 5x \le -6 \rightarrow x \le -\dfrac{6}{5}$



II) $\begin{array}{l} 3x-4 > 5 \\ \quad +4 \quad +4 \end{array} \rightarrow \dfrac{3x}{3} > \dfrac{9}{3} \rightarrow x > 3$



THEY DO NOT OVERLAP!

4 THERE IS NOT SOLUTION TO THIS DOUBLE INEQUALITY.