Math 321- Spring 2015 - Exam1

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Instructions. Technology and notes (including the formula sheets from our book) are allowed on this exam. Each problem is worth 10 points. **If you use notes or formula sheets, make a reference. When using technology describe which commands (or keys typed) you used or print out your worksheet.**

1. Let $f(x)$ be a twice differentiable function on the interval $[-2,4]$ such that $f(-2)=3$, $f'(-2)=-2$, $f(4)=3$, and $f'(4)=-1$. Determine the value of $\int_{-2}^{4} x f''(x)\,dx$.

$$\int x f''(x)\,dx \underset{\uparrow}{=} x f'(x) - \int f'(x)\,dx \Rightarrow$$

$$x = u \to dx = du \;;\; f''(x)dx = dv \Rightarrow v = \int dv = \int f''(x)\,dx = f'(x) + c$$

$$\Rightarrow \int_{-2}^{4} x f''(x)\,dx = \Big[x f'(x)\Big]_{-2}^{4} - \int_{-2}^{4} f'(x)\,dx = \Big[x f'(x) - f(x)\Big]_{-2}^{4} =$$

$$= 4f'(4) - f(4) - \big(-2f'(-2) - f(-2)\big) = 4(-1) - \cancel{3} + 2(-2) + \cancel{3}$$

$$= -8$$

2. During one year the average daily temperature in Waleska varies with a rate given by the model

$$r(t) = -\frac{8}{73}\pi \sin\left(\frac{2\pi}{365}t - 236\right)$$

Celsius degrees per day, with $t=1$ corresponding to January the 1^{st}. If February the 1^{st} and June the 1^{st} are respectively day 32 and day 153, what is the temperature variation between these two days? (*You can use technology for the computation, but first show the setup*)

$$r(t) = T'(t) \Rightarrow \Delta T = T(t)\Big]_{32}^{153} = \int_{32}^{153} r(t)\,dt =$$

$$= -\frac{8\pi}{73}\int_{32}^{153} \sin\left(\frac{2\pi}{365}t - 236\right)dt =$$

$$= -\frac{8\pi}{73}\cdot\left(-\frac{365}{2\pi}\right)\left[\cos\left(\frac{2\pi}{365}t - 236\right)\right]_{32}^{153}$$

$$= 20\left(\cos\left(\frac{306}{365}\pi - 236\right) - \cos\left(\frac{64}{365}\pi - 236\right)\right) \approx 32\,^{\circ}C$$

3. Show the substitution method to find the indefinite integral $\int xe^{3x^2+1}\,dx$.

$u = 3x^2+1 \Rightarrow du = u'\,dx = 6x\,dx \Rightarrow dx = \frac{1}{6x}\,du$, THEN

$\int x e^{3x^2+1}\,dx = \int \cancel{x} e^{u} \cdot \frac{1}{6\cancel{x}}\,du = \frac{1}{6}\int e^{u}\,du = \frac{1}{6}e^{u} + C$

$= \frac{1}{6} e^{3x^2+1} + C$

4. Show the integration by parts method to find the indefinite integral $\int x\cos x\,dx$.

$x = u \rightarrow dx = du$

$dv = \cos x\,dx \rightarrow v = \int dv = \int \cos x\,dx = \sin x$ $\Big] \Rightarrow$

$\Rightarrow \int x\cos x\,dx = uv - \int v\,du = x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

5. Use Simpson's Rule with $n = 4$ to approximate the integral $\int_2^8 \sin^2\left(\frac{x}{2\pi}\right) dx$.

$$S_4 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$\Delta x = \frac{b-a}{n} = \frac{8-2}{4} = \frac{6}{4} = \frac{3}{2} \Rightarrow x_0 = 2,\ x_1 = 3.5,\ x_2 = 5,\ x_3 = 6.5,\ x_4 = 8$$

THEN:

$$\int_2^8 \sin^2\left(\frac{x}{2\pi}\right) dx \approx S_4 = \frac{1}{2}\left[\sin^2\left(\frac{1}{\pi}\right) + 4\sin^2\left(\frac{3.5}{2\pi}\right) + 2\sin^2\left(\frac{5}{2\pi}\right)\right.$$

$$\left. + 4\sin^2\left(\frac{6.5}{2\pi}\right) + \sin^2\left(\frac{4}{\pi}\right)\right] \approx \frac{1}{2}\left(.098 + 4(.28) + 2(.51) + 4(.739)\right.$$

$$\left. + .914\right) \approx \boxed{3.053}$$

6. Compute the most general antiderivative $F(x)$ of $f(x) = x\ln(x)$, then the particular antiderivative satisfying the initial condition $F(1) = 2$

$$F(x) = \int x \ln x \, dx = uv - \int v\,du$$

$$u = \ln x \Rightarrow du = \frac{1}{x}dx$$

$$dv = x\,dx \Rightarrow v = \int dv = \int x\,dx = \frac{x^2}{2}$$

$$\Rightarrow F(x) = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}dx =$$

GENERAL

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C = \boxed{\frac{x^2}{4}\left(\ln x^2 - 1\right) + C}$$

PLUG DATA: $2 = F(1) = \frac{1}{4}(\ln 1 - 1) + C \Rightarrow 2 = -\frac{1}{4} + C$

$\Rightarrow C = 2 + \frac{1}{4} = \frac{9}{4}$. THEN

PARTICULAR: $F(x) = \frac{x^2}{4}\left(\ln x^2 - 1\right) + \frac{9}{4}$

7. A particle moves with velocity function

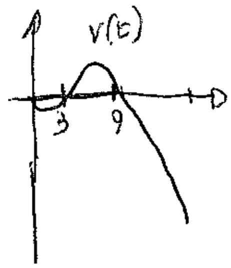
$$v(t) = -0.04x^3 + 0.48x^2 - 1.08x$$

feet per second. Compute both the *displacement* and the *total distance covered* in 20 seconds.

DISPLACEMENT $= \int_0^{20} v(t)\,dt = \left[-\frac{.04}{4}x^4 + \frac{.48}{3}x^3 - \frac{1.08}{2}x^2\right]_0^{20} = -536$

FEET, WHICH MEANS THE PARTICLE AFTER 20 SECONDS IS 536 FEET BEFORE THE INITIAL POSITION.

TOTAL DISTANCE $= \int_0^{20} |v(t)|\,dt = \int_0^3 -v(t)\,dt + \int_3^9 v(t)\,dt + \int_9^{20} -v(t)\,dt$



$= \frac{13832}{25} \approx 553.28$ FEET

8. Use the partial fractions decomposition method to compute

$$\int \frac{x+1}{(2x+1)(x-2)}\,dx.$$

$$\frac{x+1}{(2x+1)(x-2)} = \frac{A}{2x+1} + \frac{B}{x-2} = \frac{A(x-2)+B(2x+1)}{(2x+1)(x-2)} \Rightarrow$$

$\Rightarrow (x-2)A + (2x+1)B = x+1$ $\begin{cases} x=2 & 5B = 3 \Rightarrow B = 3/5 \\ x=-\frac{1}{2} & -\frac{5}{2}A = \frac{1}{2} \Rightarrow A = -\frac{1}{5} \end{cases}$. THEN

$$\int \frac{x+1}{(2x+1)(x-2)}\,dx = \int \frac{-\frac{1}{5}}{2x+1}\,dx + \int \frac{3/5}{x-2}\,dx =$$

USE $\int \frac{u'}{u}\,dx = \ln|u|$

$\hookrightarrow u' = 2$

$$= -\frac{1}{5}\cdot\frac{1}{2}\ln|2x+1| + \frac{3}{5}\ln|x-2| + C$$

$$= \ln\sqrt[10]{\frac{(x-2)^6}{|2x+1|}} + C$$

9. Compute the improper integral $\int_2^\infty xe^{-x^2+4}\,dx$.

$$\int_2^\infty x e^{-x^2+4} = \lim_{t\to\infty} \int_2^t x e^{-x^2+4}\,dx =$$

$$u = -x^2+4 \Rightarrow dx = \frac{1}{u'}\,du = -\frac{1}{2x}\,du$$

$$\hookrightarrow \begin{cases} x=2 \to u=0 \\ x=t \to u=-t^2+4 \end{cases}$$

$$= \lim_{t\to\infty} \int_0^{-t^2+4} \cancel{x} e^u \left(-\frac{1}{2\cancel{x}}\,du\right) = -\frac{1}{2}\lim_{t\to\infty} \left[e^u\right]_0^{-t^2+4}$$

$$= -\frac{1}{2}\lim_{t\to\infty}\left[e^{-t^2+4} - 1\right] = \frac{1}{2} - \frac{1}{2}\lim_{t\to\infty} e^{-t^2+4}$$

$$\hookrightarrow \left[-t^2+4 \xrightarrow[t\to\infty]{} -\infty\right]$$

$$= \frac{1}{2} - \frac{1}{2}\lim_{y\to-\infty} e^y \to 0 \quad = \frac{1}{2}$$