Math 102 - Spring 2012 - Test 2

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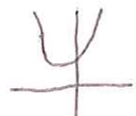
Instructions. Only calculators are allowed on this examination. Each problem is worth 10 points. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Solve the equation $x^2 + 4x + 8 = 0$.

BY GRAPH WE SEE THAT



THERE ARE NONE X-INTERCEPTS, THEREFORE WE HAVE COMPLEX SOLUTIONS:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)} = \frac{-4 \pm \sqrt{-16}}{2}$$

$$= \frac{-4 \pm \sqrt{16}\cdot\sqrt{-1}}{2} = \frac{-4 \pm 4i}{2} = \frac{2(-2 \pm 2i)}{2} \to$$

$$\to X = -2 \pm 2i \; \begin{cases} -2-2i \\ -2+2i \end{cases}$$

2. A bank loans \$375,000 to a development company to purchase three business properties. If one of the properties costs \$55,000 more than the other and the third costs twice the sum of these two properties. Write a system of three linear equations in three variables modeling the costs of these properties, then find out each cost.

VARIABLES: X, Y, Z, ARE THE VALUE OF EACH PROPERTY IN THOUSAND DOLLARS.

"ONE PROPERTY COSTS \$55000 MORE THAN THE OTHER" $\to X = Y + 55$

"THE THIRD COSTS TWICE THE SUM OF THESE TWO..." $\to Z = 2(X+Y) \to Z = 2X + 2Y$

TOTAL INVESTED \$375,000 $\to X + Y + Z = 375$

SYSTEM IN STANDARD FORM:

$$\begin{cases} X - Y = 55 \\ 2X + 2Y - Z = 0 \\ X + Y + Z = 375 \end{cases} \xrightarrow[\text{MATRIX}]{\text{AUGMENTED}} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 55 \\ 2 & 2 & -1 & 0 \\ 1 & 1 & 1 & 375 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 90 \\ 0 & 1 & 0 & 35 \\ 0 & 0 & 1 & 250 \end{array}\right]$$

SOLUTION $\to \begin{cases} X = 90 \text{ THOUSAND DOLLARS} \\ Y = 35 \text{ K\$} \\ Z = 250 \text{ K\$} \end{cases}$

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3. Solve the following systems of linear equations.

(a) $\begin{cases} 3x + 2y - z = 4 \\ 2x - 3y + z = 3 \end{cases}$

USE THE MATRIX AGAIN OR AS FOLLOWS:

ELIMINATION: $5X - Y = 7 \rightarrow Y = 5X - 7$

$+Y-7 \quad +Y-7$

$-4 + Z \quad +Z - 4$

EQ1 PLUG $Y = 5X - 7 \rightarrow 3X + 2(5X-7) - Z = 4 \rightarrow Z = 3X + 10X - 14 - 4$

$\rightarrow Z = 13X - 18$

SOLUTION: $\begin{cases} X \text{ IS FREE } (X = X) \\ Y = 5X - 7 \\ Z = 13X - 18 \end{cases}$

USING MATRIX ONE GETS $\begin{cases} X = \frac{1}{13}Z + \frac{18}{13} \\ Y = \frac{5}{13}Z - \frac{1}{13} \\ Z = Z \end{cases}$

(b) $\begin{cases} x - 2y + z - 3w = 10 \\ 2x - 3y + 4z + w = 12 \\ 2x - 3y + z - 4w = 7 \\ x - y + z + w = 4 \end{cases}$

$\xrightarrow{\text{MATRIX}} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 2 & -3 & 4 & 1 & 12 \\ 2 & -3 & 1 & -4 & 7 \\ 1 & -1 & 1 & 1 & 4 \end{array}\right] \xrightarrow{\text{RREF}}$

$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -57 \\ 0 & 1 & 0 & 0 & -70 \\ 0 & 0 & 1 & 0 & -25 \\ 0 & 0 & 0 & 1 & 16 \end{array}\right] \xrightarrow{\text{SOLUTION}} \begin{cases} X = -57 \\ Y = -70 \\ Z = -25 \\ W = 16 \end{cases}$

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4. The profit for a product can be described by the function $P(x) = 40x - 3000 - 0.01x^2$ dollars, where x is the number of units produced and sold.

   (a) To maximize profit, how many units must be produced and sold? What is the maximum possible profit?

   (b) What levels of production will yield a \$10,000 profit?

NOTE THAT: $P(x) = ax^2 + bx + c$ IN STANDARD FORM

IS $P(x) = -.01x^2 + 40x - 3000$

(a) PARABOLA DOWNWARD $(a < 0) \rightarrow$ MAX AT VERTEX (h, k), WHERE:

$h = \frac{-b}{2a} = \frac{-40}{2(-.01)} = 2000$ (2000 UNITS MAXIMIZE THE PROFIT)

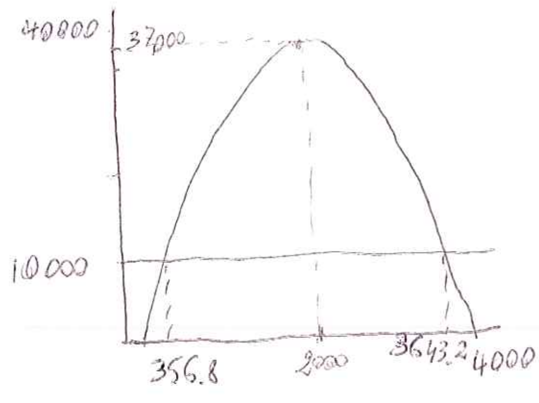
MAXIMUM PROFIT IS $k = P(h) = P(2000) = 37000$ DOLLARS

(ONE CAN USE $\boxed{2ND} + \boxed{TRACE} + \boxed{4}$ IN AN APPROPIATE WINDOW)
MAXIMUM

(b) SOLVE $P(X) = 10\,000 \rightarrow \underbrace{-.01x^2 + 40x - 3000}_{Y_1} = \underbrace{10000}_{Y_2}$, THEN TWICE

WE CAN USE THE GRAPH

$\boxed{2ND} + \boxed{TRACE} + \boxed{5}$ INTERSECT



LOOKING AT THE TABLE, THE PRODUCTION LEVELS FOR WHICH THE PROFIT IS CLOSER TO \$ 10,000 ARE

$X = 357$, $X = 3643$ UNITS

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5. Graph the solution set of the following system of linear inequalities, labeling the corner point and highlighting the contour.

$$\begin{cases} 4x - 2y < 3 \\ 2x + 3y \geq 2 \end{cases}$$

IT IS FASTER TO SOLVE THE INEQUALITIES FOR Y IN TERMS OF X :

INEQ 1 : $\frac{-2Y}{-2} < \frac{-4X+3}{-2} \to Y > 2X - \frac{3}{2}$ (REGION ABOVE)

INEQ 2 : $\frac{3Y}{3} \geq \frac{-2X+2}{3} \to Y \geq -\frac{2}{3}X + \frac{2}{3}$ (REGION ABOVE

BOUNDARY LINES:

$$\begin{cases} Y = 2X - 3/2 & \text{DASHED} \\ Y = -\frac{2}{3}X + 2/3 & \text{SOLID} \end{cases}$$

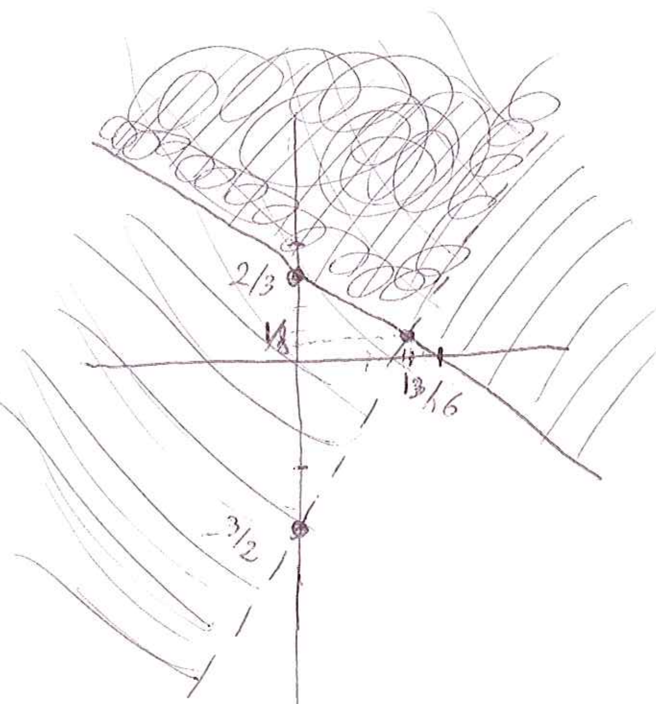
CORNER POINT (BY CALCULATOR OR)

$$\left(2X - \frac{3}{2} = -\frac{2}{3}X + \frac{2}{3}\right)\cdot(6)$$

$$6(2X) - 6\left(\frac{3}{2}\right) = 6\left(-\frac{2}{3}X\right) + 6\left(\frac{2}{3}\right)$$

$$12X - 9 = -4X + 4 \to 16X = 13$$
$$+4X + 9 \quad +4X + 9$$

$$\to X = \frac{13}{16} \to Y = 2\left(\frac{13}{16}\right) - \frac{3}{2} = \frac{1}{8}$$



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6. For the extreme weather months, Palmetto Electric charges \$10.50 plus 9.115 cents per kilowatt-hour (kWh) for the first 1400 kWh and \$138.11 plus 7.511 cents for all kilowatt-hours above 1400.

   (a) Write the function that gives the monthly charge in dollars as a function of the kilowatt-hours used.

   (b) What is the monthly charge if 1800 kWh are used?

   (c) What is the monthly charge if 820 kWh are used?

(a) Y = "MONTHLY CHARGES IN \$" ; X = "KWh USED IN A MONTH" ; RATES IN $\frac{\$}{KWh} = \frac{\frac{¢}{100}}{KWh}$

$$Y = \begin{cases} 10.50 + .09115X, & 0 \le X \le 1400 \\ 138.11 + .07511(X - 1400), & X > 1400 \end{cases}$$

(b) $X = 1800 > 1400 \to Y = 138.11 + .07511(1800 - 1400) = 168.15$ DOLLARS

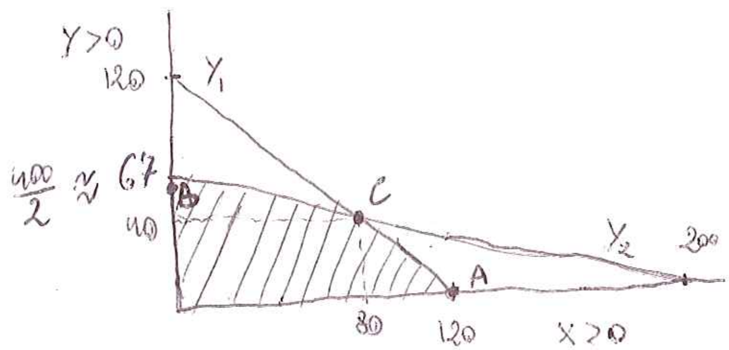
(c) $X = 820 < 1400 \to Y = 10.50 + .09115(820) = 85.24$ DOLLARS

7. Evergreen Company produces two types of printers, the inkjet and the laserjet. It takes 2 hours to make the inkjet and 6 hours to make the laserjet. The company can make at most 120 printers per day and has 400 labor-hours available per day.

   (a) Set up a system of linear inequality describing this application.

   (b) List three possible pairs of production levels.

ITEM	Q # PRINTERS	R WORK RATE HOURS/PRINT	Q·R LABOR HOURS
INK	X	2	2X
LASER	Y	6	6Y
TOTAL	≤ 120		≤ 400



$$\begin{array}{ll} Y_1 & X + Y \le 120 \\ Y_2 & 2X + 6Y \le 400 \end{array}$$

(INKJET, LASERJET): A(120, 0), B(0, 66), C(80, 40)

WITHOUT A GRAPH I SHOULD PLUG ALL OF THESE IN BOTH INEQUALITIES.

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