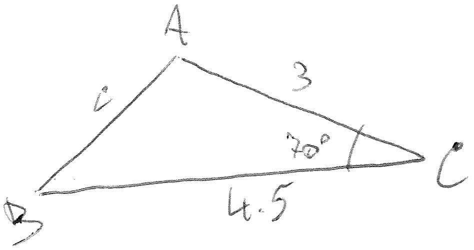
MAT 121 – Exam4 – Spring 2017

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use your own cheat sheet after I approve it, or the one on Eagleweb, or our book. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve for the remaining angles and side of the one triangle that can be created. Round to the nearest hundredth:

$$C = 70°, a = 4.5, b = 3$$



LAW COSINES:

$c^2 = a^2 + b^2 - 2ab\cos C$

$= (4.5)^2 + 3^2 - 2(4.5)(3)\cos(70°)$

$c = \sqrt{c^2} \approx 4.4739 \to c \approx 4.47$

LAW SINES: $\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin A = \frac{a}{c}\sin C \approx 0.946 \Rightarrow$

$\Rightarrow A = \operatorname{asin}(0.946)$ OR $A = 180° - \operatorname{asin}(0.946)$

$\Rightarrow A \approx 71.08°$ $\quad A \approx 108.92°$

$B = 180° - A - C \Rightarrow \begin{cases} A \approx 71.08° \to B = 38.92° \checkmark \\ A \approx 108.92° \to B = 1.08° \checkmark \end{cases}$

CHECK: $\frac{\sin C}{c} = \frac{\sin(70°)}{4.47} \approx .21$

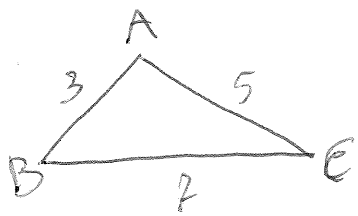
I) $\frac{\sin(38.92°)}{3} \approx .21 \checkmark$

II) $\frac{\sin(1.08°)}{3} \approx .01$ NOT POSSIBLE.

THEN: $A = 71.08°$, $B = 38.92°$, $c = 4.47$.

2. Solve for the remaining angles and side of the one triangle that can be created. Round to the nearest hundredth:

$$a = 7, b = 5, c = 3$$



LAW COSINES

$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow$

$\Rightarrow \cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{49 + 25 - 9}{70} = \dfrac{13}{40}$

$C = a\cos\left(\frac{13}{40}\right) \approx 0.38 \text{ RAD} \approx 21.79°$

LAW OF SINES:

$\dfrac{\sin A}{a} = \dfrac{\sin C}{c} \approx 0.1237$

$\sin A \approx .8659 \Rightarrow A = a\sin(.8659)$ OR $A = 180° - a\sin(.8659) \Rightarrow$

$\Rightarrow A = 59.99°$ OR $A = 120.01°$

$B = 180° - A - C = 98.22°$ OR $B = 38.2°$

CHECK:

I) $\dfrac{\sin(98.22°)}{5} \approx .2 \neq .12$

II) $\dfrac{\sin(38.2°)}{5} \approx .1237$ ✓

THEN:

$A = 120.01°$, $B = 38.2°$, $C = 21.79°$

3. Nick wants to build an herb garden in the backyard but is only using the area beyond the path running through the yard. How large would the garden be if it is triangular-shaped with sides of length 12 feet, 8 feet, and 6 feet? Round to the nearest hundredth.

HERON'S FORMULA: $s = \frac{a+b+c}{2}$, $A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{12+8+6}{2} = 13$

$A = \sqrt{13(1)(5)(7)} = \sqrt{455} \approx 21.33$ FT^2

4. Consider the following parametric equations:

$$x = 3 - \cos(\theta) \text{ and } y = \cos(2\theta)$$

(a) Eliminate the parameter θ, by writing your answer in simplest form solved for y.

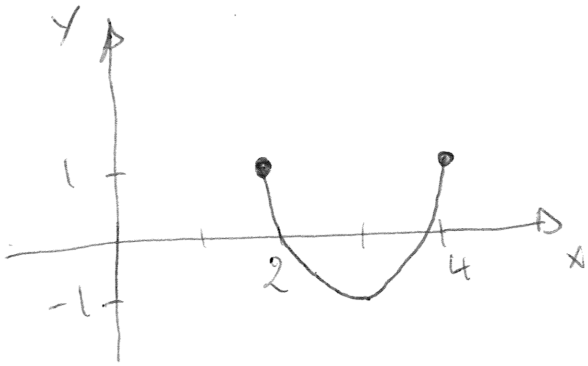
(b) Determine the domain and range of the equation obtained by eliminating the parameter. Please write your answer in interval notation.

(a) $\cos\theta = 3 - x$

$Y = \cos(2\theta) = 2\cos^2\theta - 1 = 2(3-x)^2 - 1 = 2(x^2 - 6x + 9) - 1$

$\Rightarrow Y = 2x^2 - 12x + 17$

(b) GRAPH PARAMETRIC EQUATIONS (CURVE): $-\infty < \theta < \infty$



DOMAIN $= [2, 4]$

RANGE $= [-1, 1]$

5. Use any convenient method to solve the following system of equations. Indicate the number of solutions to this system. State the solution, if one exists, as an ordered triple, and if there are infinitely many solutions, express the solution set in terms of one of the variables. Leave all fractional answers in fraction form.

$$\begin{cases} 2x - y + 4z = 1 \\ x - 3y + z = 2 \\ 3x - 4y + 5z = 3 \end{cases}$$

AUGMENTED MATRIX $= \left[\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ 1 & -3 & 1 & 2 \\ 3 & -4 & 5 & 3 \end{array}\right] \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 11/5 & 1/5 \\ 0 & 1 & 2/5 & -3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$

$\xrightarrow[\text{SYSTEM}]{\text{DEPENDENT}} \begin{cases} x + \frac{11}{5}z = \frac{1}{5} \\ y + \frac{2}{5}z = -\frac{3}{5} \\ z = z \end{cases} \Rightarrow \begin{cases} x = -\frac{11}{5}z + \frac{1}{5} \\ y = -\frac{2}{5}z - \frac{3}{5} \\ z = t \end{cases}$

SOLUTION SET $= \left\{\left(-\frac{11}{5}z + \frac{1}{5},\ -\frac{2}{5}z - \frac{3}{5},\ z\right) \text{ FOR } z \text{ IN } \mathbb{R}\right\}$

APPROXIMATIONS NOT ACCEPTED: $\begin{bmatrix} 2.2 & .2 \\ .4 & -.6 \end{bmatrix}$

6. Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{cases} 2w - x = 2 \\ w - y + z = -2 \\ -3w - 3x - z = 3 \\ x + 6y = 1 \end{cases}$$

Indicate the number of solutions to this system. State the solution, if one exists, as an ordered quadruple, and if there are infinitely many solutions, express the solution set in terms of one of the variables.

AUGMENTED MATRIX $= \begin{bmatrix} 2 & -1 & 0 & 0 & 2 \\ 1 & 0 & -1 & 1 & -2 \\ -3 & -3 & 0 & -1 & 3 \\ 0 & 1 & 6 & 0 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 0 & 27/46 \\ 0 & 1 & 0 & 0 & -19/23 \\ 0 & 0 & 1 & 0 & 7/23 \\ 0 & 0 & 0 & 1 & -105/46 \end{bmatrix} \rightarrow$

$\xrightarrow[\text{ONE SOLUTION}]{\text{SYS}} \begin{cases} w = 27/46 \\ x = -19/23 \\ y = 7/23 \\ z = -105/46 \end{cases}$

SOL. SET $= \left\{\left(\frac{27}{46}, -\frac{19}{23}, \frac{7}{23}, -\frac{105}{46}\right)\right\}$

APPROX. NOT ACCEPTED $(.59, -.83, .30, -2.28)$

7. The following are the winning times for the chicken's 50-ft race in Waleska, for selected years:

X	Year	Time (sec)	X	Year	Time (sec)
30	1930	24.2	80	1980	20.6
58	1958	20.9	89	1989	20.3
65	1965	20.7	96	1996	20.24
69	1969	20.5	100	2000	19.98
74	1974	20.8	112	2012	19.12

Consider x to be the number of years after 1900, and y to be the winning time. Use technology to answer to the following questions.

(a) What are the quadratic and the exponential models that are the best fit for these data? (Round your answer to five decimal places).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate the winning times in 1999 and 2013. Round to the hundredth of second.

(a) QUADRATIC: $Y = ax^2 + bx + c$: $Y = .00065X^2 - .14367X + 27.62334$

$R^2 = .90912$

EXPONENTIAL: $Y = be^{a}$: $Y = 24.76612\, e^{-.00232X}$

$R^2 = .82129$

(b) QUADRATIC BETTER FIT, WITH LARGER R^2

(c) $1999 \to X = 99 \to Y = 19.78$ SEC.

$2013 \to X = 113 \to Y = 19.7$ SEC.