MAT 102 - Exam4 - Fall 2013

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**Instructions.** Each problem is worth 10 points. If you solve a problem graphically then draw the graph you used. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded at least to two decimal places, unless otherwise specified. You might need some of the following formulas:

$\log_a(MN) = \log_a(M) + \log_a(N)$; $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$; $A = P\left(1 + \frac{r}{n}\right)^{nt}$; $A = Pe^{rt}$; $R = \log\left(\frac{I}{I_0}\right)$; and $D = 10\log\left(\frac{I}{I_0}\right)$

**SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

**1.** Evaluate the following expressions rounding to four decimal places.

**(a)** $\log_{2.35} 15 = \frac{\ln(15)}{\ln(2.35)} \approx 3.1695$

**(b)** $(35)(1.2)^{-4.3} \approx 15.9804$

**2.** A certain species of deer is to be introduced into a forest, and wildlife experts estimate the population will grow according to the model $P(t) = (597)4^{0.5t}$, where $t$ represents the number of years from the time of introduction. What does this model predict the population will be in 5 years? (Round to the nearest deer.)

IT IS ASKED FOR P VALUE WHEN $t = 5$, THAT IS

$P(5) = (597)(4^{(0.5)5}) = 19104$ DEER.

**3.** Joy has $4900 that she wants to invest in a mutual fund for 4.5 years, at which time she plans to close out the account and use the money as a down payment on a car. She finds one local bank offering an annual interest rate of 2.64% compounded monthly (Bank 1), and another bank offering an annual interest rate of 2.5% compounded continuously (Bank 2).
(a) Which bank would pay Joy more interest?
(b) How much will Joy have for her down payment? (Round to two decimal places.)

(a) $P = 4900$, TAKE $t = 1$ TO COMPARE

BANK 1: $r = \frac{2.64}{100} = .0264$; $n = 12 \rightarrow A = P\left(1 + \frac{r}{n}\right)^{nt} =$

$= 4900\left(1 + \frac{.0264}{12}\right)^{12} = 5030.94 \Rightarrow I = A - P = 130.94$

BANK 2: CONTINUOUSLY IS $A = Pe^{rt}$; $r = \frac{2.5}{100} = .025 \rightarrow A =$

$= 4900\, e^{.025} = 5024.04 \Rightarrow I = A - P = 124.04$

**BANK 1 IS BETTER**

(b) $A(4.5) = 4900\left(1 + \frac{.0264}{12}\right)^{(12 \cdot 4.5)} = 5517.39$ DOLLARS

**4.** Given that $I_o = 10^{-14}$ watts/meter$^2$, what is the intensity of a sound for which the decibel level of the sound measures 216?

DECIBEL FORMULA: $D = 10 \log\left(\frac{I}{I_o}\right)$

$216 = 10 \log\left(\frac{I}{10^{-14}}\right)$ DIVIDE BY 10 $\Rightarrow$ $21.6 = \log\left(\frac{I}{10^{-14}}\right)$ $\Rightarrow$ EXPONENTIATE IN BASE 10

$10^{21.6} = \frac{I}{10^{-14}} \Rightarrow I = 10^{21.6} \cdot 10^{-14} = \boxed{10^{7.6}} \approx 39,810,717.06$

**5.** Solve the following exponential equation. If you don't write your answers as an exact expression then round it to four decimal places.

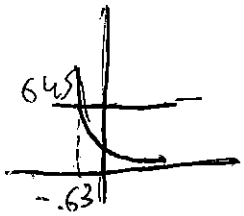
$$3^{4-3x} = 645$$

GRAPHICALLY OR ALGEBRA:

$\log_3(3^{4-3x}) = \log_3(645) \Rightarrow 4-3x = \log_3 645 \Rightarrow 4 - \log_3 645 = 3x \Rightarrow$

$\Rightarrow \boxed{x = \dfrac{4-\log_3 645}{3}} = \dfrac{4-\dfrac{\log 645}{\log 3}}{3} \approx -.6295$

GRAPH




**6.** Solve the following logarithmic equation. If you don't write your answers as an exact expression then round it off to four decimal places. **Check for extraneous solutions.**

$$\log_4(x+1) + \log_4(3x+5) = 2$$

ALGEBRA:

$\log_4\underbrace{((x+1)(3x+5))}_{\text{FOIL}} = 2 \Rightarrow$ EXPONENTIATE IN BASE 4:

$\Rightarrow 3x^2+8x+5 = 4^2 \Rightarrow 3x^2+8x-11=0 \Rightarrow$

$-16 \quad -16$

$\Rightarrow x = \dfrac{-8 \pm \sqrt{64+132}}{6} = \dfrac{-8 \pm 14}{6}$ $\begin{cases} x = -\frac{22}{6} = -\frac{11}{3} \approx -3.6667 \\ x = \frac{6}{6} = 1 \end{cases}$

CHECK: a) $x=1 \rightarrow$ LOGS ARE DEFINED ✓

aa) $x \approx -3.6667 \rightarrow \log_4(-3.6667+1)$ IS NOT DEFINED.

ONLY ONE SOLUTION: $x=1$

7. Brandon is saving up money for a down payment on a car. He currently has $1592, but knows he can get a loan at a lower interest rate if he can put down $2100. If he invests the $1592 in an account that earns 5.7% annually, compounded weekly, how long will it take Brandon to accumulate the $2100?

SIMPLE INVESTMENT: $A = P\left(1+\frac{r}{n}\right)^{nt}$

$r = \frac{5.7}{100} = .057$; $n = 52$; $P = 1592$

THEN:

$$A = 1592\left(1+\frac{.057}{52}\right)^{52t}$$

NEEDS: $A = 2100$

$$\Rightarrow 1592\left(\frac{52.057}{52}\right)^{52t} = 2100$$

BAMPH OR ALGEBRA $\frac{2100}{1592} = \frac{525}{398}$

$$\Rightarrow \left(\frac{52.057}{52}\right)^{52t} = \frac{525}{398} \Rightarrow \frac{52t}{52} = \frac{1}{52}\log_{\frac{52.057}{52}}\left(\frac{525}{398}\right) \Rightarrow$$

TAKE $\log_{\frac{52.057}{52}}$ ON BOTH SIDES

$\Rightarrow t \approx 4.86 \Rightarrow$ ALMOST 5 YEARS.

**8.** The table below gives the number of births, in thousands, to females over the age of 35 for a particular state every two years from 1980 to 1996.

| Year | 1980 | 1982 | 1984 | 1986 | 1988 | 1990 | 1992 | 1994 | 1996 |
|---|---|---|---|---|---|---|---|---|---|
| Births (thousands) | 44.5 | 36.0 | 32.1 | 40.3 | 46.7 | 50.8 | 56.9 | 51.4 | 47.2 |

Consider $x$ to be the number of years after 1980, and $y$ to be the birth. Use technology to answer to the following questions.

(a) Find the cubic and the quartic models that are the best fit for these data. (Round your answer to five decimal places).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate how many births to females over the age of 35 there were in this state in 1995. Round to the nearest newborn using the greatest integer function.

(a) CUBIC: $Y = -.05123X^3 + 1.22541X^2 - 6.37619X + 44.02424$

$R^2 = .94832$

QUARTIC: $Y = .00237X^4 - .12693X^3 + 1.97834X^2 - 8.23368X + 44.93263$

$R^2 = .9654$

(b) QUARTIC IS BETTER, WITH LARGER $R^2$

(c) STORED REGRESSION WITH [VARS] → [Y-VARS] → [FUNCTION] → [$Y_1$]

YEAR 1995 CORRESPONDS TO $X = 1995 - 1980 = 15 \Rightarrow Y = 50.4234$

→ 50,423 NEWBORNS IN 1995.