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Name KEY

**Instructions.** Each problem is worth 10 points. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded to two decimal places. You might need some of the following formulas:

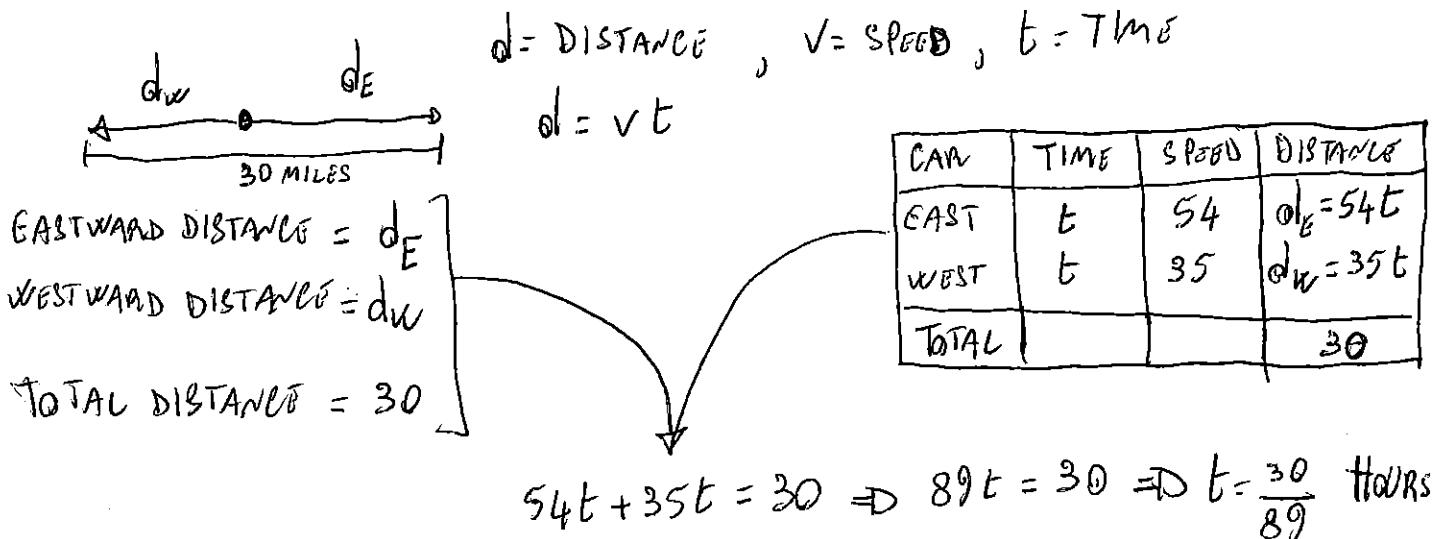
- $h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$ , with  $g = 32 \frac{ft}{sec^2} \approx 9.8 \frac{m}{sec^2}$
- $d = vt$  and  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$  and  $A^2 - B^2 = (A - B)(A + B)$

**SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Simplify the following radical expression.

$$\begin{aligned} & \sqrt[3]{125x^7y^{23}} \\ &= \sqrt[3]{5^3} \cdot \sqrt[3]{x^{2 \cdot 3 + 1}} \cdot \sqrt[3]{y^{7 \cdot 3 + 1}} = 5 \sqrt[3]{(x^2)^3} \sqrt[3]{x} \cdot \sqrt[3]{(y^7)^3} \sqrt[3]{y^2} \\ &= 5x^2y^7 \sqrt[3]{xy^2} \end{aligned}$$

2. Two cars leave a house at the same time. One travels due west at an average speed of 35 miles per hour, and the other travels due east at an average speed of 54 miles per hour. After how many minutes will the two cars be about 30 miles apart?



To get minutes multiply by 60 :

$$t = \frac{30}{89} (60) \approx 20 \text{ minutes}$$

3. Consider the following compound inequality.

$$-4 \leq 6 - 2x < 21$$

(a) Describe the solution set using interval notation, and (b) graph the solution set.

NOTE:  $-4 \leq 6 - 2x < 21$  corresponds to " $-4 \leq 6 - 2x$  AND  $6 - 2x < 21$ ".

$$\begin{array}{ccccccc} -4 \leq 6 - 2x < 21 & | & -10 \leq -2x < 15 & | & 5 \geq x > -7.5 & \text{OR} & -7.5 < x \leq 5 \\ -6 & -6 & -6 & -2 & -2 & -2 & \end{array}$$

(a)  $(-7.5, 5]$

(b)



4. Solve the following quadratic equation.

$$6x^2 - 11x - 7 = 0$$

$$\begin{array}{l} 6x^2 - 11x - 7 = 0 \\ \text{PRODUCT } 6(-7) = -42 \\ \text{SUM } -11 \\ \Rightarrow 6x^2 - 14x + 3x - 7 = 0 \end{array}$$

$$6x^2 - 14x + 3x - 7 = 0$$

$$2x(3x - 7) + 1(3x - 7) = 0$$

$$(3x - 7)(2x + 1) = 0$$

$$3x - 7 = 0 \rightarrow 3x = 7 \rightarrow x = \frac{7}{3}$$

$$2x + 1 = 0 \rightarrow 2x = -1 \rightarrow x = -\frac{1}{2}$$

5. A ball is kicked upward from a 2-foot height at a speed of 20 feet per second. How long does it take the ball to be 7 feet above the ground?

$$h = -\frac{1}{2}gt^2 + v_0 t + h_0 \quad ; \quad g = 32 \text{ ft/sec}^2; \quad h_0 = 2; \quad \text{"UPWARD" } v_0 = 20; \quad h = 7$$

$$7 = -16t^2 + 20t + 2 \rightarrow -16t^2 + 20t - 5 = 0 \rightarrow 16t^2 - 20t + 5 = 0$$

$$t = \frac{20 \pm \sqrt{20^2 - 4(16)(5)}}{2(16)} = \frac{20 \pm \sqrt{80}}{32} = \frac{20 \pm 4\sqrt{5}}{32} = \frac{5 \pm \sqrt{5}}{8} \quad \begin{cases} \frac{5-\sqrt{5}}{8} \approx .35 \\ \frac{5+\sqrt{5}}{8} \approx .9 \end{cases}$$

AFTER ABOUT .35 SEC OR .9 SEC.

6. Solve the following rational equation.

$$\frac{x^2 + 6x - 16}{x^2 - 6x + 8} - \frac{x+1}{x+4} = \frac{7x^2 + x - 20}{x^2 - 16}$$

SIMPLIFY:

$$\frac{(x+8)(x-2)}{(x-4)(x-2)} - \frac{x+1}{x+4} = \frac{7x^2 + x - 20}{(x-4)(x+4)}$$

DOMAIN:  $x \neq 4, -4, 2$

LCM:  $(x-4)(x+4)$

MULTIPLY BY LCM:

$$(x+4)(x+8) - (x-4)(x+1) = 7x^2 + x - 20$$

$$x^2 + 12x + 32 - (x^2 - 3x - 4) = 7x^2 + x - 20$$

$$x^2 + 12x + 32 - x^2 + 3x + 4 = 7x^2 + x - 20 \rightarrow \frac{7x^2 - 14x - 56}{7} = 0$$

$$x^2 - 2x - 8 = 0 \rightarrow (x-4)(x+2) = 0 \quad \begin{cases} x=4 & \leftarrow \text{CAN'T BE USED} \\ x=-2 \end{cases}$$

$$\text{CHECK: } \frac{(-2)^2 + 6(-2) - 16}{(-2)^2 - 2(-2) + 8} - \frac{-2+1}{-2+4} \stackrel{?}{=} \frac{7(-2)^2 + (-2) - 20}{(-2)^2 - 16}$$

$$\frac{4 - 12 - 16}{4 + 4 + 8} - \frac{-1}{2} \stackrel{?}{=} \frac{28 - 22}{4 - 16} \rightarrow \frac{-24}{16} + \frac{1}{2} \stackrel{?}{=} -\frac{6}{12} \rightarrow \frac{1}{2} - \frac{3}{2} \stackrel{?}{=} -\frac{1}{2} \quad \checkmark$$

7. If Lucia were to mow her backyard alone, it would take her twice the amount of time it takes her husband to mow the same area. Lucia and her husband mow their backyard together in 3 hours. How long would it take Lucia to mow her backyard alone?

$$\begin{aligned} \text{HUSBAND WORK TIME} &= X \rightarrow \text{"HUSBAND WORK RATE"} = \frac{1}{X} \\ \text{LUCIA WORK TIME} &= 2X \rightarrow \text{"LUCIA WORK RATE"} = \frac{1}{2X} \\ \text{TOGETHER WORK TIME} &= 3 \rightarrow \text{"TEAM RATE"} = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \rightarrow \frac{1}{X} + \frac{1}{2X} &= \frac{1}{3} & \text{LCD} = 3 \cdot 2X = 6X & \rightarrow 6X \left( \frac{1}{X} + \frac{1}{2X} \right) = 6X \cdot \frac{1}{3} \\ \rightarrow 6 + 3 &= 2X \Rightarrow 2X = 9 \quad (\text{THIS IS ALREADY LUCIA'S WORK TIME}) \\ \rightarrow X &= \frac{9}{2} = 4.5 \text{ HOURS (HUSBAND'S WORK TIME)} \\ &\boxed{9 \text{ HOURS}} \end{aligned}$$

8. Solve the following radical equation.

$$3\sqrt{2x+5} + 8 = x$$

$$\begin{aligned} \text{ISOLATE RADICAL: } \frac{3\sqrt{2x+5}}{3} &= \frac{x-8}{3} \rightarrow \left(\sqrt{2x+5}\right)^2 = \left(\frac{x-8}{3}\right)^2 \\ \rightarrow 2x+5 &= \frac{(x-8)^2}{9} \rightarrow 9(2x+5) = x^2 - 16x + 64 \rightarrow \\ \rightarrow x^2 - 34x + 19 &= 0 \rightarrow x = \frac{34 \pm \sqrt{34^2 - 4(19)}}{2} = \frac{34 \pm \sqrt{1080}}{2} = \\ &= \frac{34 \pm \sqrt{4 \cdot 9 \cdot 30}}{2} = \frac{34 \pm 6\sqrt{30}}{2} = \boxed{17 \pm 3\sqrt{30}} \quad \begin{array}{l} \approx 5.68 \\ \text{TO CHECK} \end{array} \quad \begin{array}{l} \approx 33.432 \\ \end{array} \end{aligned}$$

CHECK:

- )  $3\sqrt{2(5.68)+5} + 8 \approx 15.431 \neq 5.68 \quad \text{NOT SOLUTION}$

- )  $3\sqrt{2(33.432)+5} + 8 \approx 33.4318 \approx 33.432 \checkmark$

ONLY ONE SOLUTION:  $x = 17 + 3\sqrt{30}$